Pure Bound Field Theory and bound states of light hydrogenlike atoms

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Abstract

We address to the Pure Bound Field Theory (PBFT) we developed earlier (e.g., Kholmetskii AL et al. Eur. Phys. J. Plus 126, 33 (2011), Eur. Phys. J. Plus 126, 35 (2011)), which explicitly takes into account the non-radiating nature of electromagnetic field of quantum bound particles in stationary states, and which allows eliminating the available subtle deviations between experimental and theoretical data in precise physics of light hydrogen-like atoms. In the present paper we show that the specific corrections of PBFT, being introduced into the basic equations of atomic physics, allow two different solutions for stationary energy states of electrically bound system "proton plus electron". One of them corresponds to the ground state of usual hydrogen atom with the averaged radius near the Bohr radius r_B, whereas another stationary state is characterized by the much smaller averaged radius of about $2\alpha^2 r_B \approx 5$ fm (where α is the fine structure constant), and the binding energy about –255 keV. We name this bound system as the "neutronic hydrogen" and discuss possible implications of our results. In particular, we show that the interaction of neutronic hydrogen with matter can explain numerous puzzling facts of low temperature nuclear synthesis.

Keywords: Light hydrogenlike atoms; Pure bound field theory; Low temperature nuclear synthesis.

1. Introduction

In the past years, we have developed the Pure Bound Field Theory (PBFT) for the description of quantum electrically bound systems (first of all, the light hydrogen-like atoms), aimed to eliminate the available subtle deviations between QED calculations and measurement data [1-4]. The development of PBFT was motivated by the known fact that the quantum bound systems in their stationary energy states do not radiate and thus, their total electromagnetic (EM) field contains only a bound component. Hence, the commonly used inhomogeneous wave equation for the operator of vector potential

$$\Box \hat{\boldsymbol{A}} = -\frac{4\pi}{c}\boldsymbol{j} \tag{1}$$

(whose validity is tacitly implied in QED of bound states), becomes in fact non-applicable, since the bound contribution $\hat{\boldsymbol{A}}_b$ alone obeys the Poisson equation, rather than the d'Alembert equation (1); the latter, as is known, is fulfilled only for the sum of bound $\hat{\boldsymbol{A}}_b$ are radiation $\hat{\boldsymbol{A}}_f$ contributions. We would like to point out that this fact had been stressed in ref. [5] about 30 years later than the first formulations of QED in the middle of 20th century. That is why the creators of QED left unchanged eq. (1) in the case of bound systems (where $\hat{\boldsymbol{A}} = \hat{\boldsymbol{A}}_b$) in contradiction with the Bohr correspondence principle. At the same

time, such an inconsistency of QED acquired a latent form, since an EM field equation is not explicitly used in the diagram technique.

The second motivating factor for the development of PBFT is the known fact that both the bound and radiating EM field components bear momentum. Therefore, the absence of EM radiation in a stationary state signifies the lack of momentum component, associated with such a radiating field, which thus could destroy the conservation of total momentum (particles and fields) for the isolated system of bound charges.

It is obvious that the only way to recover the total momentum conservation law in the absence of EM radiation is to modify in an appropriate way the dynamical characteristics of bound particles. In order to understand how such modifications should be made, we suggested to consider, as the classical limit of electrically bound quantum systems, the orbiting hypothetical classical charges with a prohibited radiation. In this way we have found [1] that for a one-body problem, the total momentum conservation law is restored in the absence of radiation, when the rest mass m of electron is multiplied by factor

$$b = 1 + U/mc^2 \text{ , i.e. } m \to bm , \tag{2}$$

while the interactional electric energy between both charges U is multiplied by the Lorentz factor γ of orbiting electron, i.e.

$$U \to \gamma U . \tag{3}$$

Note that $b<1$, since $U<0$ for a bound system.

Introducing these modifications into the Dirac-Coulomb equation for quantum one-body problem, we determine in the limit $Z\alpha<<1$ the quantum counterparts of the factors b and γ as follows [1]:

$$b_n = 1 - (Z\alpha)^2/n^2 , \ \gamma_n = \left(1 - (Z\alpha)^2/n^2\right)^{-1/2} , \tag{4a-b}$$

where α is the fine structure constant, Z is the atomic number, and n is the principal quantum number.

This approach can be naturally extended to the quantum two-body problem with the introduction of four specific PBFT coefficients b_{mn}, b_{Mn}, γ_{mn}, γ_{Mn} (where m, M are the rest masses of bound charges), and further re-postulation of the Breit equations without external field, with the appropriate introduction of the mentioned coefficients into the structure of these equations. This gives rise to the development of PBFT in the form of an effective theory [1, 2]. Thus it does not touch the diagram technique of QED, though the modifications of dynamical parameters of electrically bound charges are also implied in QED of bound states [2]. Such a theory furnishes the same gross, as well as fine structure of atomic energy levels, as those led by the conventional approach for hydrogen-like atoms. However, the approach of PBFT does induce corrections to the energy levels at the scale of hyperfine interactions, which at once remove a number of subtle discrepancies between theory and experiment for hydrogen-like atoms:

- the elimination of the available up-to-date deviation between theory and experiment for the1S–2S interval and 1S spin-spin splitting in positronium [2];

- a perfect quantitative agreement between calculated and experimental data for meso-atoms with respect to the dependence of bound muon decay rate on the atomic number of atom Z [6];

- the derivation of the same proton charge radius r_p=0.841(6) fm for both the classic Lamb shift in hydrogen and in muonic hydrogen [4], which perfectly agrees with the latest measurements of r_p via the Lamb shift in muonic hydrogen r_p =0.84184(67) fm [7, 8]; a similar value has been recently obtained via laser spectroscopy in muonic deuterium [9].

One should stress that for any other problem of precise physics of hydrogen-like atoms, where the agreement between experimental data and QED calculations has been al-

ready achieved before the development of PBFT, the deviation between the results of CED and PBFT calculations either disappears, or furnishes values, lying beyond the measurement precision [2].

These facts allow us to state that at the present time, PBFT is the most successful theory of precise physics of hydrogen-like atoms with respect to the achieved agreement between calculated results and collected experimental data. For convenience, in section 2 we present a short introduction into PBFT and the results obtained.

One should stress that in past years, the substantial deviation between the results of CED calculations [10, 11] and experimental data (e.g., [12-14]) had been observed for the He-like atoms, which becomes more and more seriously increasing with respect to Z, in effect as faster than Z^3 [12]. Currently, we develop the PBFT for the three-body problem, and hope either to eliminate, or diminish the disagreement between theory and experiment for such He-like atoms.

This problem will be considered elsewhere; in the present contribution we disclose one more principal prediction of PBFT, beyond our results presented in refs. [1, 2]: we show in section 3 that the modifications of dynamical parameters for the one-body problem (2) and (3) (aimed to provide the conservation of total momentum in the absence of EM radiation), admit, in effect, two different stationary energy states in the bound system of the proton and the electron, which substantially differ by their properties. We demonstrate the validity of this statement in the simplest (and thus the most convincing) case of the semi-classical limit (i.e., the Bohr atom model), where the equation, determining the velocity of electron in a stationary state, becomes quadratic, as soon as the PBFT factors (2) and (3) are introduced into consideration. The first solution of such quadratic equation corresponds to the usual hydrogen atom, whereas the second solution (determined the new stationary state) gives the averaged radius of stationary orbit of about 5 fm, and is characterized by some unusual properties, discussed below. We suggest to name this bound state of the proton and electron as the "neutronic hydrogen" (nH), to highlight its property to behave in many processes as an electrically neutral particle with nucleonic mass. In section 4 we suppose that neutronic hydrogen had been already discovered in numerous experiments about the so-called "low temperature nuclear synthesis", which makes physical interpretation of such experiments clear and understandable. We finally conclude in section 5.

2. Light hydrogenlike atoms in pure bound field theory

In precise physics of light hydrogenlike atoms, we deal with the two-body problem, with the mass M and charge Ze for the first particle, and mass m and charge e for the second particle. As is shown in ref. [2], the Breit equation without external field for the Schrödinger-like wave function $\psi(\boldsymbol{r})$ is modified in PBFT to the form

$$\left[\frac{p^2}{2mb_{mn}}+\frac{p^2}{2Mb_{Mn}}-\gamma_{mn}\gamma_{Mn}\frac{Ze^2}{r}-\frac{p^4}{8m^3b_{mn}{}^3c^2}-\frac{p^4}{8M^3b_{Mn}{}^3c^2}+U_b\left(\boldsymbol{p}_m,\boldsymbol{p}_M,\boldsymbol{r}\right)\right]\Psi\left(\boldsymbol{r}\right)=W\Psi\left(\boldsymbol{r}\right), \tag{5}$$

where $\boldsymbol{p}=\boldsymbol{p}_m=-\boldsymbol{p}_M$ is the operator of momentum,

$$b_{mn}=1-\frac{(Z\alpha)^2}{n^2}\frac{M}{M+m},\ b_{Mn}=1-\frac{(Z\alpha)^2}{n^2}\frac{m}{M+m}, \tag{6a-b}$$

$$\gamma_{mn}=1-\frac{(Z\alpha)^2}{n^2}\frac{M^2}{(M+m)^2},\ \gamma_{Mn}=1-\frac{(Z\alpha)^2}{n^2}\frac{m^2}{(M+m)^2} \tag{6c-d}$$

are the correcting PBFT factors for the two-body problem, written to the accuracy $(Z\alpha)^2$, W is the energy, and the term $U_b\left(\boldsymbol{p}_m,\boldsymbol{p}_M,\boldsymbol{r}\right)$ is equal to

$$U_b(\boldsymbol{p}_m,\boldsymbol{p}_M,\boldsymbol{r})=-\frac{\pi Ze^2\hbar^2}{2c^2}\left(\frac{1}{b_{mn}{}^2m^2}+\frac{1}{b_{Mn}{}^2M^2}\right)\delta(\boldsymbol{r})-\frac{Ze^2}{2b_{mn}b_{Mn}mMr}\left(\boldsymbol{p}_m\cdot\boldsymbol{p}_M+\frac{\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p}_m)\boldsymbol{p}_M}{r^2}\right)-$$

$$\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{4b_{mn}{}^2m^2c^2r^3}(\boldsymbol{r}\times\boldsymbol{p}_m)\cdot\boldsymbol{\sigma}_m+\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{4b_{Mn}{}^2M^2c^2r^3}(\boldsymbol{r}\times\boldsymbol{p}_M)\cdot\boldsymbol{\sigma}_M-$$

$$\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{2b_{mn}b_{Mn}mMc^2r^3}((\boldsymbol{r}\times\boldsymbol{p}_m)\cdot\boldsymbol{\sigma}_M-(\boldsymbol{r}\times\boldsymbol{p}_M)\cdot\boldsymbol{\sigma}_m)+$$

$$\frac{Ze^2\hbar\gamma_{mn}\gamma_{Mn}}{4b_{mn}b_{Mn}mMc^2}\left(\frac{\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_M}{r^3}-3\frac{(\boldsymbol{\sigma}_m\cdot\boldsymbol{r})(\boldsymbol{\sigma}_M\cdot\boldsymbol{r})}{r^3}-\frac{8\pi}{3}(\boldsymbol{\sigma}_m\cdot\boldsymbol{\sigma}_M)\delta(\boldsymbol{r})\right),$$

where $\boldsymbol{\sigma}$ is the Pauli matrix. We point out that with the correcting factors b_{mn}, b_{Mn}, γ_{mn}, γ_{Mn} equated to unity, eq. (5) transforms to the common Breit equation without external field [15].

In order to solve eq. (5), one can apply the perturbation theory approach in the limit $Z\alpha$<<1, which is realized in the most convenient way via the substitution

$$\boldsymbol{r}=\boldsymbol{r}'/b_{mn}b_{Mn}\gamma_{mn}\gamma_{Mn}\,.$$

This substitution allows to transform eq. (5) to the form

$$\left[-\frac{\hbar^2\nabla^2_{r'}b_{Mn}}{2m}-\frac{\hbar^2\nabla^2_{r'}b_{mn}}{2m}-\frac{Ze^2}{r}+\frac{1}{b_{mn}b_{Mn}\gamma_{mn}{}^2\gamma_{Mn}{}^2}\left(\frac{p^4}{8m^3b_{mn}{}^3c^2}-\frac{p^4}{8M^3b_{Mn}{}^3c^2}+U_b(\boldsymbol{p}_m,\boldsymbol{p}_M,\boldsymbol{r}')\right)\right]\Psi(\boldsymbol{r}')=W'\Psi(\boldsymbol{r}') \quad (7)$$

$$W'=W/b_{mn}b_{Mn}\gamma_{mn}{}^2\gamma_{Mn}{}^2\,,$$

where we have taken into account that $p^2=-\hbar^2\nabla^2_r=-b_{mn}{}^2b_{Mn}{}^2\gamma_{mn}{}^2\gamma_{Mn}{}^2\hbar^2\nabla^2_{r'}$.

By such a way we present the Hamiltonian in eq. (7) as the sum of the Schrödinger-like term and that of the perturbation term.

Eq. (7) represents the basic PBFT equation for the quantum two-body problem [1, 2]. One can show (see, e.g., [1]) that this equation yields the same gross and fine structure of the atomic energy levels, as those furnished by the common approach. As is shown in ref. [2], the pure bound field corrections (expressed as some combination of factors b_{mn}, b_{Mn}, γ_{mn}, γ_{Mn}) to nS levels and fine structure corrections appear only in the order $(Z\alpha)^6\,m/M$; the corrections to hyperfine spin-spin interval, as well as the radiative corrections might have the order $(Z\alpha)^2$.

In particular, the correction δW_{PBFT} to the nS level reads as [2]

$$\delta W_{PBFT}=\frac{2mc^2(Z\alpha)^6}{n^5(M+m)^5}\left(2mM^4+m^2M^3+2m^3M^2\right)-\frac{mc^2(Z\alpha)^6m^2M^3}{n^6(M+m)^5}-\frac{15}{8}\frac{mc^2(Z\alpha)^6}{n^6}\frac{mM^2\left(M^2+m^2\right)}{(M+m)^5}. \quad (8)$$

We see that the terms of eq. (8) scales as n^{-5} or n^{-5}; thus, the correction (8) should be taken into account for 1S and 2S states only. It becomes especially significant for positronium (where m=M):

$$\delta W^{Ps}_{PBFT}=2.95\ \mathrm{MHz}\,,$$

so that it should certainly be considered with regards for the re-estimation of 1S-2S interval in positronium.

Besides, for positronium the Breit potential includes the additional annihilation term (e.g., [15, 16]), whose PBFT correction for orthopositronium reads [2, 4]:

$$\delta W^{Ps}_{ann}=\frac{mc^2(Z\alpha)^6}{4n^5}=4.53\ \mathrm{MHz}\,.$$

Thus the total correction to 1S-2S interval is defined as the sum of δW^{Ps}_{PBFT} and δW^{Ps}_{ann}, i.e.

$$\delta W_{total}^{Ps}(1S-2S) = \delta W_{PBFT}^{Ps} + \delta W_{ann}^{Ps} = 7.48 \text{ MHz}. \quad (9)$$

Now we remind that the modern theoretical value of this interval is equal to [17]

$$E_{1S-2S}^{Ps} = 1\,233\,607\,222.2(6)\,\text{MHz}, \quad (10)$$

and the most precise experimental result is as follows [18]:

$$(E_{ex})_{1S-2S}^{Ps} = 1\,233\,607\,216(3)\,\text{MHz}. \quad (11)$$

One can see that the deviation between the values (10) and (11) amounting to about two times than the uncertainty involved by the measurement of 1*S*-2*S* interval; we have to recall that the discrepancy in question between theory and experiment remained unanswered since a long time.

Whereas, according to PBFT, we have to subtract the value (9) from the calculated value (10), which yields [2]

$$(E_{PBFT})_{1S-2S}^{Ps} = 1\,233\,607\,214.7(6)\,\text{MHz}, \quad (12)$$

in a good agreement with the experimental result (10).

We believe that the result (12) represents the appreciable achievement of PBFT.

Addressing again to eq. (7), one can see that the PBFT corrections to hyperfine spin-spin interval have a minimal order of magnitude $(Z\alpha)^2$. The explicit calculations carried out in ref. [3] yield the following expression for spin-spin interval of *nS* level:

$$(W_{PBFT})_{s-s} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2mM}{(M+m)^2}\right) W_{s-s}, \quad (13)$$

where $W_{s\text{-}s}$ is the spin-spin interval calculated in the standard theory. Now we have to emphasize that eq. (13) still does not determine the total PBFT correction, because the term $W_{s\text{-}s}$ itself contains the ratios $g_m\boldsymbol{\sigma}_m/m$, $g_M\boldsymbol{\sigma}_M/M$ (where g_m, g_M are *g*-factors for particles *m* and *M*, correspondingly), which, in general, are also should be modified in PBFT.

Considering consequently the cases of hydrogen, positronium and muonium, we note that for the first two atoms, the PBFT corrections to the mentioned ratios $g_m\boldsymbol{\sigma}_m/m$, $g_M\boldsymbol{\sigma}_M/M$ are negligible in comparison with the term in brackets of the rhs of eq. (13) [3]. Therefore, this equation is directly applicable to the hydrogen atom and to the positronium. In the case of hydrogen, eq. (13) gives the numerical value of correction for 1*S* level less than 100 kHz, and can be ignored at the present accuracy with regards to the determination of hyperfine structure (hfs).

For 1*S* state of positronium eq. (13) yields:

$$(W_{PBFT})_{s-s}^{Ps} = \left(1 - \frac{(Z\alpha)^2}{2n^2}\right) W_{s-s}^{Ps}, \quad (14)$$

where

$$W_{s-s}^{Ps} = 203\,391.7(6)\,\text{MHz} \quad (15)$$

is the value presently calculated [17].

The corresponding experimental data are 203 389(2) [19] and 203 387(2) [20], which disagree with the value (15) by approximately 2 standard deviations.

Now eq. (14) allows us to compute the hyperfine spin-spin interval in positronium, corrected in PBFT, using the numerical value (15):

$$(W_{PBFT})_{s-s}^{Ps} = 203\,386(1)\,\text{MHz}. \quad (16)$$

This result is in a neat agreement with the experimental data and definitely represents one more appreciable achievement of PBFT.

Finally, applying eq. (13) to muonium, we have to involve additionally the PBFT correction to ratio of the muon magnetic moment to its mass. As is shown in ref. [3], the PBFT re-estimation of the magnetic dipole moment of muon μ_μ reads as

$$\left(\mu_\mu\right)_{PBFT} = \left(1 - \frac{(Z\alpha)^2}{n^2}\frac{2mM}{(M+m)^2}\right)\mu_\mu, \qquad (17)$$

where μ_μ is the magnetic dipole moment of muon calculated in the common theory.

Further we point out that the energy $W_{s\text{-}s}$ in eq. (13) is linearly proportional to the magnetic dipole moment, and introducing the PBFT correction (17), we find that the correcting factors in eq. (13) and in eq. (17) (i.e. the terms in the bracket of rhs) cancel each other. Hence, the total correction to hyperfine structure (hfs) interval in muonium disappears.

We would like to stress the known fact that the value of 1*S* spin-spin interval in muonium calculated in the common way perfectly agrees with the measured data (see, e.g. [17]). This result particularly backs up our statement made in the introduction section: in all cases, where QED results are in a quantitative agreement with the measurement data, the PBFT corrections either disappear, or give negligible contributions beyond the measurement precision.

We can add that for muonic hydrogen, the nuclear size effect is much larger than any PBFT corrections, so that the re-estimation of Zemach proton radius via hfs in muonic hydrogen leads to the difference from the common value about $1.0\cdot10^{-3}$ fm [3], which is smaller than the present uncertainty in determination of this parameter.

Finally, with respect to the radiation corrections to the atomic energy levels (where the approximation of the one-body problem is well fulfilled), the PBFT modifications (2), (3) in the input of QED expressions should be accounted for. Below we present the corrections to the Lamb shift for light hydrogen-like atoms, which emerge within PBFT. In particular, for the 2*S*-2*P* Lamb shift we obtain [2, 4]

$$\left(L_{PBFT}\right)_{2S-2P} = \gamma_{n=2}^2 L_{2S-2P} = \left(1 - \frac{(Z\alpha)^2}{4}\right)^{-1} L_{2S-2P}, \qquad (18a)$$

where L_{2S-2P} is the value of Lamb shift calculated in QED, and we have used eq. (4b) at n=2. Thus the relative PBFT correction to the 2*S*-2*P* Lamb shift is equal to

$$\frac{\delta L_{2S-2P}}{L_{2S-2P}} = \frac{\left(L_{PBFT}\right)_{2S-2P} - L_{2S-2P}}{L_{2S-2P}} = \left(1 - \frac{(Z\alpha)^2}{4}\right)^{-1} - 1 = 1.33\cdot10^{-5} \quad \text{(at } Z\text{=1)}. \qquad (18b)$$

The correction (18b) substantially exceeds the relative measurement precision of the 2*S*-2*P* Lamb shift for Doppler-free two-photon laser spectroscopy (see, e.g., [21]) and in the case of hydrogen, it has the same order of magnitude, just like that of the relative nuclear structure contribution. Therefore, the correction (18b) does affect the value of the proton charge radius r_E calculated via the classic Lamb shift.

In order to determine the proton size in the PBFT framework, it is sufficient to use the quadratic parametrization for the 2*S*-2*P* Lamb shift [2, 4], i.e.

$$L_{2S-2P} = A + B r_E^{\,2}, \qquad (19)$$

where in the case of hydrogen the coefficients A and B are as follows:

$$A\text{=}1057695.05 \text{ kHz}, \; B\text{=}195.750 \text{ kHz}. \qquad (20a\text{-}b)$$

In PBFT, eq. (19) is modified to the form (see eq. (18a))

$$L_{2S-2P} = \gamma^2 A + \gamma^2 B \left(r_{PBFT}\right)_E^{\,2}. \qquad (21)$$

Hence, combining eqs. (19) and (21), we derive the relationship between the proton charge radius, evaluated in PBFT ($\left(r_{PBFT}\right)_E$ and the commonly adopted value r_E:

$$\left(r_{PBFT}\right)_E = \left(\frac{r_E^{\,2}}{\gamma^2} + \frac{A\left(1-\gamma^2\right)}{B\gamma^2}\right)^{1/2} = \left(\frac{r_E^{\,2}}{\gamma^2} - (Z\alpha)^2\frac{A}{4B}\right)^{1/2}, \qquad (22)$$

where we have taken into account eq. (4b) for the factor γ at n=2.

Substituting into eq. (22) the numerical values (20) along with the CODATA-2014 value r_E=0.8751(61) fm [22], we obtain

$$\left(r_{PBFT}\right)_E = 0.834(6) \text{ fm}.$$

This estimation is already much closer to the proton size derived in refs. [7-9], than the CODATA value of r_E. At the same time, we recall that the CODATA value of the proton charge radius incorporates the experimental data in both particle physics and atomic physics, and, in general, is less than the proton size derived from the classic Lamb shift solely. In particular, the modern data on 2*S*-2*P* Lamb shift in hydrogen obtained by various authors within the standard evaluation (see refs. [17, 23] and references therein) define the range of variation of the value of r_E between 0.875 fm and 0.891 fm. Thus taking the midpoint r_E=0.883 fm, we obtain

$$\left(r_{PBFT}\right)_E = 0.841(6) \text{ fm},$$

which is in full harmony with the modern measurement results of the proton size [7, 8].

Now we point out that for muonic hydrogen, the relative PBFT correction to 2*S*-2*P* Lamb shift (18b) is much less than the relative contribution of the nuclear size effect. This means that this correction practically does not affect the proton size evaluated in the muonic hydrogen via the common approach, as compared to what one comes out with, though the PBFT. In particular, using the parameterization (1) of ref. [7] with the numerical coefficients multiplied by $\gamma_{n=2}^2$ times according to eq. (18a), we derive the difference between $\left(r_{PBFT}\right)_E$ and r_E to be about $3\cdot10^{-3}$ fm. This value lies below the uncertainty of the muonic experiments [7-9] and can be ignored.

Thus, the exact coincidence of the proton size obtained via the classic Lamb shift and laser spectroscopy of muonic hydrogen [7, 8] represents, in our opinion, a very important achievement of PBFT.

What is more, for the 1*S* Lamb shift in hydrogen, our approach also predicts the decreased proton size in comparison with the CODATA value. Omitting details of our calculations, which can be found in ref. [2], we would like to present straight the final result of re-estimation of r_E via the 1*S* Lamb shift:

$$\left(r_{PBFT}\right)_E = 0.846(22) \text{ fm}.$$

Note that this value well agrees with both the result of calculation of the proton size via the classic Lamb shift experiments and that obtained via muonic hydrogen spectroscopy.

Thus, we reveal that PBFT seems to be a crucial theory for the resolution of the proton size puzzle. In this respect, further experimental verification of PBFT looks topical, because the results of this theory actually can play the important role in the achievement of further progress in modern physics of simple atoms. Such verification can be done based on one more prediction of PBFT, which admits, in effect, two different stationary energy states for the bound system "proton and electron", which substantially differ by their properties, as described in the next section.

3. New bound state for hydrogenlike atoms

First of all, let us show, how eqs. (2) and (3) are derived in the classical limit of quantum one-body problem with prohibited radiation [1]. In this way, we analyze the motional equation for a charge *e* with a rest mass *m*, orbiting about a heavy host charge *Ze* with a rest mass $M\to\infty$, being immovable in the frame of observation. To handle this problem, we use the law of conservation of the total momentum

$$\boldsymbol{P}_m + \boldsymbol{P}_M + \boldsymbol{P}_{EM} = const\,, \tag{23}$$

where $\boldsymbol{P}_m$, $\boldsymbol{P}_M$ are the mechanical momenta of the orbiting and the host particle respectively, and $\boldsymbol{P}_{EM}$ is the interaction EM momentum. Denoting $\boldsymbol{v}$ the velocity of particle

e, and taking into account that for the bound EM field, $\boldsymbol{P}_{EM} = \gamma U/c^2\,\boldsymbol{v}$ [1], we obtain from eq. (23)

$$\frac{d}{dt}\gamma m\boldsymbol{v} + \frac{d}{dt}\frac{\gamma U}{c^2}\boldsymbol{v} = -\frac{d\boldsymbol{P}_M}{dt}\,; \tag{24}$$

here $U = -Ze^2/r$ is the interaction electric energy of both charges. The *rhs* of eq. (24) represents the force acting on the host particle. For a circular motion of particle *e*, its velocity $\boldsymbol{v}$ is orthogonal to the line joining both particles at any moment of time. Hence according to the Heaviside expression (see e.g. [5]), the electric field $\boldsymbol{E}_b$ of orbiting particle is equal to

$$\boldsymbol{E}_b = \gamma\, e\boldsymbol{r}/r^3\,, \tag{25}$$

at the location of host particle, where *r* being the radius of the orbit. Combining eqs. (23)-(25), we obtain

$$\frac{d}{dt}\gamma\left(m + \frac{U}{c^2}\right)\boldsymbol{v} = \gamma\frac{Ze^2\boldsymbol{r}}{r^3}\,. \tag{26}$$

The equation of motion (26) written within pure bound field constraint differs from the corresponding equation that would be written for the one-body problem in usual classical electrodynamics in two points: first, the rest mass *m* is replaced by $m_b = \left(m + U/c^2\right) = mb$ (see eq. (2)); second, the interaction energy Ze^2/r becomes $\gamma Ze^2/r$ (see eq. (3)).

For a circular motion, where $|d\boldsymbol{v}/dt| = v^2/r$, this equation acquires the simple scalar form

$$bmv^2 = Ze^2/r\,. \tag{27}$$

Next, we would like to determine a stationary energy state for the hydrogenlike atom. For simplicity, we apply a semi-classical approach, where the classical equation (27) is complemented by the second Bohr postulate, which should be also modified in comparison with its standard formulation due to the PBFT corrections (2) and (3). In order to make this, we consider the second Bohr postulate as the requirement of integer number of de Broglie wavelengths λ of electron along the length of its orbit $2\pi r$, i.e.

$$n\lambda = 2\pi r\,, \tag{28}$$

where *n*=1, 2, 3…, and

$$\lambda = h/P_m \tag{29}$$

according to the de Broglie relationship. The immediate problem is that eq. (29) is applicable, in general, to the non-relativistic limit $v<<c$ only, where $P_m \approx mv$. Thus, we should be careful, introducing PBFT corrections into this equation. Anyway, the replacement (2) does not depend on the velocity magnitude, and can be immediately introduced already in the non-relativistic limit, leading to $\lambda_b = h/bmv$.

Further, the momentum of particle *m*, moving with the relativistic velocity $\boldsymbol{v}$ in some inertial frame of observation K_0, takes in PBFT the form [1]

$$\boldsymbol{P}_{0m} = \gamma bm\boldsymbol{v}\,, \tag{30}$$

and we would like to determine conditions, where eqs. (30) and (29) can be used jointly. For this purpose we consider the Lorentz transformation from the frame K_0 to another inertial frame K with their relative velocity $\boldsymbol{u}$ being collinear to $\boldsymbol{v}$. Using the Lorentz transformation for the energy-momentum, we obtain

$$\boldsymbol{P}_m = \gamma_u \left(\boldsymbol{P}_{0m} + \frac{\boldsymbol{u} E_0}{c^2} \right), \tag{31}$$

where $\gamma_u = \left(1 - u^2/c^2\right)^{-1/2}$. Taking into account the equality $E_0 = \sqrt{m^2c^4 + {P_{0m}}^2 c^2}$, we can present eq. (31) in the form

$$\boldsymbol{P}_m = \gamma_u \boldsymbol{P}_{0m} \left(1 + \frac{\boldsymbol{u}}{c} \sqrt{1 + \frac{m^2 c^2}{{P_{0m}}^2}} \right). \tag{32}$$

One can see that only under the equality m=0 (i.e., for the photon, where $P=h\boldsymbol{k}$, $\boldsymbol{k}$ being the wave vector), eq. (32) along with the relationships $\lambda_0 = h/P_0$, $\lambda = h/P$ transforms into the well-known Doppler expression for the wavelength of light, which can be considered as the Lorentz transformation for the time component of the wave four-vector (e.g., [24]). However, at any finite rest mass m, the equality $\lambda = h/P_m$, in general, is not fulfilled in an arbitrary inertial frame. The exception is the ultra-relativistic case, where the ratio mc/P_{0m} becomes much smaller than unity and, like in the case of photon, this ratio can be equated to zero. Hence, the relationship $\lambda = h/P_m$ is again applicable to any inertial observer and we get the right to substitute the relativistic equation (30) into this relationship. Thus, in such ultra-relativistic limit we derive from eqs. (28)-(30)

$$\frac{n}{\gamma_b m v} = 2\pi r, \text{ or}$$

$$\gamma_b m v r = n\hbar, \tag{33}$$

where $\hbar = h/2\pi$.

Now it is important to emphasize that the new bound state of light hydrogenlike atoms, which we derive below, corresponds to the ultra-relativistic velocity of the electron, where γ>>1, so that eq. (33) is indeed well applicable. Of course, this equation can be always applied to the non-relativistic limit (where $\gamma \to 1$) for the determination of gross energy structure of usual hydrogenlike atoms.

At the same time, it is also important to stress that eq. (33) remains incomplete, and as it is, it says nothing, for example, about a fine structure of the atomic levels, which is defined via further development of PBFT [1, 2]. However, the obtained equation (33) is sufficient for the purpose of the present paper. In particular, we get from this equation.

$$r = n\hbar/\gamma_b m v, \tag{34}$$

and substituting this value into eq. (27) we obtain:

$$v = \frac{Ze^2\gamma}{n\hbar} = \frac{Z\alpha\gamma c}{n}. \tag{35}$$

This equation differs from the conventional Bohr's equation $v = Z\alpha c/n$ (which yields only the single solution for v) by the presence of factor γ, which stems from the replacement $Ze^2/r \to \gamma\, Ze^2/r$ applied in PBFT. Designating β=v/c, we further derive from eq. (35):

$$\beta^2 = \frac{(Z\alpha)^2 \gamma^2}{n^2} = \frac{(Z\alpha)^2}{n^2\left(1 - \beta^2\right)},$$

which yields two solutions:

$$\beta_H{}^2 = \frac{1}{2}\left(1 - \sqrt{1 - 4(Z\alpha)^2/n^2}\right) = \frac{1}{2}(1 - R), \text{ and} \tag{36}$$

$$\beta_N{}^2 = \frac{1}{2}\left(1 + \sqrt{1 - 4(Z\alpha)^2/n^2}\right) = \frac{1}{2}(1 + R), \tag{37}$$

where we designated

$$\sqrt{1-4(Z\alpha)^2/n^2}=R. \tag{38}$$

In order to disclose the physical meaning of two different solutions (36) and (37), we determine the binding energy of a light hydrogenlike atom within the PBFT approach, which in the approximation of the one-body problem represents the sum of kinetic and potential energy of the bound electron, i.e. [1]

$$E_b=(\gamma-1)bmc^2-\gamma\frac{Ze^2}{r}=(\gamma-1)bmc^2-\gamma(1-b)mc^2=mc^2(2\gamma b-\gamma-b), \tag{39}$$

where we have used the equality

$$Ze^2/rmc^2=1-b. \tag{40}$$

Further on, we involve the equalities

$$\gamma=\frac{1}{\sqrt{1-\beta^2}},\ b=\frac{1}{1+\beta^2}, \tag{41a-b}$$

where in the derivation of eq. (41b) we have used the motional equation of electron in the form $b\beta^2=Ze^2/rmc^2$ and applied eq. (40).

Thus, the first solution (36) yields

$$\gamma_H=\frac{1}{\sqrt{1-\frac{1}{2}(1-R)}}=\left(\frac{2}{1+R}\right)^{1/2},\ b_H=\frac{1}{1+\frac{1}{2}(1-R)}=\frac{2}{3-R}. \tag{42a-b}$$

For light hydrogenlike atoms, where $Z\alpha<<1$, we can present eq. (38) in the form

$$R\approx 1-2(Z\alpha)^2/n^2, \tag{43}$$

ignoring the terms of the order $(Z\alpha)^4$ and higher. Hence, to the same accuracy of calculations, eqs. (42a-b) yield:

$$\gamma\approx\left(\frac{1}{1-(Z\alpha)^2/n^2}\right)^{1/2}\approx 1+\frac{(Z\alpha)^2}{2n^2},\ b\approx\frac{2}{3-R}\approx\frac{1}{1+(Z\alpha)^2/n^2}\approx 1-\frac{(Z\alpha)^2}{n^2}. \tag{44a-b}$$

Substituting eqs. (44a-b) into eq. (39), we derive

$$(E_b)_H=-mc^2\frac{(Z\alpha)^2}{2n^2}, \tag{45}$$

which corresponds to the usual ground state of light hydrogenlike atoms, and we supplied this solution by the subscript "*H*" (hydrogen).

Finally, using eqs. (27) and (36), we show that in the case of hydrogen (Z=1), the averaged radius of stationary orbit is equal to the Bohr radius $r_B=\hbar/mc\alpha$.

Next, consider the second solution (37) for the parameter β. Substituting eq. (37) into eqs. (41a-b), we obtain:

$$\gamma_N=\frac{1}{\sqrt{1-\frac{1}{2}(1+R)}}=\left(\frac{2}{1-R}\right)^{1/2},\ b_N=\frac{1}{1+\frac{1}{2}(1+R)}=\frac{2}{3+R}. \tag{46a-b}$$

Thus, eqs. (46a-b) determine a new bound state in the system "nucleus and electron", which essentially differs from the usual hydrogenlike atom. For example, at $Z\alpha<<1$, where we can use the equality (43), the factor γ becomes much larger than unity:

$$\gamma_N \approx \frac{n}{Z\alpha} >> 1, \tag{47}$$

which corresponds to the ultra-relativistic case. For example, at Z=1 (hydrogen) and for the ground state n=1, we get γ_N=137. This result confirms the validity of application of eq. (33), which, as we have mentioned above, is legitimate in the ultra-relativistic limit.

Concurrently, the result obtained signifies that at $Z\alpha$>>1, the strong inequality γ_N>>1 is not any longer fulfilled (see eq. (47)), so that the ultra-relativistic limit becomes inapplicable. Therefore, our basic equation (33) is inapplicable, too, due to the violation of the de Broglie relationship $\lambda = h/P$ for different observers moving at intermediate relative velocities (i.e., when neither the non-relativistic limit, nor the ultra-relativistic limit are fulfilled). This means that the entire approach, which we developed, is valid only for the light hydrogenlike atoms, where $Z\alpha$<<1.

Further on, substituting eq. (43) into eq. (46b), we find

$$b_N \approx \frac{1}{2}\left(1+\frac{(Z\alpha)^2}{2n^2}\right), \tag{48}$$

and combining eqs. (39), (47) and (48), we obtain the binding energy of the new bound state as

$$(E_b)_N \approx mc^2\left(\frac{(Z\alpha)}{2n}-\frac{1}{2}-\frac{(Z\alpha)^2}{4n^2}\right) = -\frac{mc^2}{2}+mc^2\frac{(Z\alpha)}{2n}-mc^2\frac{(Z\alpha)^2}{4n^2} \approx -\frac{mc^2}{2}+mc^2\frac{(Z\alpha)}{2n} \tag{49}$$

in the lowest order in $Z\alpha$.

Now we determine the radius of stationary orbit of the new bound state, substituting eqs. (37) and (45a-b) into eq. (33):

$$r = \frac{\sqrt{1-R}(3+R)\alpha n}{2\sqrt{1+R}} r_B . \tag{50}$$

Further, using the expression $R = \sqrt{1-\frac{4(Z\alpha)^2}{n^2}} \approx 1-\frac{2(Z\alpha)^2}{n^2}-\frac{2(Z\alpha)^4}{n^4}$, we decompose eq. (50) to the accuracy $(Z\alpha)^4$:

$$r \approx 2Z\alpha^2 r_B + \frac{Z^3\alpha^4}{n} r_B . \tag{51}$$

At Z=1 (proton), the value of this radius is about 5 fm: that is, the electron "is in motion" actually very close to the proton, and with a velocity very close to c. It is also interesting to notice that r depends on n only in the term of the order α^4, and thus, it decreases very slowly with an increasing n.

From the physical viewpoint, the fact of huge (ultrarelativistic) speed of electron with the extremely low "radius" of its orbit (51) is understood, when we remind that in this bound state, the electric interactional energy of the electron and proton is multiplied in PBFT by the Lorentz factor (47) and is also huge. Thus, the difference of huge electron's energy, related to its ultrarelativistic motion, and huge interactional energy becomes finite, though still very large in the atomic energy scale (about a half of the electron's rest mass, see eq. (49)).

Further, eq. (49) discloses one more essential feature of the new bound state: unlike to the case of a usual hydrogenlike atom, the binding energy of the state decreases with the increase of n. This means that the state with the lowest energy corresponds to the limit $n\to\infty$, which, however, cannot be exactly fulfilled, because in this limit $R\to 1$ (see eq. (38)), and v becomes equal to c (see eq. (37)). We can name such a state with the lowest

energy as "asymptotic state" (AS). One can see that at the asymptotic state, we have almost exact equalities

$$E_b = -mc^2/2, \tag{52}$$

$$r = 2Z\alpha^2 r_B. \tag{53}$$

The fact that the lowest energy state corresponds to the limit $n\to\infty$ indeed looks exotic, like some other properties of the new bound state. At the same time, it does not signify the presence of any inconsistency, because the total energy of the state with $n\to\infty$ tends to a finite value. This is related to the fact already mentioned above that the increase of positive motional energy of electron by γ_N times (see eq. (47)) is counterbalanced by the corresponding increase of the value of the negative interaction electric energy by the same factor γ_N, which thus keeps finite the binding energy of the system (52).

The value (52) exceeds many orders of magnitude the typical energies of ionization of atoms; therefore, in many processes of atomic physics and solid state physics, the new bound system "proton and electron" behaves as a neutral particle with the rest mass, comparable with nucleonic mass and the size, comparable with nucleonic size. Due to this reason, we suggest to name the revealed new bound state of the proton and electron as "neutronic hydrogen", and designate it as "nH".

Finally, we point out that the binding energy of the neutronic hydrogen ($(E_b)_N = -mc^2/2 \approx$-255 keV) is much lower than the binding energy for the hydrogen ($(E_b)_\mathrm{H}=$
–13.6 eV), and its seems at the first glance that a spontaneous radiation transition between the energy levels of H and energy levels of nH becomes possible, with the photon energy

$$h\nu = (E_b)_H - (E_b)_N \approx 255 \text{ keV}. \tag{54}$$

However, one can show that such a radiation process is prohibited. First of all, this is explained by the fact that in PBFT, the electric interactional energy is multiplied by the Lorentz factor γ (see eq. (3)), which is near unity for the electron, having the averaged velocity $v\sim\alpha c$ in the ground state of hydrogen. Thus, such a non-relativistic electron cannot "know" about another set of stationary energy states for the neutronic hydrogen, realized for the ultra-relativistic bound electron with the huge interactional energy $\gamma_N U$ (where $\gamma_N>>1$ as defined by eq. (47)). This result is quite understandable in the semi-classical approach, where under no circumstances the non-relativistic electron in hydrogen could emit the photon with the energy comparable with its rest energy (54), because such a process would immediately violate the energy conservation law. In the quantum domain, we could, at the first glance, imagine that this radiation process is possible due to a huge fluctuation of the energy of the bound electron in the hydrogen atom (admitted, in general, by the Heisenberg uncertainty relation for a very short time interval). However, here we have to take into account that due to the irradiation of photon with the energy (54), the electron would acquire a huge recoil momentum $\boldsymbol{p}_{recoil} = -h\boldsymbol{k}$. One can easily calculate that the energy of electron obtained due to the recoil becomes many orders of magnitude larger than its binding energy in hydrogen atom. In these conditions, the bound state of the proton and electron would be destroyed with obvious violation of the energy conservation law.

Therefore, for the usual hydrogen the state with n=1 remains stationary, and an isolated hydrogen atom in the ground state and isolated "neutronic hydrogen" in the AS state both exist forever. In principle, the non-radiation transitions from the usual hydrogen to "neutronic hydrogen" is admitted, e.g., under the atomic collisions; at the same time, the probability of ionization of hydrogen due to such collisions is many orders of magnitude larger than the probability of formation of "neutronic hydrogen". In order to demonstrate the validity of this statement, we can consider the hydrogen gas and estimate its

temperature T_i, corresponding to its ionization energy E_i=13,6 eV (i.e., when the kinetic energy of colliding atoms becomes larger than E_i), as well as its temperature in the case, where the formation of neutronic hydrogen becomes possible due to atomic collisions. In the latter case we have to take into account that in the absence of radiation transition from the hydrogen to neutronic hydrogen, the energy release about 255 keV needed for the formation of neutronic hydrogen can happen only in such collision processes, where the transmission of the resultant energy 255 keV from one atom to another atoms becomes probable. This means, that under single-entry collisions, the kinetic energy of colliding atoms should have, as minimum, the same order of magnitude of 255 keV.

One can readily estimate that for the hydrogen gas, the ionization energy 13.6 eV corresponds to its temperature about $1.6\cdot10^5$ K, whereas the energy 255 keV corresponds to the temperature $\approx2\cdot10^9$ K, which few orders of magnitude exceeds the temperature inside the Sun. Thus, an actual formation of neutronic hydrogen in gases, in liquids and in solid materials can be expected only in highly non-equilibrium processes as a seldom event, when the kinetic energy of some individual atoms can many times exceed the average kinetic energy of other constituents. Further research on this subject is definitely needed.

4. Neutronic hydrogen: possible experiments

Exploring the problem of experimental verification of formation of neutronic hydrogen, at the moment one can suggest three methods:

spectroscopy of neutronic hydrogen;

measurement of spin of neutronic hydrogen;

registration of secondary effects, accompanied the interaction of neutronic hydrogen with matter.

A possibility of spectroscopy of nH is related to the fact that in a stationary situation, this bound system is in the asymptotic state. Therefore, the neutronic hydrogen can absorb EM radiation, corresponding to the transition from AS to any state with a finite value of n. For example, at n=1, the difference between energy levels is

$$\Delta E_{n=1} = E_{n=1} - E_{AS} = \frac{1}{2} mc^2\alpha \approx 1.9\ keV. \qquad (55)$$

Similarly, we obtain $\Delta E_{n=2} = 0.93\ keV$, $\Delta E_{n=3} = 0.63\ keV$, etc.

The estimation of a width of these energy levels lies outside the scope of the present paper. We nevertheless notice that the energy range of EM radiation $\leq$2 keV is quite inconvenient from the experimental viewpoint due to its high absorption ability in a matter.

Considering the absorption of a visible light by the neutronic hydrogen (say, at the energy near 0.5 eV), we see from eq. (48) that this corresponds to the energy levels with $n\approx4\cdot10^3$. At this energy range, the difference between two neighbour levels is equal to

$$\Delta E = E_n - E_{n+1} \approx mc^2\alpha/2n^2 \approx 4\cdot10^{-4}\ eV\ . \qquad (56)$$

This value is very small indeed, and even if a natural width of the energy levels is very small, too (and this can be actually the case, due to the time dilation effect for bound electron with the huge Lorentz factor $\gamma\approx5\cdot10^5$ at $n\approx4\cdot10^3$), the broadening of the given lines due to experimental factors (e.g., the Doppler effect) will be anyway substantially larger than the difference (56). Therefore, in real measurements, the neighbouring lines at $n\approx10^3$ and higher will be merged into a wide absorption band, covering the range of visible light and lower frequencies of EM radiation.

At the same time, the possibility to perform spectrometric measurements implies a sufficiently large number of nH atoms. At the moment, the problem of creation of such atoms needs further research, and only in the case of its successful solution, a more detailed insight to spectroscopy of neutronic hydrogen will become topical.

The same statement is relevant with respect to the possibility to measure the spin of nH, which can be equal either 0, or 1, so that the neutronic hydrogen represents a boson. In general, the intensity of the neutronic hydrogen beams in spin measurements can be much smaller than the intensity of the beam in spectroscopic measurements. Thus, one can expect to carry out such measurements in a near future, when at least a reproducible formation of nH will be achieved.

At the present stage, a possibility to discover the neutronic hydrogen via its interaction with matter looks more promising, because in such interactional processes, even single events of nH formation can be detected, at least in principle. As we have mentioned above, these atoms can be created in some highly energetic non-equilibrium processes, e.g., in sparks in a gas atmosphere, containing light atoms, in the interaction of hydrogen and other light atoms with a metal surface during, e.g., electrolytic processes, where such metals serve as electrodes, under irradiation of substances, containing hydrogen (e.g., water), with high-energy γ-quanta, etc.

The experiments of such a kind belong to the known area of empiric research of the so-called "low temperature nuclear synthesis", which attracted considerable attention at the end of 1980s and beginning of 1990s of past century. Later, this branch of experimental activity and attempts of interpretation of the results obtained became highly speculative, and publications on this subject disappeared in mainstream physics journals. As we are aware, one of the last reviews of experiments for low-temperature nuclear synthesis, published in a recognized journal, had been presented in ref. [25]. Now we stress the following standpoints of this review:

- the events of cold nuclear transformations *are actually observed*;
- these transformations can be accompanied (or not) by *low-intensity flux of neutrons and (or) γ-quanta;*
- the successful experiments occur often non-reproducible in different laboratories;
- the origin of energy, forced nuclei to approach each other and to induce their interaction and transformation, *remains non-explained*, and the available models (implying huge local electric fields inside condensed matter under some specific conditions) are not supported by any convinced calculations.

In our opinion, the latter obstacle seems the most crucial, and it is totally eliminated, if we assume the formation of neutronic hydrogen in all observed processes of low-temperature nuclear transformation. In this case, instead of adoption of the hypothesis about nuclei transformations due to local electric fields inside some materials (which, without any further speculations, *must be recognized impossible* at low temperatures), we necessarily come to deal with the interaction of nH with nuclei, which in many situations, due to its zero electric charge and nucleonic size, can be described similarly to the interaction of ordinary neutrons with nuclei, where nuclear mutation with further irradiation of γ-quanta and/or neutrons represent quite expected effects.

At the same time, a detailed analysis of the formation of neutronic hydrogen and its interaction with matter lies outside the scope of the present paper.

5. Conclusion

In a series of our previous works we have shown that the corrections of PBFT to the standard equations of atomic physics, which explicitly take into account the non-radiative nature of electromagnetic field of electrically bound quantum particles, occur very successful in the elimination of available subtle disagreements between theory and experiment for light hydrogen-like atoms. In the present paper we additionally show that the introduction of PBFT corrections into the basic equations of atomic physics allows two different solutions for the stationary energy states; one of them corresponds to the hydrogen atom, whereas another one is characterized by a very small average radius of

stationary orbit (about 5 fm) and an upmost binding energy to be almost equal to the half of electron rest mass with the reverse sign. We named this bound state as the neutronic hydrogen, because in many processes of its interaction with a matter, it behaves similarly to the neutron.

In the present paper we disclosed the principal features of the new bound state, using the simplest approach of the Bohr atom model. For more detailed analysis it is necessary, as a further step, to solve the Dirac equation for electron in the Coulomb potential with the introduction of PBFT corrections, and to determine the energy states and corresponding wave functions for the new bound state. This represents a separate problem, which lies outside the scope of the present paper. Now it is important to stress that just the formation of neutronic hydrogen and its interaction with nuclei can be (or even must be) responsible for observation of so-called "low temperature nuclear transformations", which does not require a huge energy, like in direct nuclear interactions.

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