An Alternative Explanation of Non-Newtonian Galactic Rotation Curves

Morgan W. Weinberg
(Dated: January 22, 2018)

Inspired by the continued success of MOND (Modified Newtonian Dynamics) in the prediction of galactic rotation curves, an attempt to derive the deep-MOND equation from known mechanics has resulted in a third explanation apart from MOND and dark matter. It is proposed that particle velocities follow the relation $v = \sqrt{GM}r^{-1/2} + \sqrt{a_x}r^{1/2}$, where a_x is a scalar accelerating field that is independent of mass. This yields the following relation for centripetal acceleration: $a = GMr^{-2} + 2\sqrt{a_x GM}r^{-1} + a_x$, which, at large radii, is nearly identical to the deep-MOND equation $a = \sqrt{a_0 GM}r^{-1}$. When applied to a handful of galaxies, the velocity equation prefers an a_x on the order of 10^{-14} (km s^{-2}), which gives a good fit of velocity curves to observed values. It is posited that scalar field a_x is a result of local galactic expansion, such that $a_x = cH_g$, where H_g is the rate of expansion. For the Milky Way, it is estimated that $H_g \approx 9.3\text{E}{-4}$ (km s^{-1} kpc^{-1}). This rate would predict an increase of the astronomical unit of 14 (cm yr^{-1}), which compares well with the recently reported measurement of 15±4 (cm yr^{-1}).

I. BACKGROUND

In the early 20th Century, astronomers made a startling discovery – the rotational velocity of galaxies at the outer regions far exceeded that predicted by Newton's Laws and Relativity given the amount and distribution of observable matter contained in those galaxies, a discovery that has continually been verified as distant galaxies come into view [1]. The two predominant hypotheses explaining this discrepancy are dark matter and MOND, with the latter receiving only marginal interest [2][3][4].

The dark-matter hypothesis holds that around 27% of the mass of the universe is a dark mass that doesn't interact with electromagnetic radiation [5]. For a given galaxy, percentages can go much higher. On the order of 90% of the mass of the Milky Way, for example, is theorized to be a spherical halo of dark matter that extends far beyond the visible disk [6][7][8].

MOND, on the other hand, holds that gravitational acceleration, below a certain limit, doesn't adhere to the inverse-square law. In this 'deep-MOND' regime, typically seen at the outer edges of galaxies, acceleration tapers inversely proportional distance, not distance squared. This renders a velocity curve that mathematically flattens out in congruence with observed curves [3][9][10][11].

Under MOND, at the inner regions of galaxies, acceleration still adheres to Keplerian decline, as observed in our own solar system and predicted by Kepler and Newton, with the transition to the more gradual decline governed by an empirically-derived interpolating function [12][13]. The lack of a rigorous, logic-based framework that produces the MOND equations, along with its specific relevance to galactic mechanics versus cosmology at large, has left dark matter as the prevailing hypothesis [3][4][14].

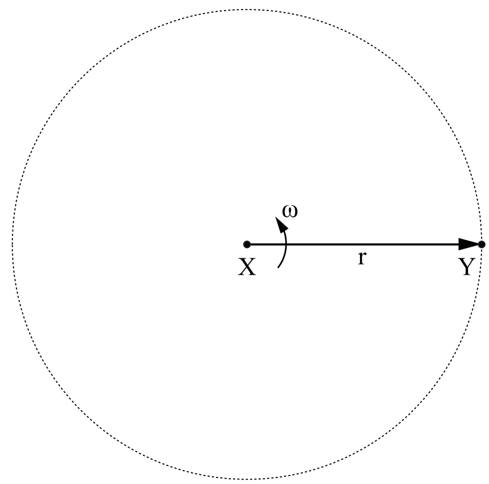

FIG. 1. Particle Y orbiting particle X at radius r.

II. DERIVATION OF THE DEEP-MOND VELOCITY-CURVE EQUATION

In this section, using non-relativistic mechanics (applicable to the outer regions of galaxies) the deep-MOND-regime equation is derived with introduction of a single free parameter, a_x, similar to the parameter a_0, which is historically associated with MOND [15].

Fig. 1 shows particle Y of mass m orbiting particle X at radius r at angular speed ω. Factoring out m and taking the derivative of the non-relativistic specific kinetic energy, E of particle Y with respect to r and ω gives the following:

$$dE = \cancel{m}d\left(\frac{1}{2}r^2\omega^2\right) = \cancel{m}r\omega^2 dr + \cancel{m}r^2\omega d\omega\ . \quad (1)$$

Eq. (1) can be re-written as Eq. (2) below, which shows the differential change in kinetic energy dE being equal to the differential radial work dW applied to the particle plus the differential inertial work dU

$$dE = dW + dU \ , \tag{2}$$

where dW is defined as

$$dW = r\omega^2 dr \ , \tag{3}$$

and dU, specifically, is the change in specific angular momentum dL/dt through angle $d\theta$ at a constant r (the derivative of r was taken in the dW term), which is derived as follows:

$$L = r^2\omega \ , \tag{4}$$
$$\frac{dL}{dt} = r^2\frac{dw}{dt} \ , \tag{5}$$
$$dU = \frac{dL}{dt}d\theta = r^2\frac{d\theta}{dt}d\omega \ , \tag{6}$$
$$dU = r^2\omega d\omega \ . \tag{7}$$

dW is an energy input: the differential work needed to urge particle Y, radial distance dr against its centripetal acceleration $r\omega^2$. dU is not an energy input as no external torque is applied to the particle. dU is only a measure of inertial torque.

From Newton, we know that the centripetal acceleration of an orbiting particle is inversely proportional to the square of the orbit radius r as follows:

$$a = r\omega^2 = \frac{\mu}{r^2}, \text{where } \mu = GM \ , \tag{8}$$

which reduces to

$$r^3\omega^2 = \mu \ . \tag{9}$$

Dividing both sides of this equation by r^2, gives this:

$$r\omega^2 = \mu r^{-2} \ . \tag{10}$$

Plugging this result into Eq. (3), yields the differential work dW in terms of μ and r:

$$dW = \mu r^{-2} dr \ . \tag{11}$$

Here, a new differential-work term is introduced, ($a_x dr$), where a_x is an accelerating field that is constant with respect to r and independent of mass. The physical reasoning for this hypothesis will be examined in Section IV; however, the objective of this section is to show that a scalar field can mathematically produce the deep-MOND equation, which has demonstrated significant predictive success [16][17][18][3][9][16][10][11][13][4].

Adding in this term, gives

$$dW = \mu r^{-2} dr + \mathbf{a_x dr} \ . \tag{12}$$

With Eq. (3), the centripetal acceleration becomes

$$r\omega^2 \cancel{dr} = \mu r^{-2}\cancel{dr} + a_x \cancel{dr} \ . \tag{13}$$

Dividing both sides by r, gives an equation for ω^2:

$$\omega^2 = \mu r^{-3} + a_x r^{-1} \ . \tag{14}$$

Taking the derivative, gives this:

$$\omega d\omega = -\frac{3}{2}\mu r^{-4} dr - \frac{1}{2}a_x r^{-2} dr \ . \tag{15}$$

Multiplying both sides of the equation by r^2 and plugging the result into Eq. (7), gives an equation for dU:

$$dU = r^2\omega d\omega = -\frac{3}{2}\mu r^{-2} dr - \frac{1}{2}a_x dr \ . \tag{16}$$

Plugging Eqs. (12) and (16) into Eq. (2), yields an equation for dE:

$$dE = \mu r^{-2} dr + a_x dr + -\frac{3}{2}\mu r^{-2} dr - \frac{1}{2}a_x dr \ , \tag{17}$$
$$dE = -\frac{1}{2}\mu r^{-2} dr + \frac{1}{2}a_x dr \ . \tag{18}$$

Integration gives the following result:

$$\int dE = \int -\frac{1}{2}\mu r^{-2} dr + \frac{1}{2}a_x dr \ , \tag{19}$$
$$E = \frac{1}{2}\mu r^{-1} + \frac{1}{2}a_x r + C \ . \tag{20}$$

Integration introduces the constant C. If E is purely a function of μ and a_x, the instinct would be to zero out C to avoid introducing an additional free parameter that would have an unknown physical basis. However, if C could be derived such that it were a function of μ and a_x, avoiding the introduction of a new parameter, there would be an argument for leaving C in place. Ultimately, it will be shown that C is critical to the derivation at hand.

Solving for v^2, gives this:

$$E = \frac{1}{2}v^2 = \frac{1}{2}\mu r^{-1} + \frac{1}{2}a_x r + C \ , \tag{21}$$
$$v^2 = \mu r^{-1} + a_x r + 2C \ . \tag{22}$$

With the aim to derive C as function of μ and a_x, parameter a_x is temporarily eliminated by taking the square root of Eq. (22) as follows:

$$v = \sqrt{\mu} r^{-1/2} + \frac{C}{\sqrt{\mu}} r^{1/2} \ . \tag{23}$$

Re-squaring v gives the result:

$$v^2 = \mu r^{-1} + 2C + \frac{C^2}{\mu} r \ . \tag{24}$$

Setting Eqs. (22) and (24) equal to each other, gives the following:

$$\cancel{\mu r^{-1}} + a_x r + \cancel{2C} = \cancel{\mu r^{-1}} + \cancel{2C} + \frac{C^2}{\mu} r \ , \tag{25}$$

$$a_x \cancel{r} = \frac{C^2}{\mu} \cancel{r} \ . \tag{26}$$

Solving for C, gives

$$C = \sqrt{a_x \mu} \ , \tag{27}$$

which confirms the suspicion that C can be derived as a function of μ and a_x.

With this definition for C, Eq. (23) can be rewritten as such:

$$v = \sqrt{\mu} r^{-1/2} + \sqrt{a_x} r^{1/2} \ . \tag{28}$$

Breaking Eq. (28) into its two terms, leaves the following:

$$v_1 = \sqrt{\mu} r^{-1/2} \ , \tag{29}$$

$$v_2 = \sqrt{a_x} r^{1/2} \ . \tag{30}$$

Eq. (28) shows that the velocity of an orbiting body has two components: the Newtonian component, v_1, which is a function of μ and a second component, v_2, which is a function of a_x. Squaring v and dividing by r, yields the equation for centripetal acceleration:

$$a = \frac{v^2}{r} = \frac{\mu}{r^2} + \frac{2\sqrt{a_x \mu}}{r} + a_x \ . \tag{31}$$

At large radii, Eq. (31) converges to:

$$a \approx \frac{2\sqrt{a_x GM}}{r} + a_x \ \ (\text{where } \mu = GM) \ , \tag{32}$$

which is nearly identical to the MOND equation for acceleration in the deep-MOND regime:

$$a = \frac{\sqrt{a_0 GM}}{r} \ . \tag{33}$$

As noted, the nomenclature a_x was chosen to avoid confusion with a_0, which is historically associated with MOND. While the two constants are similar, they are defined differently, which will be discussed below. At large-radii, both approaches show accelerations that decline inversely proportional to distance. The key difference, however, is the inclusion of last the term in Eq. (31), the constant offset a_x, which does not exist in MOND. This term results in a value for a_x that is an order of magnitude lower than a_0, but in the end, both approaches produce nearly identical galactic rotation curves that match observed velocities.

Referring back to Eq. (20), congruence with MOND offers empirical evidence that the inclusion and derivation of C as a function μ and a_x, as shown by Eq. (27), is correct. Eq. (27) shows that for C to be zero, a_x would also have to be zero, as μ is clearly nonzero. Hence, it is argued that if a_x is nonzero, then C, as defined, must be included in Eq. (20).

Before proceeding, a paradox needs to be resolved regarding the centripetal-acceleration equation, Eq. (31), which has an additional term, introduced via constant C, as compared to the differential-work equation, Eq. (12). The differential work dW is the energy needed to urge an orbiting particle against its centripetal acceleration, and thus Eqs. (12) and (31) must be congruent. This extra term needs to be added to Eq. (12), but it needs to be shown that doing so will not alter equations derived therefrom – Eqs. (18) and (31), as a new term in Eq. (31) would create somewhat of a feedback loop. Redefining dW, gives this:

$$dW = \mu r^{-2} dr + 2\sqrt{a_x \mu} r^{-1} dr + a_x dr \ . \tag{34}$$

Following the derivation above, Eq. (3) gives the relation for $r\omega^2$:

$$r\omega^2 \cancel{dr} = \mu r^{-2} \cancel{dr} + 2\sqrt{a_x \mu} r^{-1} \cancel{dr} + a_x \cancel{dr} \ . \tag{35}$$

Dividing both sides by r, gives this:

$$\omega^2 = \mu r^{-3} + 2\sqrt{a_x \mu} r^{-2} + a_x r^{-1} \ . \tag{36}$$

Taking the derivative, gives

$$\omega d\omega = -\frac{3}{2} \mu r^{-4} dr - 2\sqrt{a_x \mu} r^{-3} - \frac{1}{2} a_x r^{-2} dr \ . \tag{37}$$

Multiplying both sides of the equation by r^2 and plugging the result into Eq. (7), gives an updated equation for dU:

$$dU = r^2 \omega d\omega = -\frac{3}{2} \mu r^{-2} dr - 2\sqrt{a_x \mu} r^{-1} - \frac{1}{2} a_x dr \ . \tag{38}$$

Plugging Eqs. (34) and (38) into Eq. (2), yields the same equation for dE as above:

$$
\begin{aligned}
dE =& \mu r^{-2} dr + \cancel{2\sqrt{a_x \mu} r^{-1}} + a_x dr + \\
& -\frac{3}{2}\mu r^{-2} dr - \cancel{2\sqrt{a_x \mu} r^{-1}} - \frac{1}{2} a_x dr \ , \qquad (39) \\
dE =& -\frac{1}{2}\mu r^{-2} dr + \frac{1}{2} a_x dr \ . \qquad (40)
\end{aligned}
$$

As shown, the additional differential-work term is canceled by its additive inverse in dU, resulting in a consistent equation for dE. In fact, any term of the form Ar^{-1} in the equation for dW would be canceled when added to dU. This leaves Eqs. (34) and (38) as the complete equations for dW and dU, respectively.

III. COMPARISON TO MOND

Under MOND, orbital velocity eventually flattens out according to the following relation:

$$
\begin{aligned}
\frac{v^2}{\cancel{r}} &= \frac{\sqrt{a_0 GM}}{\cancel{r}} \ , \qquad (41) \\
v^4 &= a_0 GM \ . \qquad (42)
\end{aligned}
$$

This comports with the Tully-Fisher Relation, which holds that $v^4 \propto M$ [19][20][17][21]. In the 1990's, it was found that even low-surface-brightness (LSB) galaxies follow this relation, contrary to assumptions at the time. This was seen as an important predictive victory for MOND, which asserts that the asymptotic velocity, $v^4 \propto M$ is universal [16][10].

Whereas MOND velocity curves mathematically flatten out, v in Eq. (28) flattens only as a consequence of the Keplerian decline – v_1 being matched by the increase in v_2. This is illustrated by Fig. 2, which shows the velocity curves for M33 with an assumed mass-light (M/L) ratio of .5 for the stellar disk, the low end of the estimated contribution to $v(r)$ [22]. Setting a_x to 1.7E−14 (km s^{-2}), results in the (v_1+v_2) curve shown in the figure, which does flatten out in agreement with observed velocities. Notice that $v(r)$ in M33 doesn't flatten to a constant value; there is a slight upward slope. In contrast, MOND curves mathematically flatten to a constant $v(r)$.

Under MOND, a_0 is the acceleration at which Newtonian mechanics transitions to the deep-MOND regime — where Keplarian decline is no longer the governing term [15][12][14]. In the present thesis, a_x is a scalar field that augments the Newtonian component, which is to say that a_0 and a_x correspond to different physical properties. As noted, the two parameters are within an order of magnitude of each other. MOND generally finds $a_0 \approx$ 1.2E−13 (km s^{-2}) [23][24][3], which corresponds to an $a_x \approx$ .9E−14 (km s^{-2}).

The higher a_x for M33 is attributable to the low M/L ratio input of .5. A ratio closer to the upper bound of 1.5, would result in a lower a_x, but 1.5 results in a $v(r)$ that falls out of line with observations, thereby constraining M/L to a lower value. This highlights the fact that curve fits require a balance between assumed M/L ratio and a_x. Giving a_x some berth as a free parameter may be appropriate as will be argued below. It is argued by some that a_0 may not be fixed either [25][9][26].

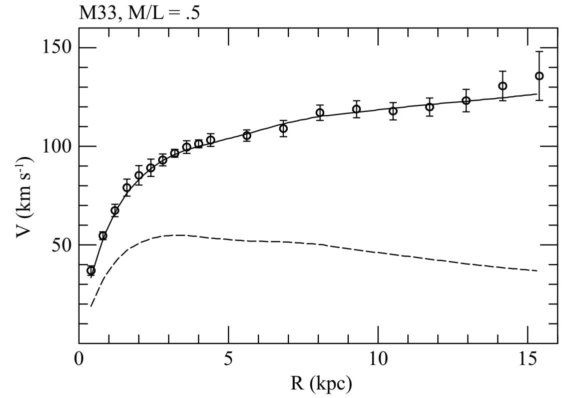

FIG. 2. Rotation curve for M33. The solid line is $(v_1 + v_2)$. The dashed line is the expected velocity based on luminous mass (stellar + gas discs), v_1. Data are from Corbelli et al. (2000).

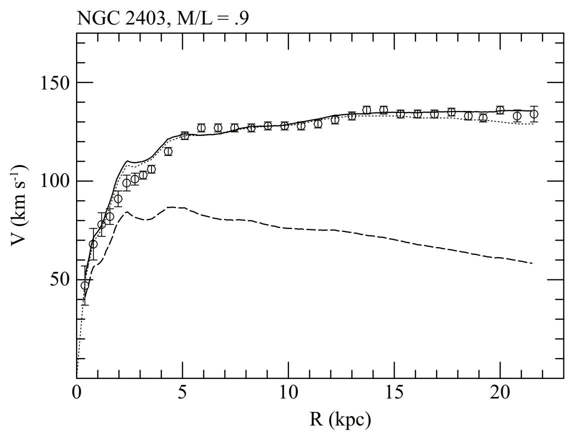

FIG. 3. Rotation curve for NGC 2403. The solid line is $(v_1 + v_2)$. The dotted line is the MOND-based curve. The dashed line is the expected velocity based on luminous mass (stellar + gas discs), v_1. Data are from Sanders (2009).

Fig. 3 shows the velocity curves for NGC 2403 with an assumed mass-light ratio of .9 [3]. The MOND curve is indicated by a dotted line. This curve and $(v_1 + v_2)$, indicated by the solid line, trace each other with minimal separation until about 12 kpc, where the MOND curve dips slightly while (v_1+v_2) stays mostly flat, both within the margin of error. Here, $(v_1 + v_2)$ was calculated with an $a_x =$ .9E−14 (km s^{-2}). It is assumed the MOND curve was derived with an $a_0 =$ 1.2E−13 (km s^{-2}).

Fig. 4 shows the velocity curves for NGC 3198, which is a spiral galaxy in UMa considered to be an exemplary test for any theory aiming to resolve the discrepancy be-

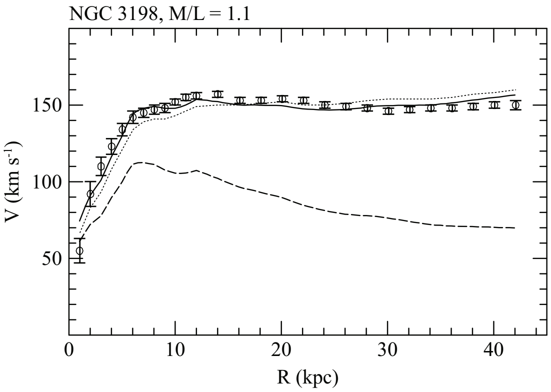

FIG. 4. Rotation curve for NGC 3198. The solid line is $(v_1 + v_2)$. The dotted line is the MOND-based curve. The dashed line is the expected velocity based on luminous mass (stellar + gas discs), v_1. Data are from Bottema et al. (2002).

tween visible mass and observed velocities. NGC 3198 is marginally troublesome for MOND [25]. The MOND curve, indicated by a dotted line, is based on a mass-light ratio of 1.1, as derived from the Cepheid-based distance of 13.8 Mpc. The curve falls below observed velocity error bars at the inner region of the galaxy and above the error bars at the outer region, with a separation of up to 10 (km s^{-1}).

As noted, MOND curves are typically based on an $a_0 = 1.2\text{E}{-13}$ (km s^{-2}). This curve from Bottema *et al.* [25], however, is based on an $a_0 = .9\text{E}{-13}$ (km s^{-2}). The authors argue that the value of a_0 is dependent on distance scale. If the distance to UMa is assumed to be 15.5 Mpc, MOND curve fits by Sanders and Verheijen [24] prefer an $a_0 = 1.2\text{E}{-13}$ (km s^{-2}). Tully and Pierce [27] and Bottema *et al.* [25] argue that 18.8 Mpc is the correct distance to UMa, in which case MOND curve fits put $a_0 = .9\text{E}{-13}$ (km s^{-2}).

At 12.5 Mpc, with an implied M/L of 1.3, the MOND curve for 3198 more closely traces observed velocities – within 5 (km s^{-1}). This distance leaves discrepancies between MOND and observations within reason while staying within 10% of the Cepheid prediction, the maximum tolerance. At 10.0 Mpc and an M/L of 2.1, the MOND curve directly traces observations, but this distance is beyond the error budget of the Cepheid method.

In Fig. 4, the $(v_1 + v_2)$ line, computed with an $a_x = .5\text{E}{-14}$ (km s^{-2}), traces the error bars well up to 30 kpc, where it diverges slightly, deviating no more than 5 km (s^{-1}) from observed velocities.

IV. SCALAR FIELD a_x

In Sections II and III, it was shown that scalar field a_x in combination with the Newtonian field can produce the deep-MOND equation (with a constant offset) that yields nearly identical rotation curves, which comport with observations. If a_x exists, what could be causing it?

Apart from galactic rotation curves that ostensibly show mass discrepancy, the other startling discovery made by astronomers in the 20th century was that nearly all visible galaxies are progressively redshifted where the amount of redshift increases linearly with distance.

The Friedmann-Robertson-Walker (FRW) metric, which models the universe as a homogeneous and isotropic fluid, shows that comoving observers at rest with respect to this cosmic fluid, the 'Hubble flow', would see the distance growing distance between each other over time with observers down the line seeing progressive redshift [28][29].

On local scales, space is no longer homogeneous, and particles are not at rest with respect to the cosmic flow, so the FRW metric no longer applies. Galaxies can roughly be modeled as a central mass, where the metric is Schwarzchild, surrounded by a sphere of Minkowski spacetime (which does not expand) and would be unaffected by global expansion [29][30]. Clearly, any local expansion would be non-relativistic, at least as currently understood.

If there were local expansion, there should be some evidence thereof. The recession of the moon is easily explained by tidal acceleration [31], but recession of the earth from the sun is harder to explain; multiple research groups have found a secular increase of the astronomical unit [32][33]. Krasinsky and Brumberg [32] calculate this rate at 15±4 (cm yr^{-1}). Neither classical Newtonian mechanics nor general relativity can explain the phenomenon. Krasinsky and Brumberg [32] rule out cosmic expansion and offer as an exotic explanation, a secular decrease in the gravitational constant, but generally leave the mystery as unsolved. Miura *et al.* [34] argue that tidal interactions on the sun could be transferring angular momentum to the planets similar to the effect between the earth and the moon.

Other phenomena exist in the solar that are left unexplained by Newtonian mechanics and general relativity such as the flyby anomaly, where spacecraft see unexplainable accelerations as they fly by planets. The effect was recently confirmed again by the Juno mission to Jupiter where analysis of the craft is showing anomalous acceleration during flybys [35].

Ostensibly, something, yet to be explained, is adding energy to systems locally in the solar system; to particles within galaxies, as evidenced by elevated rotation curves; and to galaxies within the cosmos that are receding at an accelerating rate. In the context of the universe, the energy in question is called 'dark energy'. In as much as MOND is motivated by the search for an alternative explanation of galactic rotation curves by stepping outside of Newtonian mechanics and relativity, it is argued that local expansion does exist and, if looked at from a different angle, could possibly explain anomalous phenomena in the solar system and in galaxies.

To be clear, the term 'expansion' is not meant to imply that space itself is expanding – being progressively

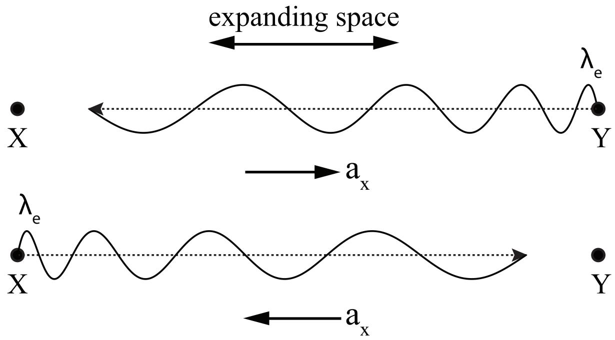

FIG. 5. Scalar field a_x between two particles in expanding space.

filled with new space or that the fabric of space is being stretched such that a steel ruler would grow larger over time. The term 'expansion' is only meant as a convenient descriptor of what is observed – photons being progressively redshifted and bodies moving freely in space growing further apart; while being completely agnostic as to the cause and what, if any, changes in space are occurring between those particles.

As a photon travels through expanding space, it is progressively stretched resulting in a redshift that increases with distance. Fig. 5 shows two comoving particles, X and Y, in expanding space. Referring back to Fig. 1, if one particle is in stable circular orbit around the second, they can be considered comoving as the change in proper distance between the two is a function of space expansion. From the perspective of X, light from Y is continuously red shifted such that the wavelength λ increases with distance from Y. The opposite is also true – from the perspective of Y, λ increases with distance from X.

At any instant in time without benefit of prior observations, observers on X and Y could conclude that redshift was a result of space expanding between the two particles, but they could also reason that there was a scalar accelerating field a_x that progressively de-energizes photons traveling between the two, akin to gravitational redshift. Here, it's posited that both conclusions would be correct. With spatial expansion, there is a scalar field that opposes the expansion. The effect is similar to eddy currents in a conductor opposing the movement of a magnetic field. Over time, observers on X and Y would notice both effects: increasing distance at an accelerating rate, with the plotted velocity-distance rotation curve showing the contribution of a_x.

To calculate a_x, the redshift of a photon traveling in expanding space is equated with the redshift of a photon traveling against field a_x.

The redshift z of a photon emitted from Y, where λ_e is the emitted wavelength is as follows [36]:

$$z = \frac{(\lambda - \lambda_e)}{\lambda_e} \ . \tag{43}$$

The recessional velocity V of Y relative to X is

$$V = cz \ , \tag{44}$$

which equates to

$$V = c\left(\frac{\lambda - \lambda_e}{\lambda_e}\right) \ . \tag{45}$$

Taking the derivative of Eq. (45) with respect to r, yields this:

$$\frac{dV}{dr} = \frac{d\lambda}{dr}\frac{c}{\lambda_e} \ . \tag{46}$$

Imagine a ruler of length r_0 connecting X and Y. As Y recedes from X at velocity V_0, if the ruler expands homogeneously, the recessional velocity of each hatch mark must vary linearly from 0 to V_0 as such:

$$H_g = \frac{V_0}{r_0} \ , \tag{47}$$

$$V = H_g r \ , \tag{48}$$

where H_g is the rate of space expansion at galactic scales. Note, the nomenclature H_g was used to avoid confusion with H_0, which refers to intergalactic expansion.

The velocity gradient H_g becomes increasingly constant with respect to time as $r_0 \gg V_0$. At galactic scales (kpc), H_g can be considered as constant.

Taking the derivative of Eq. (48) with respect to r, gives the following:

$$\frac{dV}{dr} = H_g \ . \tag{49}$$

Plugging this into Eq. (46), gives the result:

$$H_g = \frac{d\lambda}{dr}\frac{c}{\lambda_e} \ , \tag{50}$$

$$\frac{d\lambda}{dr} = \frac{\lambda_e}{c}H_g \ . \tag{51}$$

The momentum of a photon is given by the Planck-Einstein relation, which equates momentum p to wavelength λ as such:

$$p = \frac{h}{\lambda} \ . \tag{52}$$

Taking the derivative with respect to r gives the result:

$$\frac{dp}{dr} = -\frac{-h}{\lambda^2}\frac{d\lambda}{dr} \ . \tag{53}$$

Plugging Eq. (51) into this, yields Eq. (54) below:

$$\frac{dp}{dr} = -hH_g \frac{\lambda_e}{c} \frac{1}{\lambda^2} \ . \tag{54}$$

Multiplying Eq. (54) by the speed of light gives this:

$$\frac{dr}{dt} = c \ , \tag{55}$$

$$\frac{dp}{\cancel{dr}} \frac{\cancel{dr}}{dt} = -hH_g \frac{\lambda_e}{\cancel{c}} \frac{1}{\lambda^2} \cancel{c} \ , \tag{56}$$

$$\frac{dp}{dt} = -hH_g \lambda_e \frac{1}{\lambda^2} \ . \tag{57}$$

Dividing both sides of Eq. (52) by the speed of light c, gives the ratio of momentum to speed, which is the 'effective mass' m_e of a photon. As above, λ_e is the emitted wavelength:

$$m_e = \frac{p}{c} = \frac{h}{c\lambda_e} \ . \tag{58}$$

The change in momentum with respect to time of m_e in field a_x is thus:

$$\frac{dp}{dt} = m_e a_x \ , \tag{59}$$

$$\frac{dp}{dt} = -\frac{h}{c\lambda_e} a_x \ . \tag{60}$$

Notice the sign reversal in Eq. (60), signifying that from the perspective of a photon leaving the point of emission, a_x is a decelerating field. Setting Eqs. (57) and (60) equal to each other gives this:

$$-\frac{h}{c\lambda_e} a_x = -hH_g \lambda_e \frac{1}{\lambda^2} \ . \tag{61}$$

Solving for a_x, produces this:

$$a_x = cH_g \frac{\lambda_e^2}{\lambda^2} \ . \tag{62}$$

Re-arranging the terms in Eq. (45) gives an equation for λ_e/λ:

$$V = c\left(\frac{\lambda - \lambda_e}{\lambda_e}\right) \ , \tag{63}$$

$$\frac{V}{c} + 1 = \frac{\lambda}{\lambda_e} \ , \tag{64}$$

$$\frac{\lambda_e}{\lambda} = \frac{c}{V + c} \ . \tag{65}$$

Plugging Eq. (65) into Eq. (62) yields:

$$a_x = cH_g \frac{c^2}{(V + c)^2} \ . \tag{66}$$

Assuming that $c \gg V$, leaves a_x essentially constant with respect to V and thus r. Eq. (66) can therefore be simplified as follows:

$$a_x = cH_g \frac{c^2}{(\cancel{V} + c)^2} \ , \tag{67}$$

$$a_x = cH_g \ . \tag{68}$$

With this relation, Eq. (28) can be expanded as so:

$$v = \sqrt{GM} r^{-1/2} + \sqrt{cH_g} r^{1/2} \ , \tag{69}$$

leaving the final relation for energy E as:

$$E = \frac{1}{2} m v^2 \ , \tag{70}$$

$$E = \frac{1}{2} GMmr^{-1} + \frac{1}{2} cH_g m r + m\sqrt{GMcH_g} \ . \tag{71}$$

Table I shows H_g for M33, NGC 2403, and NGC 3198, ordered by distance. At galactic scales, (km s^{-1} kpc^{-1}), relatively low rates of expansion can yield significant accelerating fields. For example, at the outer edge of NGC 2403 (20 kpc), with an H_g = 9.3E−4 (km s^{-1} kpc^{-1}), the recessional velocity is .019 (km s^{-1}), verifying the assumption that $c \gg V$.

An H_g = 9.3E−4 (km s^{-1} kpc^{-1}) corresponds to an a_0 = 1.2E−13 (km s^{-2}), which is appropriate for the Milky Way in term of MOND [37][13]. Scaling kpc to km, gives an H_g = 3.0E−20 (km s^{-1} km). Multiplying the distance between the earth and sun of 149.6E6 (km), by H_g, gives a recessional velocity = $4.5E - 012$ (km s^{-1}), which translates to 14 (cm yr^{-1}). This is in strong agreement with the value calculated by Krasinsky and Brumberg [32] of 15±4 (cm yr^{-1}). Of course, this is only one data point, and if any of other planets in the solar system were found to not be receding as predicted, the present hypothesis would largely be falsified.

At cosmic scales, (km s^{-1} Mpc^{-1}), H_g for NGC 2403 recalculates to .93 (km s^{-1} Mpc^{-1}), which is two orders of magnitude lower than the Hubble constant H_0, most recently calculated at 70.0 km s^{-1} Mpc^{-1} [38]. As described, the FRW metric, which does predict cosmic expansion, does not apply at galactic scales, making any possible reconciliation of H_g with H_0 beyond the scope of this paper.

Coincidentally, the original MOND papers by Milgrom (1983) noted the similarity in magnitude between a_0 and cH_0 [15][39][40].

Eqs. (69) and (71) only make sense in the context of two or more particles. In other words, cH_g doesn't connect particles to random points in space. A two-particle

TABLE I. recessional velocity V, a_x, and H_g for M33, NGC 2403, and NGC 3198.

Galaxy	Distance (Mpc)	V (km s^{-1}) @15 kpc	a_x (km s^{-2})	H_g (km s^{-1} kpc^{-1})
M33	.73-.94	0.026	1.7E−14	17E−4
NGC 2403	2.5	0.014	.9E−14	9.3E−4
NGC 3198	13.8	0.0077	.5E−14	5.1E−4

system is already connected by a gravitational field between the particles. In essence, cH_g can only augment this existing gravitational field. For example, particle Y, in orbit around particle X, doesn't slowly grind to halt as it's continuously redshifted. Redshifted with respect to what? There is no field between particle Y and an infinite number of points in space along its path. The field only exists between particles X and Y. This means further that the second term in Eq. (69) cannot exist without the first term. Interestingly, the second term, ($\sqrt{cH_g}r^{1/2}$), requires that H_g be non negative, implying that space can never contract.

V. MATTER WAVES

The analysis, so far, has focused on photons. If, however, expanding space can redshift light, wave-particle duality suggests that matter should be redshifted as well; which begs the question: does the redshift of a slow-moving massive particle translate to scalar field cH_g? Or is the magnitude of the field velocity dependent such that $a_x = vH_g$? If this were the case, two particles with zero relative velocity would see the magnitude of a_x drop to zero. To explore this, let us start with the de Broglie relation for the frequency of a particle of rest mass m_0 and velocity v, where v is the group velocity of the wave:

$$f = \frac{m_0 c^2}{h}\left(1-\frac{v^2}{c^2}\right)^{-1/2} . \tag{72}$$

Taking the derivative with respect to r gives this:

$$\frac{df}{dr} = -\frac{1}{2}\frac{m_0 \cancel{c^2}}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{\cancel{c^2}}\right)\frac{dv}{dr} , \tag{73}$$

$$\frac{df}{dr} = \frac{m_0 v}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\frac{dv}{dr} . \tag{74}$$

Substituting dr/dt for v, gives the following:

$$\frac{df}{dr} = \frac{m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\frac{dv}{\cancel{dr}}\frac{\cancel{dr}}{dt} , \tag{75}$$

$$\frac{df}{dr} = \frac{m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\frac{dv}{dt} . \tag{76}$$

Assuming a non-relativistic velocity for v and substituting a_x for dv/dt, gives this relation for df/dr:

$$\frac{df}{dr} = \frac{m_0}{h}\left(1-\cancel{\frac{v^2}{c^2}}\right)^{-3/2}\frac{dv}{dt} , \tag{77}$$

$$\frac{df}{dr} = \frac{m_0}{h}a_x . \tag{78}$$

Eq. (78) shows that a frequency gradient can induce the acceleration of a particle. Because frequency is a measure of energy or time, df/dr can be thought of an energy gradient or a gradient of increasing time dilation. An observer on particle X would perceive a clock running on particle Y slightly behind a local clock and vice versa – an observer on particle Y would perceive particle X's clock as running slower. Furthermore, the observers would see each others clocks not just running slower but decelerating.

Referring back to Eq. (43), the redshift z of a photon traveling between Y and X can be rewritten in terms of frequency through the wave-speed equation for light $f\lambda = c$ as such:

$$\lambda = \frac{c}{f} , \tag{79}$$

$$z = \frac{(\lambda - \lambda_e)}{\lambda_e} , \tag{80}$$

$$z = \frac{(c/f - c/f_e)}{c/f_e} , \tag{81}$$

$$z = \frac{f_e}{f} - 1 . \tag{82}$$

Plugging z into Eq. (44), gives the recessional velocity V in terms of frequency f, where f_e is the emitted frequency:

$$V = c\left(\frac{f_e}{f} - 1\right) . \tag{83}$$

Taking the derivative with respect to r, yields this:

$$\frac{dV}{dr} = \frac{-cf_e}{f^2}\frac{df}{dr} . \tag{84}$$

Plugging the velocity gradient H_g into dV/dr and rearranging terms, gives this relation for df/dr:

$$\frac{df}{dr} = \frac{-f^2}{cf_e} H_g \ . \tag{85}$$

While a photon traveling from Y to X loses energy according to Eq. (85), does the same relation hold for a slow-moving particle of mass? To explore this, we start by re-writing Eq. (51) as a function of emitted frequency f_e via Eq. (79):

$$\frac{d\lambda}{dr} = \frac{\lambda_e}{c} H_g \ , \tag{86}$$

$$\frac{d\lambda}{dr} = \frac{1}{f_e} H_g \ . \tag{87}$$

For a photon, the rate of change of λ over time can be found by multiplying Eq. (87) by the speed of light (Eq. (55)):

$$\frac{d\lambda}{\cancel{dr}} \frac{\cancel{dr}}{dt} = \frac{1}{f_e} H_g c \ , \tag{88}$$

$$\frac{d\lambda}{dt} = \frac{c}{f_e} H_g \ . \tag{89}$$

Here, it is posited that the change in λ over time, for a given particle frequency (energy) is irrespective of particle speed. When a photon traveling from Y to X is redshifted, it is redshifted over a span of time [36]. Because light propagates, that time period translates to a distance, giving the redshift vs. distance correlation. Along these lines, it is presumed that two particles of identical energy (and thus frequency) traveling at different speeds in expanding space would see identical redshifts over a period of time but different redshifts over distance, with the slower moving particle seeing more redshift in a given distance as a consequence of moving slower.

With this in mind, a second relation for df/dr, for a massive particle, as a function of H_g can be derived. The frequency f of a matter wave is related to its wavelength λ as so:

$$f = \frac{v_p}{\lambda} \ , \tag{90}$$

where v_p is the phase velocity of the wave. Taking the derivative with respect to time, gives this:

$$\frac{df}{dt} = \frac{-v_p}{\lambda^2} \frac{d\lambda}{dt} \ . \tag{91}$$

Plugging in Eq. (89), which, as argued, should be true for both photons and particles of mass, gives the following:

$$\frac{df}{dt} = \frac{-v_p}{\lambda^2} \frac{c}{f_0} H_g \ . \tag{92}$$

Here, f_e is re-written as f_0, which refers to the frequency of a matter wave vs. that of a photon. Plugging Eq. (90) into λ yields this:

$$\frac{df}{dt} = \frac{-v_p c}{v_p^2} \frac{f^2}{f_0} H_g \ , \tag{93}$$

$$\frac{df}{dt} = \frac{-c}{v_p} \frac{f^2}{f_0} H_g \ . \tag{94}$$

A particle Y moving relative to particle X at velocity v, where v is the group velocity of the wave can be described as follows:

$$\frac{dt}{dr} = \frac{1}{v} \ . \tag{95}$$

Multiplying Eq. (94) times this relation yields an equation for df/dr:

$$\frac{df}{\cancel{dt}} \frac{\cancel{dt}}{dr} = \frac{-c}{v_p v} \frac{f^2}{f_0} H_g \ . \tag{96}$$

From de Broglie, the phase velocity times the group velocity of a matter wave is the speed of light squared: $v_p v = c^2$. With this, a final equation for df/dr for a massive particle is as follows:

$$\frac{df}{dr} = \frac{-c}{c^2} \frac{f^2}{f_0} H_g \ , \tag{97}$$

$$\frac{df}{dr} = \frac{-f^2}{cf_0} H_g \ . \tag{98}$$

Setting Eqs. (78) and (98) equal to each other gives this:

$$\frac{m_0}{h} a_x = \frac{-f^2}{cf_0} H_g \ . \tag{99}$$

The rest frequency f_0 of m_0 is given by Eq. (72) as $v \to 0$:

$$m_0 = \frac{f_0 h}{c^2} \ . \tag{100}$$

Plugging this into Eq. (99), yields this relation for a_x:

$$\frac{f_0 \cancel{h}}{c^2 \cancel{h}} a_x = \frac{-f^2}{cf_0} H_g \ , \tag{101}$$

$$a_x = -cH_g \frac{f^2}{f_0^2} \ . \tag{102}$$

Eq. (102) can be rewritten via Eq. (90) as this:

$$a_x = -cH_g \frac{(v_p/\lambda)^2}{(v_p/\lambda_0)^2} , \quad (103)$$

$$a_x = -cH_g \frac{\lambda_0^2}{\lambda^2} , \quad (104)$$

which is equivalent to Eq. (62). The negative sign can be ignored with the understanding that a_x is an attractive field between particles.

As Eq. (72) shows, for non-relativistic velocities (which is the case for recessional velocity V), $f \approx f_0$, allowing a_x to simplify to:

$$a_x = cH_g , \quad (105)$$

which shows that slow-moving massive particles see the same scalar field cH_g as do photons.

Eqs. (102) and (105) hinge on the duality of particles and waves, which is well understood to be the case. If particles of mass travel in waves, it's plausible that they are redshifted in the same manner as photons. A core argument is that the magnitude of particle redshift is a function of time, but not particle velocity, as described by Eq. (89). Slow-moving and fast-moving particles of a given energy are equally redshifted over time, with slower-moving particles redshifted more per unit distance.

VI. CONCLUSIONS

In the early 20th century, astronomers made two remarkable discoveries: the velocity curves for galaxies do not follow Keplerian decline, and galaxies show increasing redshift with distance. This paper suggests that the two phenomena are interconnected. While general relativity explains celestial mechanics in the realm of extremely high gravitational potentials, with Newtonian gravitation filling in the middle, an explanation of mechanics at the low end of the spectrum has remained elusive.

The most widely accepted hypothesis, dark matter, is nearly impervious to falsification as models are given free parameters to define halo shape and density distribution [3][21]. Direct evidence could end the debate, but to date, experiments designed to detect dark-matter particles have come up empty handed [41].

The competing hypothesis, MOND, doesn't rely on hidden matter and is very predictive at galactic scales. Since being first proposed by Mordehai Milgrom in 1983, MOND has been tested against numerous galaxies with broad success. Even borderline results haven't risen to the level of falsification as a consequence of uncertainties in assumptions and observations. Importantly, MOND is easily falsifiable, even if the value of a_0 is allowed a certain tolerance, which is appropriate if a_0 is a function of time and perhaps other metrics.

One impediment to MOND has traditionally been its empirical nature. The aim of this paper was to present a line of physical reasoning that produces the deep-MOND-regime equation. The resulting equation is close enough in form to produce nearly identical velocity curves. The hypotheses diverge in the interpretation of a_0, which is labeled a_x with respect to the thesis herein to avoid confusion.

Under MOND, a_0 marks the acceleration at which regimes transition from Newtonian to deep MOND. This paper argues that a_x is not a marker but a distinct scalar field that exists on top of the Newtonian component. This allows a single equation, Eq. (69), to be derived that produces velocity curves without need of an interpolating function.

The second objective of the paper was to offer an hypothesis as to the origin of a_x, which is argued to be a consequence of expanding space, such that $a_x = cH_g$, where H_g is the local (galactic) rate of expansion. The implication of Eq. (69) is that space can never contract. Defining a_x as a separate field vs. a fundamental physical constant, as implied by MOND, gives a_x some berth as free parameter, as local expansion may vary between galaxies. More data points are needed to establish the relationship between H_g and other parameters such as cosmological distance and time, and galaxy mass. Also, minor variability in a_x could explain possible variance in a_0 as noted by Bottema *et al.* [25], Gentile *et al.* [9], and Bottema and Pestana [26].

As the FRW metric does not account for local expansion, evidence thereof would imply a non-relativistic phenomenon. Such evidence may exist in the solar system, as measurements show the astronomical unit AU increasing at a rate of 15±4 (cm yr^{-1}). An $H_g = 9.3\text{E}{-4}$ for the Milky Way translates to an AU increase of 14 (cm yr^{-1}), which is in close agreement with the observed value. Given, however, that this is only one data point, other planets found not to be receding in concert at a rate predicted by H_g would constitute falsification of the present hypothesis. With that said, additional phenomena exist in the solar system that are unexplained by Newtonian Mechanics and General Relativity, such as the flyby anomaly. Of course, the most salient phenomenon unaccounted for by classical physics and relativity are elevated galactic rotation curves.

[1] V. C. Rubin and W. K. Ford, Jr., Astrophysical Journal, Letters **159**, 379403 (1970).
[2] E. J. Lerner and J. B. Almeida, eds., *1st Crisis in Cosmology Conference*, AIP Conference Proceedings No. 822 (American Institute of Physics, 2006).
[3] R. H. Sanders, Advances in Astronomy (2009), 752439.

[4] S. S. McGaugh, Canadian Journal of Physics **93**, 250 (2015).
[5] P. A. R. Ade *et al.*, Astronomy and Astrophysics **571** (2014).
[6] G. Battaglia, A. Helmi, H. Morrison, P. Harding, E. W. Olszewski, M. Mateo, K. C. Freeman, and J. Norris, Monthly Notices of the RAS **364**, 433 (2005).
[7] P. J. McMillan, Monthly Notices of the RAS **414**, 2446 (2011).
[8] P. R. Kafle, S. Sharma, G. F. Lewis, and J. Bland-Hawthorn, Astrophysical Journal, Letters **794** (2014), 59.
[9] G. Gentile, B. Famaey, and W. J. G. de Blok, Astronomy and Astrophysics **527**, 441 (2011).
[10] S. S. McGaugh, Physical Review Letters **106** (2011), 121303.
[11] P. Kroupa, Publications of the Astron. Soc. of Australia **29**, 395 (2012).
[12] M. Milgrom, New Astronomy Review **46**, 741 (2002).
[13] F. Iocco, M. Pato, and G. Bertone, Physical Review D **92**, 28 (2015).
[14] M. Milgrom, Canadian Journal of Physics **93**, 107 (2015).
[15] M. Milgrom, Astrophysical Journal, Letters **270**, 365 (1983).
[16] S. S. McGaugh and W. J. G. de Blok, Astrophysical Journal, Letters **499**, 66 (1998).
[17] R. H. Sanders and S. S. McGaugh, Annual Review of Astron and Astrophys **40**, 263 (2002).
[18] O. Tiret, F. Combes, G. W. Angus, B. Famaey, and H. S. Zhao, Astronomy and Astrophysics **476**, L1 (2007).
[19] R. B. Tully and J. R. Fisher, Astronomy and Astrophysics **54**, 661673 (1977).
[20] S. S. McGaugh, Astrophysical Journal, Letters **533**, L99 (2000).
[21] S. S. McGaugh, Astronomical Journal **143** (2012), 40.
[22] E. Corbelli and P. Salucci, Monthly Notices of the RAS **311**, 441 (2000).
[23] K. G. Begeman, A. H. Broeils, and R. H. Sanders, Monthly Notices of the RAS **249**, 523 (1991).
[24] R. H. Sanders and M. A. W. Verheijen, Astrophysical Journal, Letters **503**, 97 (1998).
[25] R. Bottema, J. L. G. Pestana, B. Rothberg, and R. H. Sanders, Astronomy and Astrophysics **393**, 453 (2002).
[26] R. Bottema and J. L. G. Pestana, Monthly Notices of the RAS **448**, 2566 (2015).
[27] R. B. Tully and M. J. Pierce, Astrophysical Journal, Letters **533**, 744 (2000).
[28] A. Friedmann, General Relativity and Gravitation **31**, 1991 (1999).
[29] M. J. Francis, L. A. Barnes, J. B. James, and G. F. Lewis, Publications of the Astron. Soc. of Australia **24**, 95 (2007).
[30] J. A. Peacock, (2008), arXiv:0809.4573 [astro-ph].
[31] D. E. Cartwright, "Lunar acceleration, earth retardation and tidal friction," in *Tides: a scientific history* (Cambridge University Press, 2001) Chap. 10, pp. 144–146.
[32] G. A. Krasinsky and V. A. Brumberg, Celestial Mechanics and Dynamical Astronomy **90**, 267 (2004).
[33] L. Iorio, Journal of Cosmology and Astroparticle Physics **2005** (2005).
[34] T. Miura, H. Arakida, M. Kasai, and S. Kuramata, Publications of the Astronomical Society of Japan **61**, 1247 (2009).
[35] L. Acedo, P. Piqueras, and J. A. Morao, (2017), arXiv:1711.08893 [astro-ph].
[36] E. Harrison, Astrophysical Journal, Letters **403**, 28 (1993).
[37] S. S. McGaugh, Astrophysical Journal, Letters **638**, 137 (2008).
[38] B. P. Abbott *et al.*, Nature **551**, 85 (2017).
[39] M. Milgrom, Astrophysical Journal, Letters **270**, 371389 (1983).
[40] M. Milgrom, Astrophysical Journal, Letters **270**, 384 (1983).
[41] D. S. Akerib *et al.*, Physical Review Letters **118** (2017), 021303.