Time Transformation between Inertial Reference Frames

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Time in an inertial reference frame can be obtained from the definition of velocity in that inertial reference frame. Velocity depends on coordinate and time. Therefore, coordinate transformation and velocity transformation between inertial reference frames can lead to time transformation. Based on this approach, the time transformation between two arbitrary inertial reference frames in one dimensional space is derived. The result shows that the elapsed time is identical in all inertial reference frames.

I. INTRODUCTION

Two identical inertial reference frames can be separated by the application of a temporary acceleration to one of these two frames. Such acceleration relates coordinate and velocity from one inertial reference frame to the other inertial reference frame. According to the definition of velocity, elapsed time can be identified if both velocity and coordinate translation can be determined.

First step is to derive the coordinate transformation between two inertial reference frames. Second step is to derive the velocity transformation. Finally, time transformation can be derived from the definition of velocity.

II. COORDINATE TRANSFORMATION

Consider one-dimensional motion.

A. Acceleration

Based on the definition of acceleration, a stationary object put under constant acceleration \( A \) for a duration \( T \) will move a distance \( D \) and increase its velocity to \( V \).

\[
D = X_f - X_i
\]  
\[
V = A \cdot T
\]  
\[
X_f = X_i + \frac{A \cdot T^2}{2}
\]

\( X_i \) is the initial position of the object before application of constant acceleration \( A \).
\( X_f \) is the final position of the object after application of constant acceleration \( A \) for a duration \( T \).
\( T \) is the total elapsed time for the application of acceleration.
\( V \) is the final velocity of the object.

Place two identical objects at two different locations, \( X_1 \) and \( X_2 \). Both objects are at rest initially. Put both objects under identical constant acceleration \( A \) at the same time for a duration \( T \).

Their final locations, \( X_{1f} \) and \( X_{2f} \), can be calculated according to the definition of acceleration. From equation (3),

\[
X_{1f} = X_{1i} + \frac{A \cdot T^2}{2}
\]
\[
X_{2f} = X_{2i} + \frac{A \cdot T^2}{2}
\]

Both objects will move at the same velocity of \( V \) at the end of duration \( T \).

\[
V = A \cdot T
\]

The distance between these two objects is \( R \). From equation (4) and (5),

\[
R = X_{2f} - X_{1f} = X_{2i} - X_{1i}
\]

\( R \) remains constant during acceleration.

The acceleration is terminated at the end of duration \( T \). Therefore, for any time \( t \) greater than \( T \),

\[
X_{1f} = X_{1i} + \frac{A \cdot T^2}{2} + (t - T) \cdot V
\]
\[
X_{2f} = X_{2i} + \frac{A \cdot T^2}{2} + (t - T) \cdot V
\]

\[
X_{2f} - X_{1f} = X_{2i} - X_{1i} = R
\]

\( R \) remains constant after acceleration is terminated.

B. Reference Frame

Both objects are stationary to each other at all time. They form a reference frame \( F_2 \) that moves at the velocity \( V \) relative to a reference frame \( F_1 \) in which both objects are initially at rest.

\[
V = A \cdot T
\]

The speed of \( F_2 \) relative to \( F_1 \) is \( V \).

Let the initial location of object 1 be the origin of both \( F_1 \) and \( F_2 \). The location of object 2 becomes a representation of the coordinate in both \( F_1 \) and \( F_2 \).
Let $x'$ be the location of object 2 in $F_2$. Let $x$ be the location of object 2 in $F_1$.

$$x' = X2_f$$  \hspace{1cm} (12)

$$x = X2_f$$  \hspace{1cm} (13)

Therefore, the coordinate transformation between $F_1$ and $F_2$ is, from equation (9),

$$x = x' + \frac{AstarT^2}{2} + (t - T) * V$$  \hspace{1cm} (14)

C. Conservation of Length

Place a stationary object of length $L$ in $F_2$. The positions of both ends of this object in $F_2$ are $x'_a$ and $x'_b$.

$$L = x'_b - x'_a$$  \hspace{1cm} (15)

Based on coordinate transformation between $F_1$ and $F_2$, equation (14),

$$x_a = x'_a + \frac{AstarT^2}{2} + (t - T) * V$$  \hspace{1cm} (16)

$$x_b = x'_b + \frac{AstarT^2}{2} + (t - T) * V$$  \hspace{1cm} (17)

$x_a$ and $x_b$ are the positions of both ends of this object in $F_1$. The length of this object in $F_1$ is $x_b - x_a$.

$$x_b - x_a = x'_b - x'_a = L$$  \hspace{1cm} (18)

The length of this object is $L$ in both $F_1$ and $F_2$. The length is independent of the relative motion between $F_1$ and $F_2$.

III. VELOCITY TRANSFORMATION

Consider one-dimensional motion

A. Identical Reference Frames

Let an inertial reference frame $F_3$ be stationary relative to inertial reference frame $F_1$. Let an object $W_1$ in $F_3$ moves at a speed of $v'$. The speed of $W_1$ in $F_1$ is $v'$. The speed of $W_1$ in $F_3$ is $v'$

B. Acceleration

Put $F_3$ under constant acceleration $A$ relative to $F_1$ for a duration $T$.

According to the definition of acceleration, this temporary acceleration produces a difference in the relative speed between $F_1$ and $F_3$ and accelerates all objects in $F_3$ by $A*V$ in $F_1$.

The speed of $W_1$ in $F_1$ is $v' + A*V$

The speed of $W_1$ in $F_3$ is $v'$.

The speed of $F_3$ relative to $F_1$ is $A*V$. The speed of $F_2$ relative to $F_1$ is also $A*V$. Therefore, $F_3$ is stationary relative to $F_2$.

The speed of $W_1$ in $F_1$ is $v' + A*V$.

The speed of $W_1$ in $F_3$ is $v'$.

The speed of $W_1$ in $F_2$ is $v'$.

Let the speed of $W_1$ in $F_1$ be $v$. The velocity transformation between $F_1$ and $F_2$ is

$$v = v' + A*V$$  \hspace{1cm} (19)

Therefore, a moving object in $F_2$ will move in $F_1$ at a speed equal to the sum of its speed in $F_2$ and the relative speed between $F_2$ and $F_1$. This is the velocity transformation from $F_2$ to $F_1$. It is independent of the speed of light.

IV. TIME TRANSFORMATION

Let the time in $F_1$ be $t$. Let the time in $F_2$ be $t'$. According to the definition of velocity,

$$v = \frac{dx}{dt}$$  \hspace{1cm} (20)

in $F_1$. While in $F_2$,

$$v' = \frac{dx'}{dt'}$$  \hspace{1cm} (21)

From equation (14),

$$dx = dx' + (dt) * V$$  \hspace{1cm} (22)

$$\frac{dx}{dt} = \frac{dx'}{dt'} + V$$  \hspace{1cm} (23)

From equations (19), (11), (20), (21),

$$\frac{dx}{dt} = \frac{dx'}{dt'} + V$$  \hspace{1cm} (24)

From equations (23) and (24),

$$dt' = dt$$  \hspace{1cm} (25)

$$t' = t + C$$  \hspace{1cm} (26)

C is a constant in time.

V. CONCLUSION

Elapsed time is identical in all inertial reference frames. Time does not run slower nor faster in any particular inertial reference frame. Two simultaneous events in one inertial reference frame are simultaneous in all other inertial reference frames.
The velocity transformation between two inertial reference frames exclusively depends on the relative speed between two inertial reference frames. It is independent of the speed of light.

The coordinate transformation between two inertial reference frames also exclusively depends on the relative speed between two inertial reference frames. It is also independent of the speed of light.

For more than a century, there have been speculation that the speed of light is a factor in transformation of time, coordinate, and velocity. This is clearly incorrect as in the proof of this paper.

Therefore, any proposed transformation that incorporates the speed of light is invalid in physics. One particular example is Lorentz Transformation[1][2] which is based on the assumption that the speed of light is independent of inertial reference frame.

As a result of its incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation. Therefore, Lorentz Transformation is not a proper transformation in physics.

Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[4][8]