

Ilija Barukčić
Horandstrase
DE-26441 Jever
Germany.
Email: Barukcic@t-online.de

Saturday, January 20, 2018

Open Letter To Professor Saburou Saitoh

Kawauchi-cho, 5-1648-16, Kiryu 376-0041, Japan

Dear Professor Saburou Saitoh,

it is with gratefulness and joy that I accept this opportunity of communication. In accordance with the historically very high tradition in science as such to test a theory that had been published or completed I am none the less very glad to express my personal thanks to other which are trying to contribute to the solution of indeterminate forms in mathematics. In this context, if different persons with different ideology and believe should arrive at the same logical conclusions with regard to such a difficult topic as indeterminate forms are, they will have to agree at least upon some view fundamental laws (axioms) as well as the methods by which other laws can be deduced therefrom. At this point, clarifying some fundamental axioms or starting points of investigations is therefore an essential part of every scientific method and any scientific progress. Thus far, in our everyday hunt for progress in science it is helpful if any attempt to build a scientific picture of complex phenomena out of some relatively simple proposition is based on principles which the scientific community can accept without any hesitation or critique. Clearly, such axioms or principles are rare. In support of this, please allow me to bring some important remarks to your attention which are worth being considered for future investigations.

Yours faithfully,

Ilija Barukčić

The nature of independence

Ilija Barukčić

Internist: Horandstrase, DE-26441, Jever, Germany.

Email: Barukcic@t-online.de

How to cite this paper: Ilija Barukčić (2018) The nature of independence. *Vixra*, 1, 1-xxxxx. http://vixra.org/author/ilija_barukcic

Received: 2018 01, 20

Accepted: 2018 01, 20

Published: 2018 01, 20

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Abstract

Objective: Accumulating evidence indicates that zero divided by zero equal one. Still it is not clear what number theory is saying about this.

Methods: To explore relationship between the problem of the division of zero by zero and number theory, a systematic approach is used while analyzing the relationship between number theory and independence.

Result: The theorems developed in this publication support the thesis that zero divided by zero equals one. It is possible to define the law of independence under conditions of number theory.

Conclusion: The findings of this study suggest that zero divided by zero equals one.

Keywords

Zero, One, Zero divide by zero, Independence, Number theory

1. Introduction

The question of the nature of independence and the plausibility of scientific methods and results with respect to some theoretical or experimental investigations of objective reality is many times so controversial that no brief account of it will satisfy all those with a stake in the debates concerning the nature of truth and its role in accounts of classical logic and mathematics. Independent of the issue about the relationship between objective reality and a theory of objective reality scientific conclusions of investigations should at least be truly independent of anyone's beliefs, anyone's ideological position or mind. Many times scientific conclusions rests on mathematics which itself is not free of assumptions.

There are several distinct ways in which a great deal of debate of the relationship between mathematics and objective reality can be analyzed. Mathematics as such may enjoy a special esteem within scientific community and is more or less above all other sciences due to the common believe that the laws or mathematics are absolutely indisputable and certain. In a slightly different way and first and after all, mathematics is a product of human thought and mere human imagination and belongs as such to a world of human thought and mere human imagination. Human thought and mere human imagination which produces the laws of mathematics is able to produce erroneous or incorrect results with the principal consequence that even mathematics or mathematical results valid since thousands of years are in constant danger of being overthrown by newly discovered facts. In addition to that, acquiring general scientific knowledge by deduction from basic principles, does not guarantee correct results if the basic principles are not compatible with objective reality or classical logic as such. In other words, if mathematics has to be regarded as a science and not as religion formulated by numbers, definitions et cetera, the same mathematics must be open to a potential revision. In general and from a theoretical point of view, a mathematics or a mathematical theorem characterized by denial(ism) and resistance to the facts which do not offer itself to a potential refutation would not allow us to distinguish scientific knowledge from its look-alikes. From a practical point of view, it is not enough to (mathematically) define how objective reality has to be, even mathematics itself must discover how nature really is. Due to the high status of science in present-day society, even mathematics itself must pass the test of reality and does not stand above all and outside of reality. The principles of mathematics should be logically compatible and receive strong experimental confirmation as much as possible. In this context, objective reality or practical or theoretical experiments as such are a demarcation line between science and fantastical pseudo-science. The conflict between science and pseudoscience is best understood with respect to the notion of independence. What is objective reality? What is human perception, human mind and human consciousness?

What is independence?

The concept of independence is of fundamental importance in philosophy, in mathematics and in science as such. In fact, it is insightful to recall Kolmogorov's theoretical approaches to the concept of independence.

“In consequence, one of the most important problems in the philosophy of the natural sciences is - in addition to the well-known one regarding the essence of the concept of probability itself - to make precise the premises which would make it possible to regard any given real events as independent.” [1]

Due to Kolmogorov, the concept of independence is still of strategic and central importance in science as such.

“The concept of mutual independence of two or more experiments holds, in a certain sense, a central position in the theory of probability.” [2]

Historically, one of the first documented mathematically approaches to the concept of independence was provided to us by De Moivre.

“Two Events are independent, when they have no connexion one with the other, and that the happening of one neither forwards nor obstructs the happening of the other.” [3]

In defining independence of events De Moivre refers one event to another event. These general considerations of De Moivre about the nature of independence is derived from the position of the ancient Greeks which demanded to describe a motion of a body while referring to another body. As was mentioned earlier, Einstein’s position concerning the concept of independence is very clear.

“Ohne die Annahme einer ... Unabhängigkeit der ... Dinge voneinander ... wäre physikalisches Denken ... nicht möglich.” [4]

Einstein's position in broken English:

“Without the assumption of ... independence of ... things from each other ... physical thinking ... would be impossible.” [4]

Einstein is elaborating on the principle of independence as follows:

“Für die relative Unabhängigkeit räumlich distanter Dinge (A und B) ist die Idee charakteristisch: äussere Beeinflussung von A hat keinen unmittelbaren Einfluss auf B; dies ist als ‚Prinzip der Nahewirkung‘ bekannt, das nur in der Feld-Theorie konsequent angewendet ist. Völlige Aufhebung dieses Grundsatzes würde die Idee von der Existenz (quasi-) abgeschlossener Systeme und damit die Aufstellung empirisch prüfbarer Gesetze in dem uns geläufigen Sinne unmöglich machen.” [5]

Einstein's position in English:

“For the relative independence of spatially distant things (A and B) the following principle is characteristic: any external influence of A has no direct influence on B; This is known as a 'principle of locality' which is only applied consistently in field theory. This principle completely abolished would disable the possibility of the existence of (nearly-) closed systems and the establishment of empirically verifiable laws in the common sense.”

A further position Einstein's is the following:

“But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S2 is independent of what is done with the system S1, which is spatially separated from the former ... the real situation of S2 must be independent of what happens to S1 ... One can escape from this conclusion only by either assuming that the measurement of S1 ((telepathically)) changes the real situation of S2 or by denying independent real situations as such to things which are spatially separated from each other. Both alternatives appear to me entirely unacceptable.” [6]

However, over recent years attempts to meet the difficulties associated with the concept of independence in quite different ways have met with little success. Still, number theory and algebra are quite silent about generally valid concept of independence.

2. Material and methods

2.1. Definitions

DEFINITION 0. (NUMBER +0).

Let c denote the speed of light in vacuum, let ε_0 denote the *electric constant* and let μ_0 the *magnetic constant*. Let i denote the imaginary. The number +0 is defined as the expression

$$\left(c^2 \times \varepsilon_0 \times \mu_0\right) - \left(c^2 \times \varepsilon_0 \times \mu_0\right) \equiv +1 - 1 \equiv +i^2 - i^2 \equiv +0 \quad (1)$$

DEFINITION 1. (NUMBER +1).

Let c denote the speed of light in vacuum, let ε_0 denote the *electric constant* and let μ_0 the *magnetic constant*. Let i denote the imaginary. The number +1 is defined as the expression

$$c^2 \times \varepsilon_0 \times \mu_0 \equiv -i^2 \equiv +1 \quad (2)$$

DEFINITION 2. (BERNOULLI TRIAL).

Let t denote a Bernoulli trial thus that

$$t = +1, \dots, +N \quad (3)$$

2.2. Methods

In the spring 1953, a graduate Student of history J. S. Switzer wrote Einstein a letter and requested Einstein's opinion on non-science and science. Einstein replied to Switzer on 23 Apr 1953 in a letter as follows:

“Development of Western science is based on two great achievements: the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility to find out causal relationships by systematic experiment (during the Renaissance). In my opinion, one has not to be astonished that the Chinese sages have not made these steps. The astonishing thing is that these discoveries were made at all.” [7]

Classical logic and systematic experiment can help us to demark science from non-science not only in physics but in mathematics as such too.

2.2.1. Thought Experiments

Thought experiments [8] play a central role both in natural sciences and in the philosophy and are valid devices of the scientific [9] investigation. One of the most common features of thought experiments is that thought experiments can be taken to provide evidence in favor of or against a theorem, a theory et cetera. In particular, there have been attempts to define a “thought experiment”, still there is no standard definition for thought experiments and the term is loosely characterized. More precisely, general acceptance of the importance of thought experiments can be found in almost all disciplines of scientific inquiry and are going back at least two and a half millennia and have practiced since the time of the Pre-Socratics [10]. A surprisingly large majority of impressive examples of thought experiments can be found in physics among some of its most brilliant practitioners are Galileo, Descartes, Newton and Leibniz [8]. Many famous physical publications have been characterized as thought experiments and include Maxwell's demon, Einstein's elevator (and train, and stationary lightwave), Heisenberg's microscope, Schrödinger's cat et cetera. Thought experiments are conducted for diverse reasons in a variety of areas and are equally common in pure, applied and in experimental mathematics.

2.2.2 Counterexamples

The relationship between an axiom and a conclusion derived in a technically correct way from such an axiom determines the validity of such a conclusion. In particular, it is impossible for an axiom to be true and a conclusion derived in a technically correct way from the same axiom to be false. A conclusion derived in a technically correct way must follow with strict necessity from an axiom and must be free of contradictions. In point of fact, a logical contradiction is not allowed in this context. It is necessary to point out that one single real or theoretical experiment can provide a logical contradiction and prove a theory wrong. Due to Einstein:

“No amount of experimentation can ever prove me right; a single experiment can prove me wrong.” [11]

A *counterexample* [12] is a simple and valid proof technique which philosophers and mathematicians use extensively to disprove a certain philosophical or mathematical [13] position or theorems as wrong and as not generally valid by showing that it does not apply in a certain single case. By using counterexamples researchers may avoid going down blind alleys and stop losing time, money and effort.

2.3. Axioms

There have been many attempts to define the foundations of logic in a generally accepted manner. However, besides of an extensive discussion in the literature it is far from clear whether the truth as such is a definable notion. As generally known, axioms and rules of a publication have to be chosen carefully especially in order to avoid paradoxes and inconsistency. Thus far, for the sake of definiteness and in order to avoid paradoxes the theorems of this publication are based on the following axiom.

2.2.1. Axiom I (**Lex identitatis. Principium Identitatis. Identity Law**)

In general, it is

$$+1 \equiv +1 \quad (4)$$

Lex identitatis or the *identity law* or *principium identitatis* is expressed mathematically in the very simple form as $+1 = +1$. A detailed of the history of the identity law (principium identitatis). In the following it is useful to point to other attempts of mathematizing the identity law. The identity law was used in Plato's dialogue Theaetetus, in Aristotle's *Metaphysics* (Book IV, Part 4) and by many other authors too. In particular, multiplying the axiom above by A we obtain $A = A$ or “A est A”. Multiplying the axiom above by B it is $B = B$ or “B est B”. Especially, Gottfried Wilhelm Leibniz (1646–1716) expressed the law of identity as everything is that what it is.

“Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra A est A, B est B.”[14].

Axiom [15]-[30] I is the most general, the most simple and the most far reaching axiom we have today.

3. Results

3.1. Theorem (Number theory and independence I)

Let +1 denote the number 1 at a certain Bernoulli trial t. Let +0 denote the number +0 at a certain Bernoulli trial t.

CLAIM.

In general, it is

$$\frac{+0}{+0} = +1 \quad (5)$$

PROOF BY INDUCTION.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1 = +1 \quad (6)$$

If the number +1 stay that what it is, the number +1, independent of the relation to any other number, there is at least one operation which assures this identity. We obtain

$$+1 \times (1) = +1 \quad (7)$$

The base case.

We prove that the statement before holds for the first natural number 1 at the first Bernoulli trial t.

$$+1 \times \left(\frac{+1}{+1} \right) = +1 \quad (8)$$

The inductive step.

We assume that the above equation is valid even after t=n runs of an experiment every time with a different number. To prove that the equation above is valid in general, we perform another, last experiment. The number at the experiment t= n+1 is equal to 0. We obtain

$$+1 \times \left(\frac{+0}{+0} \right) = +1 \quad (9)$$

QUOD ERAT DEMONSTRANDUM.

According to number theory, 0/0=1.

3.2. Theorem (Number theory and independence II)

Let $+1$ denote the number 1 at a certain Bernoulli trial t . Let $+\infty$ denote the positive infinity at a certain Bernoulli trial t . Let $+0$ denote the number $+0$ at a certain Bernoulli trial t .

CLAIM.

In general, it is

$$+\infty \times 0 = +1 \quad (10)$$

PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1 = +1 \quad (11)$$

If the number $+1$ stay that what it is, the number $+1$, independent of the relation to any other number, there is at least one operation which assures this identity. We obtain

$$+1 \times (1) = +1 \quad (12)$$

The statement before holds even for infinity $+\infty$ at a certain Bernoulli trial t .

$$+1 \times \left(\frac{+\infty}{+\infty} \right) = +1 \quad (13)$$

Changing, we obtain

$$+\infty \times \left(\frac{+1}{+\infty} \right) = +1 \quad (14)$$

Under conditions where $(1/\infty) = 0$ [30] we obtain

$$+\infty \times 0 = +1 \quad (15)$$

QUOD ERAT DEMONSTRANDUM.

3.3. Theorem (Probability theory and independence)

Let $p({}_0A_t)$ denote the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$. Let $p({}_RB_t)$ denote the probability $p({}_RB_t)$ as associated with an event ${}_RB_t$. Let $p({}_0A_t \cap {}_RB_t)$ denote the joint distribution of ${}_0A_t \cap {}_RB_t$ at a certain Bernoulli trial t .

CLAIM.

In general, it is

$$\frac{+0}{+0} = +1 \quad (16)$$

DIRECT PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1 = +1 \quad (17)$$

Classical logic defines the number 1 in a mathematically correct way as

$$1 \times p({}_0A_t) = 1 \times p({}_0A_t) \quad (18)$$

or equally as

$$p({}_0A_t) = p({}_0A_t) \quad (19)$$

The probability that an event ${}_0A_t$ will occur is equal to $p({}_0A_t)$. If we assume that the occurrence of *the event* ${}_0A_t$ is *independent* of anything else, of any other event, of any other probability of the occurrence of another event ${}_RB_t$ denoted by $p({}_RB_t)$ which itself occurs with the probability $p({}_RB_t)$, the probability of an event ${}_0A_t$ will and must stay that what it is, i.e. $p({}_0A_t)$. This must not mean that the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$, is and must be constant. A probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ stays only that what it is, a third has no influence on this fact. There is at least one algebraic operation which assures the independence of the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ from a third. We obtain the following equation

$$p({}_0A_t) \times (1) = p({}_0A_t) \quad (20)$$

In other words, the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ is multiplied by 1, the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ stays that what it is, the probability $p({}_0A_t)$. Thus far, an event ${}_RB_t$, with its own probability of occurrence of $p({}_RB_t)$ can but must not have any influence of the probability of $p({}_RB_t)$. Under conditions of independence of event ${}_0A_t$ and event ${}_RB_t$, the equation before is respected only under circumstances where we accept that $(p({}_RB_t) / p({}_RB_t)) = 1$. Only under these conditions an event ${}_RB_t$, with its own probability of occurrence of $p({}_RB_t)$ has no influence on the occurrence of the event ${}_0A_t$. The equation before changes to

$$p({}_0A_t) \times \left(\frac{p({}_R B_t)}{p({}_R B_t)} \right) = p({}_0A_t) \quad (21)$$

In other words, especially under conditions of independence and due to probability theory, it is

$$\frac{p({}_0A_t \cap {}_R B_t)}{p({}_R B_t)} = \frac{p({}_0A_t) \times p({}_R B_t)}{p({}_R B_t)} = p({}_0A_t) \quad (22)$$

According probability theory, every single even can possess a probability between 0.0 and 1.0, including 0.0 and 1.0. In other words, even if the probability of the occurrence of an event ${}_R B_t$, is equal to $p({}_R B_t)=0$, the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ is independent of this fact, the same probability stays that what it is, $p({}_0A_t)$, and should not change at all since the same is independent of $p({}_R B_t)$. The equation before is and must be valid for any probability value and even in the case if $p({}_R B_t) = 0$, since the same is derived from axiom I. Thus far, let $p({}_R B_t) = 0$, we obtain

$$p({}_0A_t) \times \frac{0}{0} = p({}_0A_t) \quad (23)$$

Whatever the result of the operation $(0/0)$ may be, under conditions of independence, the same operation must ensure that $p({}_0A_t) = p({}_0A_t)$. If an event ${}_0A_t$ is independent of any other event ${}_R B_t$, then this is the case even under conditions if $p({}_0A_t) = 1$. In other words, even if the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ takes the value $p({}_0A_t) = 1$, this has no influence on the independence of events. Under conditions where $p({}_0A_t) = 1$ we obtain

$$1 \times \frac{0}{0} = 1 \quad (24)$$

Thus far, if the law of independence of the probability theory is accepted as generally valid, we must accept that

$$\frac{+0}{+0} = +1 \quad (25)$$

QUOD ERAT DEMONSTRANDUM.

4. Discussion

Today, the division of zero by zero is commonly not used and completely misleading. Does a possible solution of the division of zero by zero exist? Of course, yes [30].

The aforementioned view is associated with the demand of a realistic approach to the solution of problems as associated with indeterminate forms. Relying on axiom I as the starting point of further deduction it is assured, that the results are logically consistent. What are we to make of this? Against this, there is a long tradition of defining the result of the division of 0 by 0 and similar operations. It is uncontroversial (though remarkable) that this approach has not lead to the solution to the problem of indeterminate forms through centuries. In general, it will be helpful to begin any theorem with regards to indeterminate forms with axiom I. In its simplest formulation, this should lead to the desired goal.

5. Conclusion

In summary, $+0/+0=+1$.

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