A Logical Disproof of Relativistic Doppler Effect of Light and an Alternative Theory

Henok Tadesse, Electrical Engineer, BSc. Ethiopia, Debrezeit, P.O Box 412
Tel: +251 910 751339 or +251 912 228639
e-mail: entkidmt@yahoo.com or wchmar@gmail.com
21 January 2018

Abstract

The Special Theory of Relativity (STR) provides a formula for the Doppler effect of light that differs from conventional formulas based on ether theory and emission theory. The relativistic Doppler effect formula becomes undefined for source-observer relative velocities greater than the speed of light. Since STR asserts that no (relative) velocity can ever reach or exceed the speed of light, this has never been seen as a problem. However, it will be shown in this paper that relative velocities greater than the speed of light can be attained even within the framework of STR, i.e. without violating the universal light speed limit. Imagine a light source and an observer at rest relative to each other and located close to each other. Light emitted by the source travels to a distant mirror and reflects back to the observer, while the mirror is moving towards the observer and the source with velocity equal to (close to) the speed of light. In this case the relative velocity between the mirror image of the source and the observer is equal to twice the speed of light, 2c. Since it is the relative velocity of the mirror image of the source and the observer that determines the Doppler effect, which is equal to 2c in this case, the relativistic Doppler effect formula becomes undefined because $\beta = V/c = 2c/c = 2$, which is greater than 1, resulting in a square root of negative number in the relativistic Doppler effect formula. This disproves relativistic Doppler effect, and hence the Special Theory of Relativity. According to STR the usual analysis is that the mirror image of the source never attains the speed of light because of the relativistic velocity addition formula. Since STR has been disproved logically, experimentally and theoretically, this assertion is not valid.

Introduction

The Special Theory of Relativity (STR) provides a formula for the Doppler effect of light that differs from conventional formulas based on ether theory and emission theory. The relativistic Doppler effect formula becomes undefined for source-observer relative velocities greater than the speed of light. Since STR asserts that no (relative) velocity can ever reach or exceed the speed of light, this has never been seen as a problem. However, it will be shown in this paper that relative velocities greater than the speed of light can be theoretically attained and physically possible even within the framework of STR, i.e. without violating the universal light speed limit.

Relative velocity greater than the speed of light

Although special relativity imposes that no (relative) velocity is greater than the speed of light, it is physically possible (theoretically) to obtain relative velocities greater than the speed of light even without violating the universal light speed limit. This can be done by an arrangement
consisting of light source, observer and a moving mirror. Theoretically, it is possible to get an ‘infinite’ relative velocity between the observer and the mirror image of the source by using multiple moving and fixed mirrors.

Suppose that a light source $S$ and an observer $O$ are at rest relative to each other and are located close to each other. Light emitted from the source goes to a distant mirror and is reflected back to the observer, while the mirror is moving towards the source and the observer with velocity equal to (close to) the speed of light $c$.

It is straightforward to show that the relative velocity of the mirror image of the source and the observer will be equal to $2c$.

Let the total distance between the source and the observer be $L$, along the light path.

Therefore,

$$L = 2D$$
From which

\[
\frac{dL}{dt} = 2 \frac{dD}{dt} = 2V \approx 2c
\]

It can be seen that the relative velocity between the mirror image of the source and the observer is twice the speed of light, for which the relativistic Doppler effect formula becomes undefined.

The relativistic Doppler effect formula for source and observer approaching each other is [2]:

\[
f' = \sqrt{\frac{1 + \beta}{1 - \beta}} f, \text{ where } \beta = \frac{V}{c}
\]

In the above case,

\[
\beta = \frac{V}{c} = \frac{2c}{c} = 2
\]

Therefore,

\[
f' = \sqrt{\frac{1 + 2}{1 - 2}} f = \sqrt{3} f
\]

which is undefined.

**Super luminal source-observer relative velocity**

We have seen that superluminal relative velocity is physically possible by the use of reflection of light from a mirror moving close to the speed of light. In this case Doppler effect arises from the relative velocity between the mirror image of the source and the observer.

Superluminal relative velocity between a real source and an observer is also physically possible. According to the theory proposed in [3], no material object can attain or exceed the speed of light. However, unlike STR, the speed limit is a limit to the absolute velocity, not relative velocity, of physical objects. The light speed limit applies only to physical, material objects. For example, the speed of electrostatic fields is infinite. The group velocity of light can also be superluminal. This theory differs from STR, which asserts that nothing (no physical object, no information) travels faster than the speed of light.

Since the light speed limit applies only to absolute velocities of physical objects, superluminal relative velocities are possible between a light source and an observer, which are subjected to absolute velocity limit as physical objects themselves. Imagine two ions moving with absolute
velocities of (close to) the speed of light c in opposite directions, approaching each other. The relative velocity between the two ions will be twice the speed of light, 2c. According to the new theory, therefore, the maximum relative velocity between two physical objects is twice the speed of light 2c.

Suppose that two particles (ions or elementary particles) are moving in opposite directions, approaching each other, each with absolute velocities of 0.9c. Therefore, the relative velocity between the two ions will be 1.8c. The problem is: what is the formulation of Doppler effect for relative velocities greater than the speed of light? No existing theory (STR, ether theory, emission theory) has anything to say about this question.

The argument is, therefore, if superluminal relative velocities exist, then there must also exist a law governing the Doppler effect of light for all possible relative velocities, including V > c.

**Relativistic analysis of the moving mirror thought experiment**

The relativistic analysis asserts that the mirror image of the source never attains or surpasses the speed of light because of the relativistic velocity addition formula.

In the reference frame of an observer ‘A’ co-moving with the mirror, the mirror image of the source moves towards A with velocity V. To obtain the velocity of the mirror image of the source in the reference frame of observer O, relativistic velocity addition formula is applied:

\[ u = \frac{v + u'}{1 + \left(\frac{vu'}{c^2}\right)} \]

where

- \( u \) is the velocity of the mirror image of the source in the reference frame of observer O
- \( u' \) is the velocity of the mirror image of the source in the reference frame of observer A
- \( v \) is the velocity of the mirror (and co-moving observer A) relative to the observer O.

We have assumed \( v = c \). The velocity of the mirror image of the source as seen by observer A is equal to the velocity of the mirror itself.

\[ u' = v \]

Therefore, the velocity of the mirror image of the source in the reference frame of observer O is:
which is different from the classical value of $2v = 2c$

Since STR has already been disproved logically, experimentally and theoretically [3], this analysis is not valid.

Therefore, superluminal velocity of the mirror image of the source is physically, theoretically possible.

The main argument of this paper is that a new theory governing the Doppler effect of light is needed because existing theories (STR, ether theory, emission theory) cannot handle superluminal relative velocities, which are theoretically possible. The universal light speed limit is accepted in this paper as applying only to material objects. Since mirror image of a light source is not a material object, it can attain superluminal velocities.
Exponential Law of Doppler Effect of Light

The new theory (formula) of the Doppler effect of light should satisfy the following criteria.

1. It should be defined for and handle all values of relative velocity, including $V > c$.

2. It should satisfy the condition

$$f \cdot \lambda = f' \cdot \lambda' = c$$

which is a requirement of constancy of the phase velocity of light, irrespective of source-observer relative velocity[1].

3. It should explain the Ives-Stilwell experiment.

An alternative theory [1] governing the Doppler effect of light that fulfills the above conditions has been proposed by this author. It is introduced here briefly.

The mysterious exponential law governing the Doppler effect of light is proposed as:

$$f' = f e^{\frac{V}{c}} \quad \text{and} \quad \lambda' = \lambda e^{-\frac{V}{c}}$$

where $V$ is positive for source and observer approaching each other, and $e$ is Euler’s constant.

Next we will show that this formula reduces to the conventional formula of Doppler effect for $V \ll c$ and that this formula explains the Ives-Stilwell experiment.

We know that

$$e^x = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \ldots \approx 1 + x + \frac{1}{2} x^2$$

Therefore,

$$e^\frac{V}{c} = 1 + \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2 + \frac{1}{6} \left( \frac{V}{c} \right)^3 + \ldots \approx 1 + \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2$$

and

$$e^{-\frac{V}{c}} = 1 - \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2 - \frac{1}{6} \left( \frac{V}{c} \right)^3 + \ldots \approx 1 - \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2$$
We show that the new formula reduces to the conventional formula for $V \ll c$, as follows.

$$
\frac{V}{c} e^{\frac{V}{c}} \approx 1 + \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2 \approx 1 + \frac{V}{c} = \frac{c + V}{c}
$$

Again for $V \ll c$

$$
\frac{c + V}{c} \approx \frac{c}{c - V}
$$

Therefore, for $V \ll c$

$$
f' = f \frac{V}{c} e^{\frac{V}{c}} \approx f \frac{c}{c - V}
$$

where $V$ is positive for source and observer approaching each other.

**The Ives-Stilwell experiment**

Next we apply the new formula to the Ives-Stilwell experiment.

Doppler frequency of approaching ion is:

$$
\lambda_{\text{app}}' = \lambda e^{\frac{V}{c}}
$$

Doppler frequency of receding ion is:

$$
\lambda_{\text{rec}}' = \lambda e^{\frac{V}{c}}
$$

The average wavelength is:

$$
\Lambda = \frac{1}{2} \left( \lambda_{\text{app}}' + \lambda_{\text{rec}}' \right) = \frac{1}{2} \left( \lambda \frac{e^{-\frac{V}{c}}}{V} + \lambda \frac{\frac{V}{c}}{} \right)
$$

$$
\approx \frac{1}{2} \lambda \left( 1 - \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2 + 1 + \frac{V}{c} + \frac{1}{2} \left( \frac{V}{c} \right)^2 \right)
$$

$$
\approx \frac{1}{2} \lambda \left( 2 + \left( \frac{V}{c} \right)^2 \right) = \lambda \left( 1 + \frac{1}{2} \left( \frac{V}{c} \right)^2 \right)
$$
From which

\[\Delta \lambda = \lambda - \lambda = \lambda \left(1 + \frac{1}{2} \left(\frac{V}{c}\right)^2\right) - \lambda\]

\[= \frac{1}{2} \left(\frac{V}{c}\right)^2 \lambda\]

\[= \frac{1}{2} \beta^2 \lambda\]

which is the same value predicted by STR and confirmed by the Ives-Stilwell experiment.

**Conclusion**

We have shown in this paper that the relativistic Doppler effect formula becomes invalid for theoretically possible and valid physical phenomenon of light. It is theoretically possible for the velocity of the image of a light source to be greater than the speed of light relative to an observer, in which case the relativistic Doppler effect formula becomes undefined. We have proposed a more natural alternative theory of the law of Doppler effect of light.

Thanks to God and Saint Virgin Mary the Mother of God

**References**

1. Exponential Law of Doppler Effect of Light – an Explanation of Ives-Stilwell Experiment, by Henok Tadesse, Vixra
   


3. Absolute/Relative Motion and the Speed of Light, Electromagnetism, Inertia and Universal Speed Limit c - an Alternative Interpretation and Theoretical Framework, by Henok Tadesse, Vixra
   