Question 424: Double Integral for π^2

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abstract

This note presents a double integral for π^2 .

Double Integral for π^2

$$\frac{\pi^2}{24}=\int_0^1\int_0^1 \frac{1-x}{x+y+\sqrt{(x+y)^2+4xy(1-x)(1-y)}}\,dxdy \qquad (1)$$

Related integrals:

$$\frac{\pi^2}{24}=\int_1^\infty\int_1^\infty \frac{x-1}{x^2y\left(x+y+\sqrt{(x+y)^2+4(x-1)(y-1)}\right)}\,dxdy \qquad (2)$$

$$\frac{\pi^2}{24}=\int_0^\infty\int_0^\infty \frac{x}{(x+1)^2(y+1)\left(2+x+y+\sqrt{(2+x+y)^2+4xy}\right)}\,dxdy \qquad (3)$$

$$\frac{\pi^2}{24}=\int_0^\infty\int_0^\infty \frac{\left(1-e^{-x}\right)e^{-x-y}}{e^{-x}+e^{-y}+\sqrt{\left(e^{-x}+e^{-y}\right)^2+4e^{-x-y}\left(1-e^{-x}\right)\left(1-e^{-y}\right)}}\,dxdy \qquad (4)$$

$$\frac{\pi^2}{12}=\int_0^\infty\int_0^\infty \frac{x+y+2xy}{(x+1)^2(y+1)^2\left(2+x+y+\sqrt{(2+x+y)^2+4xy}\right)}\,dxdy \qquad (5)$$

References

1. Guillera, J. and Sondow, J. : Double Integrals and Infinite Products for Some Classical Constants Via Analytic Continuations of Lerch's Transcendent. 2005. http://arxiv.org/abs/math.NT/0506319.