Does Heisenberg’s Uncertainty Principle Predict a Maximum Velocity for Anything with Rest-Mass below the Speed of Light?

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Abstract

In this paper we derive a maximum velocity for anything with rest-mass from Heisenberg’s uncertainty principle. The maximum velocity formula we get is in line with the maximum velocity formula suggested by Haug in a series of papers. This supports the assertion that Haug’s maximum velocity formula is useful in reconsidering the path forward in theoretical physics. In particular, it predicts that the Lorentz symmetry will break down at the Planck scale, and shows how and why this happens. Further, it shows that the maximum velocity for a Planck mass particle is zero. At first this may sound illogical, but it is a remarkable result that gives an new and important deep insight in this research domain.

Key words: Heisenberg’s uncertainty principle, maximum velocity of matter, reduced Compton wavelength

1 Introduction

Haug [2, 3, 4, 5, 6] has recently suggested a new maximum velocity for subatomic particles (anything with mass) that is just below the speed of light. The formula is given by

\[ v_{\text{max}} = c \sqrt{1 - \frac{l_p^2}{\lambda^2}} \] (1)

where \( \lambda \) is the reduced Compton wavelength of the particle we are trying to accelerate and \( l_p \) is the Planck length [7, 8]. This formula can be derived from special relativity by simply assuming that the maximum frequency one can have is the Planck frequency \( \frac{c}{l_p} \), or that the shortest wavelength possible is the Planck length. We will also get the same formula if we assume that the ultimate fundamental particle has a spatial dimension equal to \( l_p \) and always is traveling at the speed of light; this is a model outlined by [1, 2].

This maximum velocity for anything with rest-mass was first predicted by Haug in 2014. It was first derived from two postulates in atomism. The theory leads to the same mathematical end results as special relativity theory, as long as one use Einstein-Poincaré synchronized clocks. However, Haug had not yet linked his theory up to key concepts of Max Planck. Here, the key understanding given in 2014 will lead to the same formula as described above.

In this paper we will show that the same formula implicitly agrees with to our knowledge a new result that comes out of Heisenberg’s uncertainty principle when combined with key insight from Max Planck.

2 Heisenberg’s Uncertainty Principle Leads to a Maximum Velocity for Anything with Rest-Mass

Heisenberg’s uncertainty principle [9] is given by

\[ \sigma_x \sigma_p \geq \hbar \] (2)

where \( \sigma_x \) is considered to be the uncertainty in the position, \( \sigma_p \) is the uncertainty in the momentum, and \( \hbar \) is the reduced Planck constant. Here we use \( \hbar \) instead of the modern and more common \( \frac{\hbar}{2} \) version.

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1Back then I only derived my maximum velocity formula with a link to one-sided relativistic Doppler shift based on a slightly different clock synchronization procedure and therefore got slightly different results than those given here.

2See also Kennard [10], who was the first to “prove” this modern inequality based on the work of Heisenberg.
This is something I likely will return to in an updated version of this paper in the months to come.\(^3\) The rest-mass of an elementary subatomic particle is given by

\[
m = \frac{\hbar}{\lambda c}
\]  

(3)

To look at the mass mathematically in this way will be key to the analysis here. For an electron, for example, we have

\[
m = \frac{\hbar}{\lambda e c} \approx 9.10938 \times 10^{-31} \text{ kg}
\]  

(4)

We will assume that the minimum uncertainty in the position of any elementary particle (or any other object) is the Planck length. Setting \(p = l_p\) and the momentum \(p\) to the relativistic momentum we get

\[
\begin{align*}
\frac{\sigma_x \sigma_p}{\hbar} & \geq 1, \\
\frac{m \bar{v}}{\bar{v}^2} & \geq \hbar, \\
\frac{\bar{v}}{\sqrt{1 - \frac{v^2}{c^2}}} & \geq \hbar,
\end{align*}
\]

Solved with respect to \(v\) this gives

\[
\begin{align*}
\frac{v^2}{1 - \frac{v^2}{c^2}} & \geq \frac{\lambda^2}{l_p^2} c^2, \\
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & \geq \frac{\lambda}{l_p}, \\
1 & \leq \frac{\lambda^2}{l_p^2} \left( \frac{1}{v^2} - \frac{1}{c^2} \right), \\
v^2 & \leq \frac{\lambda^2}{l_p^2} c^2 \left( 1 - \frac{v^2}{c^2} \right), \\
v^2 \left( 1 + \frac{\lambda^2}{l_p^2} \right) & \leq \frac{\lambda^2}{l_p^2} c^2, \\
v^2 & \leq \frac{\lambda^2}{l_p^2} c^2 \left( 1 + \frac{\lambda^2}{l_p^2} \right), \\
v & \leq c \sqrt{1 + \frac{l_p^2}{\lambda^2}}.
\end{align*}
\]  

(5)

For example, for an electron we have \(\lambda_e \approx 3.861593 \times 10^{-13}\) and the Planck length \(l_p \approx 1.616199 \times 10^{-35}\). We then see that the velocity of an electron must be

\[
v \leq c \sqrt{1 + \frac{l_p^2}{\lambda^2}} \leq c \times 0.99999999999999999999999999999999999999999999912
\]

This is almost identical to the maximum velocity we get from Haug’s maximum velocity formula

\[
v_{max} = c \sqrt{1 - \frac{l_p^2}{\lambda^2}} \approx c \times 0.99999999999999999999999999999999999999999999912
\]

\(^3\)This is the first working paper on this.
For all observed particles we have $\lambda >> l_p$ and then we can use the Taylor series expansion for
$$\frac{1}{\sqrt{1 - \frac{l_p^2}{\lambda^2}}} \approx 1 - \frac{l_p^2}{2\lambda^2} + \frac{3l_p^4}{8\lambda^4} ...$$ Further, we have the Taylor series expansion for $\sqrt{1 - \frac{l_p^2}{\lambda^2}} \approx 1 - \frac{l_p^2}{2\lambda^2} + \frac{3l_p^4}{8\lambda^4} ...$.

In most cases, using only the first term of the Taylor series expansion will be more than accurate enough, and then we see that they are the same, which explains why we got the same numerical value for an electron.

This indicates that embedded in the Heisenberg’s uncertainty principle (when combined with key insight from Max Planck) we find an indication that Haug’s maximum velocity formula for anything with rest-mass likely is correct and this supports the analysis provided in Haug’s other papers on this topic.

3 The Planck Mass Particle Must Stand Absolute Still

The rest-mass of the Planck mass particle is given by
$$m_p = \frac{\hbar}{l_p c} \approx 2.1765 \times 10^{-8} \text{ kg} \quad (7)$$

That is the reduced Compton wavelength of a Planck mass particle is $l_p$. Further, we know that the Planck mass particle momentum is $m_p c$. Now let us put this in into formula into the Heisenberg’s uncertainty principle, where we again set $x = l_p$. In this special case, we think it makes sense to set it only “equal to” rather than “greater than or equal to,” because unlike for any other particle the Planck mass particle momentum we claim always must be $m_p c$, then there is no uncertainty per se in the momentum. However, the uncertainty principle limit must hold, and this is exactly what we see here

$$\frac{\sigma_x \sigma_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \hbar$$

This can only happen when $v = 0$. That is to say, only a Planck mass particle must have a velocity of zero. At first this sounds absurd. But actually this is not so strange at all. The Planck mass particle, according to Haug, can only last for one Planck second. This is the collision point between two light particles. What is the speed of a light particle at the very turning point of light? It is zero. This means that light has two invariant “velocities”: when it is energy, it always moves at speed $c$ as measured with Einstein-Poincaré synchronized clocks, no matter what reference frame may be. And the velocity is zero when the particle is colliding and stands still for one Planck second, also as measured with Einstein-Poincaré synchronized clocks. As we see, at the deepest level the world is likely binary: we only have the Planck mass particle lasting for one Planck second (colliding indivisible particles), and energy (non-colliding indivisible particles).

We predict that the Planck mass particle, the Planck length, and the Planck second are invariant and the same as observed from any reference frame. This means that Lorentz symmetry is broken at the Planck scale; this view is consistent with has been predicted by several scientists in relation to quantum gravity.

4 Conclusion

It looks like the Heisenberg’s uncertainty principle predicts an exact maximum velocity for anything with rest-mass that is below the speed of light. For any practical purpose this seems to be the same limit as given by Haug’s earlier suggested maximum velocity formula for anything with rest-mass.

This could have major implications how we look at light particles in the very collision point with other light particles. This also indicates that Lorentz symmetry breaks down at the Planck scale; the Planck mass particle stands absolutely still and is invariant and the same as observed across different reference frames.

This is a first draft; comments and constructive criticisms are welcome.
References


