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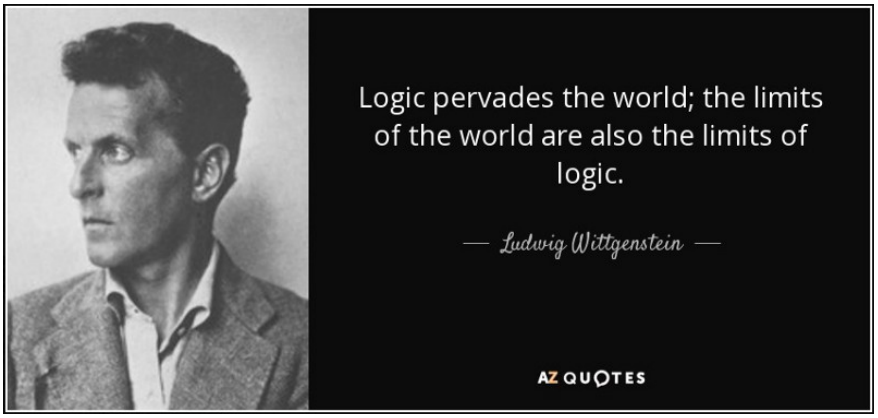
Physics as the limits of logic



Essentially, we formalize this intuition into a consistent theory of physics.

Introduction

Here, I present the discovery of a derivation of the laws of physics - along with a resulting massive simplification thereof- from pure reason, showing that the laws of physics are connected to the limits of logic. So far, I have explicitly obtained many of the most fundamental laws, including: the Dirac equation, the Schrödinger equation, special relativity, general relativity, and dark energy. In this derivation, the world is represented as a very simple Gibbs ensemble of statistical physics that has only time and space as its quantities. This equation is, to my knowledge, the simplest equation in the literature with such a wide physical scope.

Parallel to its physical interpretation, this same Gibbs ensemble also describes what I have come to understand as *feasible mathematics*; this dual interpretation is the key to deriving physics as the limits of pure reason. What is feasible mathematics? Feasible mathematics is an alternative to (and in some permissive sense, a generalization of) computational complexity theory (CT). Let us first see what CT is. CT is the study of the inherent difficulty of computational problems; as such, CT classifies problems by the increase in difficulty associated with an increase in input size. For example, a binary search algorithm will have a difficulty of O(log n), and thus it has a logarithmic complexity. In this example, the number of steps required to find an item amongst n sorted items grows proportionally to the logarithm of n.

Why bother with an alternative? The problem is that CT does not correctly distinguish between all indicators of complexity. As an example, the difficulty between, say, an exponential problem with a small multiplication constant (O(2^n)*0.001) and a polynomial problem with a large multiplication constant ($O(n^2)*1^{0999999999}$) is incorrectly classified. As far as CT is concerned, the latter problem is much simpler than the first as it only grows in n^2 versus 2^n. However, in practice, the latter might never be solved because there might not be enough resources in the observable universe to do so (even for n=1). Therefore, although this idea is very interesting, something is missing from CT.

This is where feasible mathematics comes in. Feasible mathematics treats computational complexity according to the absolute difficulty of problems. Thus, it is not "fooled" by, say, a mere multiplication constant. Its domain is defined as an extension to the halting probability Ω of computer science. Using a similar construction, we define a probability Z that represents the probability that a random program will halt within some available computing resources. These resources can be time, memory, clock speed, etc. Z does for *feasible mathematics* what Ω does for "*abstract mathematics*;" it can decide all the programs that halt within the available computing resources. When the limits are made to vanish, Z converges to Ω.

Recall that both interpretations are represented by the same Gibbs ensembles, and thus feasible mathematics and the universe are both described by the same statistical system. As a path to feasible mathematics exists within pure reason, the beauty and consequence of the dual interpretation is that the laws of physics can also be recovered from pure reason up to an isomorphism connecting feasible mathematics and physics. Why are both interpretations described by the same statistical system? The embedding argument explains why this is no coincidence. We will see that the embedding argument bounds any theory of everything in physics to the limits of feasible mathematics (which are conveniently the same as the laws of physics).

In what follows, we will briefly present both interpretations of the Gibbs ensemble, starting with a) the statistical physics interpretation and then b) the feasible mathematics interpretation. Afterward, we will present the embedding argument, which explains why and how the two are connected.

Interpreted as statistical physics…

The world constructed as a partition function (2018)

We propose a simple partition function that unifies a surprisingly large amount of physical laws. The…

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Summary of the paper:

The paper proposes this Gibbs ensemble:

$$Z = \sum_{q \in Q} e^{-\beta[E(q) - Pt(q) + Fx(q)]}$$

where every micro-state is represented by a space (x) and time (t) quantity.

It is the simplest possible Gibbs ensemble that can represent micro-states via both a time and a space quantity.

Then, we show how both the “quantum world” (specifically, the Schrödinger and Dirac equations) and the “classical world” (specifically, the law of inertia, general relativity, and dark energy) are both emergent from a more fundamental “thermo-statistical world” described by said equation.

The laws are derived by first taking the equation of state of the Gibbs ensemble

$$TdS = dE - Pdt + Fdx$$

and then independently studying its regimes. For example, posing dS=0 and dE=0, we get the fundamental relation of special relativity connecting time to space:

$$dx = \frac{P}{F}dt$$

The fundamental relation of special relativity where c:=P/F

This relation is sufficient to derive all of special relativity. All laws are derived in such a manner and as different regimes. A continuum expansion over the partition function (see paper for rigorous treatment) yields the law of inertia, general relativity, and dark energy as:

$$TdS = \frac{F}{2\pi}dx$$

The fundamental relation of inertia

$$TdS = \frac{F}{16\pi L}dA$$

The fundamental relation of general relativity

$$TdS = \frac{3F}{4\pi A}dV$$

The fundamental relation of dark energy

Each fundamental relation corresponds to a different scale of growth in entropy; linear in the case of dx, square in the case of dA, and cubic in the case of dV. All fundamental relations represent the minimum required to derive their respective equations; the law of inertia in the dx case, the Einstein field equations in the dA case, and a negative pressure in the dV case.

The laws of quantum mechanics are shown to be the consequences of fluctuations along the dx, dA, dV, and dt parameters that are inherent to any thermo-statistical system. Using results from other authors that connect the Dirac equation to a random walk in time and space, I was able to show that the Dirac equation is a thermo-statistical analogue to special relativity; the Schrödinger equation is also obtained as a thermo-statistical analogue, but instead to the law of inertia.

The conclusion is that the world may be purely emergent from thermodynamics and entropy.

Interpreted as feasible mathematics…

Feasible Mathematics (2018)

From algorithmic information theory (and using notions of algorithmic thermodynamics), we…

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Summary of the paper:

This paper extends the parallel between statistical physics and algorithmic thermodynamics as originally proposed by Kohtaro Tadaki, John Baez, and Mike Stay.

In it, I show how algorithmic thermodynamics essentially describes feasible mathematics. The thermodynamic notion of "a system in contact with a heat bath" can be interpreted as "a program executed with constantly available resources allocated to it by a supercomputer."

Feasible halting probabilities are constructed by grouping the programs of interest under a Gibbs ensemble; this forms a statistical ensemble where each program is a micro-state. The Lagrange multipliers that are introduced are the constant computing resources available to the system. Constructing a feasible halting probability based on program halt time and input length yields the following partition function:

$$Z = \sum_{q \in Q} e^{-\beta[E(q) - Pt(q) + Fx(q)]}$$

where q is a program, t(q) is its halting time, and x(q) is its length in bits.

In this case, P takes the role of a compute-power and F takes the role of a halting-density. Both quantities (and others) are further discussed in the paper.

Here, Z describes the system of all programs that halt within some available computing resources. As it is the same Gibbs ensemble as the physical case, the laws of physics derived in the previous paper are also recoverable here—but here they are instead interpreted as laws governing the limits of feasible computation.

The embedding argument

The equations are the same, but do they represent the same thing? The embedding argument answers this question. First, however, let us start from the basics.

What is a formal theory?

A formal theory is a list of axioms and a list of rules. The rules transform a sequence of symbols into another sequence. When applied to the axioms and within the context of the theory, the rules are *truth-preserving*.

We will say that a theory is universal if it is Turing complete. What does this mean? For it to be Turing complete, its rules and axioms must be sufficiently general to import the rules and axioms of any other theory, and then it must be sufficiently flexible to correctly apply said imported rules to said imported axioms. Let's unpack this a little… first, we can construct such a theory relatively easily. For example, Peano's axioms (PA) are such a theory and this is how, using numbering, Gödel was able to prove his famous theorems. Second, a universal theory can embed any other theory. Using the example of PA (arithmetic), it suffices to associate a prime to each sentence of a language, and then we define equations on these primes so as to repeat the behavior of the rules we want to import. As arithmetic contains both addition and multiplication, any rules can be imported; thus, PA is universal.

As a universal theory is Turing complete, its scope will be the same as any other universal theory. Therefore, one universal theory cannot be more universal than the others—universality reaches a maximum.

In physics, however, we rarely look at the properties of universal theories as a whole. Theories of physics are constructed by introducing a set of axioms/rules that may or may not break the universality. The laws of physics are often meant to take the roles of the axioms themselves and they govern what allegedly can and cannot happen in the universe. Each set so chosen must then be experimentally tested to verify its physical applicability. This is the standard approach.

As we will argue, there is a better approach.

The problem of the preferred set of axioms

By definition, all axioms are unprovable; therefore, there can be no set of axioms that are logically preferred over another. Therefore, selecting a preferred set of axioms to describe reality is immediately a first-class logical error. A more philosophically honest approach is to consider the case of **axiomatic-invariance**—now possible with feasible mathematics. In this case, the laws of physics must be introduced as meta-logical limits that are widely applicable to all universal theories. The philosophical pill is thus much easier to swallow as no logically-unjustifiable set of axioms is required to get the process started.

The problem of embedding

I originally became convinced that the standard approach is ultimately incorrect when I realized that, due to the anthropic principle, the universe must embed an observer capable of at least feasible mathematics.

The existence of such an observer can be proven irrefutably: simply repeat Descartes' cogito in your head and solve a few equations—then *you are* this observer! More generally, said observer is able to investigate any field of logic up to its available resources and/or abilities. Thus, it has what is sometimes called experimental freedom.

With a bit of imagination, we can call this observer a *mathematician*, and with less imagination *a feasibly-bounded universal Turing machine*. Thus, as the mathematician is part of the universe, the theory of everything in physics that embeds its description must be a universal theory up to at least the feasible bound accessible by said mathematician. Limiting the theory of everything in physics to something less than a universal theory through the use of axioms would reduce the scope of inquiry available to the mathematician, and therefore he would no longer be capable of the full scope of feasible mathematics.

What is then the correct approach?

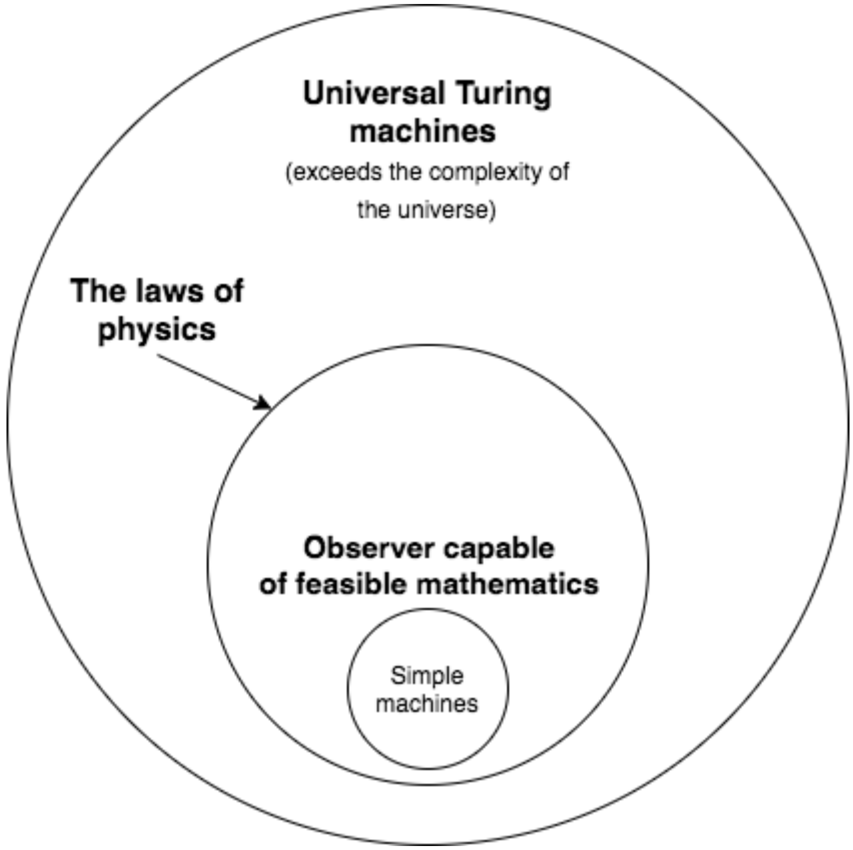
Let us now combine everything we have so far. First, as a recap, we have:

- The theory of everything in physics must embed the description of an observer capable of feasible mathematics. Thus, the theory must be universal within the feasible boundary.
- As they are Turing complete, universal theories share a common logical equivalence. Therefore, as long as the choice of axioms is sufficiently general to recover universality, the choice is inconsequential for feasible mathematics.
- The limits of feasible mathematics are identical in structure to the laws of physics.

The correct approach to construct a theory of physics is to embed feasible mathematics into any universal theory. The requirement that the theory of physics be universal means that the axiomatic structure of the chosen universal theory is ultimately inconsequential. Therefore, the observer is not bounded by any specific set of axioms or rules, but instead by a feasible bound that is generally applicable to all fields of logic. Feasible mathematics, as a meta-theory, equally applies to all sets of axioms; hence, the theory of physics that it describes is **axiom-invariant**—in other words, its axiomatic-load is reduced to NIL.

Visualization

One way to visualize reality in this context is as follows:



1. The existence of an observer capable of feasible mathematics (e.g., you and me) necessitates that any physical theory be universal within the feasible bounds.

2. The otherwise finite resources that are available for computation restricts the universality of the theory to a feasible limit.

3. The boundary between the universal and the feasible is defined by feasible mathematics and corresponds to the laws of physics. From this, the laws of physics are **explicitly** derivable.

4. This provides us with a derivation of the laws of physics that is axiom-invariant. Thus, any universal theory will be equally restricted by the feasible bound. Therefore, the standard approach of parachuting a set of axioms and then experimentally validating it can now be upgraded to a derivation from pure reason, which is applicable to all universal theories and independently from their axiomatic constructions.

5. As an interesting side-note, a universal Turing machine that could exceed the feasible bound would not be constructible in the

universe as it would violate the laws of physics.

Conclusion

The laws of physics are **axiom-invariant** and are applicable to all universal theories as the limits of pure reason.

Ultimately, the proof is found in the pudding:

1. The present derivation does produce an equation from pure reason that unifies, under one umbrella, dark energy, the Schrödinger and Dirac equations, special and general relativity, and more.

2. Furthermore, it groups the laws under the umbrella of thermodynamics, the most general field of physics. This result has long been suspected by many physicists.

3. It also connects physics purely to information and to computation —again, as long suspected by many physicists.

4. It further reduces the axiomatic load to zero by being axiom-invariant—as suspected (or hoped for?) by many philosophers.

5. On top of that, the equation is extremely simple for its scope (perhaps the simplest in its class in the literature).

What's not to like ?—tell me in the comments.