Does Heisenberg's Uncertainty Principle Predict a Maximum Velocity for Anything with Rest-Mass below the Speed of Light ?

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Abstract

In this paper we derive a maximum velocity for anything with rest-mass from Heisenberg's uncertainty principle. The maximum velocity formula we get is in line with the maximum velocity formula suggested by Haug in a series of papers. This supports the assertion that Haug's maximum velocity formula is useful in considering the path forward in theoretical physics. In particular, it predicts that the Lorentz symmetry will break down at the Planck scale, and shows how and why this happens. Further, it shows that the maximum velocity for a Planck mass particle is zero. At first this may sound illogical, but it is a remarkable result that gives a new and important insight into this research domain. We also show that the common assumed speed limit of $v < c$ for anything with rest-mass is likely incompatible with the assumption of a minimum length equal to the Planck length. So one either has to eliminate the idea of the Planck length as something special, or one has to modify the speed limit of matter slightly to obtain the formula we get from Heisenberg's uncertainty principle.

Key words: Heisenberg's uncertainty principle, maximum velocity of matter, reduced Compton wavelength

1 Introduction

Haug [1, 2, 3, 4, 5] has recently suggested a new maximum velocity for subatomic particles (anything with mass) that is just below the speed of light. The formula is given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{1}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the particle we are trying to accelerate and l_p is the Planck length [7, 8]. This formula can be derived from special relativity by simply assuming that the maximum frequency one can have is the Planck frequency $\frac{c}{l_p}$, or that the shortest wavelength possible is the Planck length. We will also get the same formula if we assume that the ultimate fundamental particle has a spatial dimension equal to l_p and always is traveling at the speed of light; this is a model outlined by [6, 1].

This maximum velocity for anything with rest-mass was first predicted by Haug in 2014. It was first derived from two postulates in atomism. The theory leads to the same mathematical end results as special relativity theory, as long as one use Einstein-Poincaré synchronized clocks. However, at that time Haug had not yet linked his theory up to key concepts of Max Planck. Here the key understanding given in 2014 will lead to the same formula described above.[1]

In this paper we will show that the same formula implicitly agrees with a new result that comes out of Heisenberg's uncertainty principle when combined with a key insight from Max Planck.

2 Heisenberg's Uncertainty Principle Leads to a Maximum Velocity for Anything with Rest-Mass

The original Heisenberg's uncertainty principle formulation [9] is given by

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[1] Back then I only derived my maximum velocity formula with a link to one-sided relativistic Doppler shift, based on a slightly different clock synchronization procedure and therefore got slightly different results than those given here.

$$\sigma_x \sigma_p \geq \hbar \tag{2}$$

where σ_x is considered to be the uncertainty in the position, σ_p is the uncertainty in the momentum, and $\hbar$ is the reduced Planck constant. Alternatively, one could use the more modern Kennard [10] version of the Heisenberg's uncertainty principle formula as we have done in section 4. The difference is $\hbar$ or $\frac{\hbar}{2}$ in the formulation. Even if this results in a very small difference in maximum velocity, it still leads to exactly the same important main conclusions. The rest-mass of an elementary subatomic particle is given by

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{3}$$

For an electron, for example, we have

$$m = \frac{\hbar}{\bar{\lambda}_e} \frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg} \tag{4}$$

We will assume that the minimum uncertainty in the position of any elementary particle (or any other object) is the Planck length. Setting $\sigma_p = l_p$ and the momentum σ_p to the relativistic momentum we get

$$\begin{aligned}
\sigma_x \sigma_p &\geq \hbar \\
l_p \sigma_p &\geq \hbar \\
l_p \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \hbar \\
l_p \frac{\frac{\hbar}{\bar{\lambda}} \frac{1}{c} v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \hbar \\
\frac{\frac{1}{c} v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{l_p} \\
\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{l_p} c
\end{aligned} \tag{5}$$

Solved with respect to v this gives

$$\begin{aligned}
\frac{v^2}{1-\frac{v^2}{c^2}} &\geq \frac{\bar{\lambda}^2}{l_p^2} c^2 \\
v^2 &\leq \frac{\bar{\lambda}^2}{l_p^2} c^2 \left(1 - \frac{v^2}{c^2}\right) \\
v^2 \left(1 + \frac{\bar{\lambda}^2}{l_p^2}\right) &\leq \frac{\bar{\lambda}^2}{l_p^2} c^2 \\
v^2 &\leq \frac{\frac{\bar{\lambda}^2}{l_p^2} c^2}{\left(1 + \frac{\bar{\lambda}^2}{l_p^2}\right)} \\
v &\leq \frac{c}{\sqrt{1 + \frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{6}$$

We could also have done the derivation below without decomposing the mass in this way; this alternative, which gives exactly the same result, is shown in the Appendix.

For example, for an electron we have $\bar{\lambda}_e \approx 3.861593 \times 10^{-13}$ and the Planck length $l_p \approx 1.616199 \times 10^{-35}$. We then see that the maximum velocity of an electron must be

$$v \leq c \frac{1}{\sqrt{1 + \frac{l_p^2}{\bar{\lambda}_e^2}}} \leq c \times 0.99999999999999999999999999999999999999999999912416$$

This is basically identical to the maximum velocity we get from Haug's maximum velocity formula

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}_e^2}} \approx c \times 0.99999999999999999999999999999999999999999999912416$$

For all observed particles we have $\bar{\lambda} >> l_p$ and then we can use the Taylor series expansion

$$\frac{v}{c} \leq \frac{1}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}} \approx 1 - \frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2} + \frac{3}{8}\frac{l_p^4}{\bar{\lambda}^4} - \frac{5}{16}\frac{l_p^6}{\bar{\lambda}^6} + \cdots \tag{7}$$

Further, we have the Taylor series expansion for

$$\frac{v_{max}}{c} = \sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}} \approx 1 - \frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2} + \frac{1}{8}\frac{l_p^4}{\bar{\lambda}^4} - \frac{1}{16}\frac{l_p^6}{\bar{\lambda}^6} + \cdots \tag{8}$$

In most cases, using only the first term of the Taylor series expansion will be more than accurate enough, and then we see that they are the same, which explains why we obtained the same numerical value for an electron.

This indicates that embedded in Heisenberg's uncertainty principle (when combined with key insight from Max Planck) we find an indication that Haug's maximum velocity formula for anything with rest-mass likely is correct and this supports the analysis provided in Haug's other papers on this topic.

Below we show some possible choices of assumptions, where only the theory presented above seems to be compatible with the Planck length being a minimum length. The idea of the velocity of a mass having to be below c, but coming as close to c as we want, is actually not compatible with accepting the Planck length as a minimum length.

1. $\sigma_x \leq l_p$ and v$\leq \frac{c}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}}$. What we have derived above.
2. $\sigma_x > 0$ and $v < c$. This is the current theory on velocity, but it is incompatible with idea of Planck length as a minimum. If modern physicists want to hold on to $v < c$, then they must reconsider and likely eliminate the Planck length. This is improbable and something I think would be very unwise; instead they should replace $v < c$ with the formula above.
3. $\sigma_x \geq 0$ and v$\leq c$. Impossible for anything with rest-mass, as the relativistic mass could become infinite, so this can easily be excluded.

Choice three can be excluded based on Einstein's analysis that a mass traveling at the speed of light will attain an infinitely large relativistic mass, so it is impossible to have the limit on v for anything with rest-mass. Choice two has the drawback that one must claim the Planck length is nothing special. Choice one is, in our view, an alternative that should be considered seriously by the physics community.

3 The Planck Mass Particle Must Stand Absolutely Still

The rest-mass of the Planck mass particle is given by

$$m_p = \frac{\hbar}{l_p}\frac{1}{c} \approx 2.1765 \times 10^{-8} \text{ kg} \tag{9}$$

That is to say, the reduced Compton wavelength of a Planck mass particle is l_p. Further, we know that the Planck mass particle momentum is $m_p c$. Now let us combine this formula with Heisenberg's uncertainty principle, where we will set $\sigma_x = l_p$ again. In this special case, we think it makes sense to set it only "equal to" rather than "greater than or equal to," because, unlike any other particle, we claim that the Planck mass particle momentum must always be $m_p c$, and then there is no uncertainty per se in the momentum. However, the uncertainty principle limit must hold, and this is exactly what we see here

$$\begin{aligned}
\sigma_x\sigma_p &\geq \hbar \\
l_p\frac{m_p c}{\sqrt{1-\frac{v^2}{c^2}}} &= \hbar \\
l_p\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} &= \hbar \\
l_p\frac{\frac{\hbar}{l_p}\frac{1}{c}c}{\sqrt{1-\frac{v^2}{c^2}}} &= \hbar \\
\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= 1
\end{aligned} \tag{10}$$

This can only happen when $v = 0$. That is to say, only a Planck mass particle must have a velocity of zero. This is the same result as given by Haug's maximum velocity formula for anything with rest-mass in the special case of a Planck mass particle

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{l_p^2}} = 0 \tag{11}$$

A velocity of zero (no matter the reference frame from which it is observed) sounds absurd at first. But actually this is not so strange at all. The Planck mass particle, according to Haug, can only last for one Planck second. This is the collision point between two light particles. What is the speed of a light particle at the very turning point of light? It is zero. This means that light has two invariant "velocities": when it is energy, it always moves at speed c as measured with Einstein-Poincaré synchronized clocks, no matter what the reference frame may be. And the velocity is zero when the particle is colliding and stands still for one Planck second, also as measured with Einstein-Poincaré synchronized clocks. As we see, at the deepest level the world is likely binary: we only have the Planck mass particle lasting for one Planck second (colliding indivisible particles), and energy (non-colliding indivisible particles).

We predict that the Planck mass particle, the Planck length, and the Planck second are invariant and the same as observed from any reference frame. This means that Lorentz symmetry is broken at the Planck scale; this view is consistent with has been predicted by several scientists in relation to quantum gravity.

4 $\hbar$ or $\frac{\hbar}{2}$?

Heisenberg [9] himself postulated his uncertainty principle as

$$\sigma_x \sigma_p \geq \hbar \tag{12}$$

Kennard [10] was the first to come up with and partly prove this version of the Heisenberg uncertainty principle

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \tag{13}$$

We could also have derived the limit on the velocity based on this the Kennard version of the Heisenberg principle, when assuming the minimum uncertainty in the position is the Planck length; this gives

$$\begin{aligned}
\sigma_x \sigma_p &\geq \frac{\hbar}{2} \\
l_p \sigma_p &\geq \frac{\hbar}{2} \\
l_p \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\hbar}{2} \\
l_p \frac{\frac{\hbar}{\bar{\lambda}} \frac{1}{c} v}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\frac{\frac{1}{c} v}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{2 l_p} \\
\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} &\geq \frac{\bar{\lambda}}{2 l_p} c
\end{aligned} \tag{14}$$

Solved with respect to v this gives

$$
\begin{aligned}
\frac{v^2}{1-\frac{v^2}{c^2}} &\geq \frac{\bar{\lambda}^2}{2^2 l_p^2} c^2 \\
v^2 &\leq \frac{\bar{\lambda}^2}{4 l_p^2} c^2 \left(1-\frac{v^2}{c^2}\right) \\
v^2\left(1+\frac{\bar{\lambda}^2}{4 l_p^2}\right) &\leq \frac{\bar{\lambda}^2}{4 l_p^2} c^2 \\
v^2 &\leq \frac{\frac{\bar{\lambda}^2}{4 l_p^2} c^2}{\left(1+\frac{\bar{\lambda}^2}{4 l_p^2}\right)} \\
v &\leq \frac{c}{\sqrt{1+4\frac{l_p^2}{\bar{\lambda}^2}}}
\end{aligned} \tag{15}
$$

For example, for an electron this would mean a speed limit of

$$
v \leq \frac{c}{\sqrt{1+4\frac{l_p^2}{\bar{\lambda}_e^2}}} \approx c \times 0.9999999999999999999999999999999999999999999964966
$$

Interestingly, when $\bar{\lambda} >> l_p$, as it is for all observed particles, numerically this is almost identical to the maximum velocity limit that one would get from the maximum velocity formula given by Haug in 2014 [6]. Haug's 2014 formula shows that when we set the diameter of the indivisible particle to l_p we get

$$
v_{max} = c\left(\frac{\bar{\lambda}^2 - l_p^2}{\bar{\lambda}^2 + l_p^2}\right) \tag{16}
$$

For an electron we have

$$
v_{max} = c\left(\frac{\bar{\lambda}_e^2 - l_p^2}{\bar{\lambda}_e^2 + l_p^2}\right) \approx c \times 0.9999999999999999999999999999999999999999999964966
$$

In addition, if we take a Taylor series expansion of formula 15 we get

$$
\frac{v}{c} \leq \frac{1}{\sqrt{1+4\frac{l_p^2}{\bar{\lambda}^2}}} \approx 1 - 2\frac{l_p^2}{\bar{\lambda}^2} + 6\frac{l_p^4}{\bar{\lambda}^4} - 20\frac{l_p^6}{\bar{\lambda}^6} \cdots \tag{17}
$$

Using only the first term of the series expansion and we have $v \leq c\left(1 - 2\frac{l_p^2}{\bar{\lambda}^2}\right)$. And also a series expansion of $\frac{1-x^2}{1+x^2} \approx 1 - 2x^2 + 2x^4 - 2x^6 \cdots$. This means when we have $\bar{\lambda} >> l_p$, we can write Haug's maximum formula as

$$
v_{max} = c\left(\frac{\bar{\lambda}^2 - l_p^2}{\bar{\lambda}^2 + l_p^2}\right) = c\left(\frac{1-\frac{l_p^2}{\bar{\lambda}^2}}{1+\frac{l_p^2}{\bar{\lambda}^2}}\right) \approx c\left(1 - 2\frac{l_p^2}{\bar{\lambda}^2}\right) \tag{18}
$$

Yet another alternative is to indicate that half the Planck length is the minimum possible uncertainty in position, instead of the full Planck length. This would lead to the same formula for the maximum limit on velocity when calculated using the Kennard version of the Heisenberg principle formula, just as we calculated using the Planck length from the original Heisenberg principle formula, that is formula 6.

We could introduce further discussion on the Heisenberg principle, including the linkage to atomism, why both versions of the Heisenberg principle make sense, and how they are closely related. However, we will leave the more philosophical discussions for a separate paper.

However, the fact that two postulates in atomism lead to all the known equations in special relativity theory plus the result shown derived from combining the theory from Heisenberg and Max Planck indicates that this theory should be considered as an alternative. Atomism is quantized from the very beginning by returning to a spatial dimension for one unique particle that makes up all other masses and energy. However, it is outside the scope of this paper to discuss this in much detail; we may elaborate on these points in future work.

5 Conclusion

It looks like Heisenberg's uncertainty principle predicts an exact maximum velocity that is below the speed of light for anything with rest-mass. For any practical purpose, this seems to be the same limit as given by Haug's earlier suggested maximum velocity formula for anything with rest-mass.

This could have major implications how we look at light particles in the very collision point with other light particles. This also indicates that Lorentz symmetry breaks down at the Planck scale. The Planck mass particle stands absolutely still and is invariant and the same as observed across different reference frames.

Appendix

This is almost the same derivation as in section 2. However, here we will complete the derivation without initially decomposing the mass:

$$\begin{aligned}
\sigma_x\sigma_p &\geq \hbar \\
l_p\sigma_p &\geq \hbar \\
l_p\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \hbar \\
\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{l_p m} \\
\frac{v^2}{\left(1-\frac{v^2}{c^2}\right)} &\geq \frac{\hbar^2}{l_p^2 m^2} \\
v^2 &\leq \frac{\hbar^2}{l_p^2 m^2}\left(1-\frac{v^2}{c^2}\right) \\
v^2 &\leq \frac{\hbar^2}{l_p^2 m^2} - \frac{\hbar^2}{l_p^2 m^2}\frac{v^2}{c^2} \\
v^2 + \frac{\hbar^2}{l_p^2 m^2}\frac{v^2}{c^2} &\leq \frac{\hbar^2}{l_p^2 m^2} \\
v^2\left(1+\frac{\hbar^2}{l_p^2 m^2}\frac{1}{c^2}\right) &\leq \frac{\hbar^2}{l_p^2 m^2} \\
v^2 &\leq \frac{\frac{\hbar^2}{l_p^2 m^2}}{\left(1+\frac{\hbar^2}{l_p^2 m^2}\frac{1}{c^2}\right)} \\
v^2 &\leq \frac{1}{\frac{l_p^2 m^2}{\hbar^2}+\frac{1}{c^2}} \\
v^2 &\leq \frac{c^2}{\frac{l_p^2 c^2 m^2}{\hbar^2}+1} \\
v &\leq \frac{c}{\sqrt{\frac{l_p^2 c^2 m^2}{\hbar^2}+1}} \\
v &\leq \frac{c}{\sqrt{\frac{m^2}{\frac{\hbar^2}{l_p^2 c^2}}+1}} \qquad (19)
\end{aligned}$$

and since $\frac{\hbar}{l_p}\frac{1}{c}$ is equal to the Planck mass, m_p, we can also write this as

$$v \leq \frac{c}{\sqrt{1+\frac{m^2}{m_p^2}}} \qquad (20)$$

Which is naturally the same formula we derived earlier since

$$\frac{m^2}{m_p^2} = \frac{\left(\frac{\hbar}{\bar{\lambda}}\frac{1}{c}\right)^2}{\left(\frac{\hbar}{l_p}\frac{1}{c}\right)^2} = \frac{l_p^2}{\bar{\lambda}^2} \qquad (21)$$

Similarly, if we had derived it this way from the Kennard version of the Heisenberg uncertainty formula we would have gotten

$$v \leq \frac{c}{\sqrt{1+4\frac{m^2}{m_p^2}}} \tag{22}$$

It is also worth pointing out that if we look at Newton's gravitational constant as a composite constant, $G = \frac{l_p^2 c^3}{\hbar}$, as previously suggested by Haug, then we can write the maximum velocity as follows, when using the original Heisenberg uncertainty formula

$$\begin{aligned} v &\leq \frac{c}{\sqrt{1+\frac{l_p^2 c^2 m^2}{\hbar^2}}} \\ v &\leq \frac{c}{\sqrt{1+G\frac{mm}{c\hbar}}} \end{aligned} \tag{23}$$

or when using the Kennard version of the Heisenberg uncertainty formula we will have

$$v \leq \frac{c}{\sqrt{1+4G\frac{m^2}{c\hbar}}} \tag{24}$$

Further, many will recognize $G\frac{m^2}{c\hbar}$ as the small gravitational coupling constant, $_g$. So we can also write the maximum limit on velocity simply as

$$v \leq \frac{c}{\sqrt{1+\alpha_g}} \tag{25}$$

These different ways to write the maximum limit on the velocity are essentially all the same, except for a small difference that will emerge depending on whether we derive it from the original Heisenberg uncertainty formulation, or from the Kennard formulation.

Solved with respect to big G we get

$$\begin{aligned} v &\leq \frac{c}{\sqrt{1+G\frac{mm}{c\hbar}}} \\ v\sqrt{1+G\frac{mm}{c\hbar}} &\leq c \\ v^2\left(1+G\frac{mm}{c\hbar}\right) &\leq c^2 \\ G\frac{mm}{c\hbar} &\leq \frac{c^2}{v^2}-1 \\ G &\leq \frac{mm}{c\hbar}\left(\frac{c^2}{v^2}-1\right) \end{aligned} \tag{26}$$

The above can only be true when $v = \frac{c}{\sqrt{1+\frac{l_p^2}{\bar{\lambda}^2}}}$; this gives

$$\begin{aligned} G &\leq \frac{mm}{c\hbar}\left(\frac{c^2}{\left(\frac{c}{1+\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}-1\right) \\ G &\leq \frac{mm}{c\hbar}\left(\frac{1}{\frac{1}{1+\frac{l_p^2}{\bar{\lambda}^2}}}-1\right) \\ G &\leq \frac{mm}{c\hbar}\left(\frac{l_p^2}{\bar{\lambda}^2}\right) \\ G &\leq \frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{c\hbar}\left(\frac{l_p^2}{\bar{\lambda}^2}\right) \\ G &\leq \frac{l_p^2 c^3}{\hbar} \approx 6.67384\times 10^{-11} m^3\cdot kg^{-1}\cdot s^{-2} \end{aligned} \tag{27}$$

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