AN IDENTITY-BASED DIRECT PROOF OF FLT V7

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ABSTRACT. By inspection, for some positive integral n (e.g., $n = 1, 2$) there clearly exist positive integers A, B, C satisfying $A^n + B^n = C^n$, an algebraic identity that we have devised with one unrestricted real variable. We find a way around the circularity inherent to this identity, allowing us to show that the positive coprime triple (A, B, C) is equal to the positive coprime triple (x, y, z) that satisfies $x^n + y^n = z^n$. Our identity discloses, for $n \geq 3$, that no value of coprime (A,B,C) satisfies $A^n + B^n = C^n$. So, we prove FLT directly.

1. Introduction

With n always denoting a positive integer : Fermat's last theorem (FLT) states, for $n \geq 3$, that no positive integral triple (x, y, z) *satisfies* $x^n + y^n = z^n$.

The expression "(x, y, z) *satisfies* $x^n + y^n = z^n$", and, more generally, "a triple of values satisfies an equation", means, in this example, that there exist positive integral values of (x, y, z) that are the values x, y, z for which the equation holds. We ignore values of (x^n, y^n, z^n) that, per this definition, also would *satisfy* $x^n+y^n = z^n$.

No one has yet devised a simple proof of FLT for every positive integer $n \geq 3$.

Specifically, FLT has not been proven simply for an infinity of primes $n > 2$.

It is a *fact* that, for *some* values of n (at minimum, $n = 1, 2$), the equation $x^n + y^n = z^n$ is satisfied by a positive integral triple (x, y, z) that is *primitive* (what we call a coprime-such-triple from which we can derive an integral multiple).

This fact is essential to our proposed *direct proof* of FLT. We call a *direct proof* one in which we make no assumptions for the purpose of deriving a contradiction.

2. Our Argument

LEM. : The {coprime x, y, z} is equal to the {coprime A, B, C} satisfying (1).

PROOF of LEM. :

(1)

$$\left(\left(\left(\frac{j}{j-1}+\frac{k}{k-1}\right)g^n\right)^{\frac{1}{n}}\right)^n+\left(\left(h-\frac{k}{k-1}g^n\right)^{\frac{1}{n}}\right)^n=\left(\left(h+\frac{j}{j-1}g^n\right)^{\frac{1}{n}}\right)^n.$$

Let A, B, C, respectively, denote $((\frac{j}{j-1}+\frac{k}{k-1})g^n)^{\frac{1}{n}}$; $(h-\frac{k}{k-1}g^n)^{\frac{1}{n}}$;$(h+\frac{j}{j-1}g^n)^{\frac{1}{n}}$.

For any given positive integral n, *algebraic identity* (1) holds for all positive rational values of g, holds for all positive integral values of $j, k > 1$, and holds for unrestricted positive real values of h such that $h > \frac{j}{j-1}g^n$ and $h > \frac{k}{k-1}g^n$.

Equation (1) is satisfied by positive coprime values of (A, B, C), as follows :

Let us restrict integral values of j, k to $j, k = 2$, for which the identity still holds.

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Therefore, (A, B, C) reduces to $((4g^n)^{\frac{1}{n}};\ (h - 2g^n)^{\frac{1}{n}};\ (h + 2g^n)^{\frac{1}{n}})$.

It is a *fact* that for *some* n (at minimum, $n = 1, 2$) a primitive positive triple (A, B, C) satisfies $A^n + B^n = C^n$. With $n = 1$, when $g = \frac{3}{4}$, $h = \frac{11}{2}$, equation (1) is $3^1 + 4^1 = 7^1$; with $n = 2$, when $g = \frac{3}{2}$, $h = \frac{41}{2}$, equation (1) is $3^2 + 4^2 = 5^2$.
This fact is another necessary basic for our proposed, direct proof of FLT.

The argument is not affected by restricting real j, k, g to $j, k = 2$, with g taking any given positive rational value, since h is an unrestricted positive real value :
For unrestricted real h, when (A, B, C) is positive coprime, $\left(\frac{(4g^n)(h-2g^n)}{(h+2g^n)}\right)^{\frac{1}{n}}$, or $\frac{AB}{C}$, takes any given positive value of $\frac{xy}{z}$ for positive coprime values of (x, y, z).

PROP. : Coprime $\frac{xy}{z}$ corresponds one-to-one with values of primitive (x, y, z).
PROOF of PROP : For any given positive primitive triple (x, y, z), as maximally occurring. by definition, for $x^n + y^n = z^n$, we notate the taken-as-known rational $\frac{xy}{z}$ by $\frac{v}{w}$ for which v, w are the taken-as-known positive integers, with $v \neq w$.
The non-coprime values of v, w can be ignored since we take, as a fact, for "some" n, that x, y, z are the coprime values which satisfy primitive $x^n + y^n = z^n$.
So, we determine the unique set of positive integers for $w = z$, and for $v = xy$. Solving $xy = v$ simultaneously with $x^n + y^n = z^n$ yields $(y^n)^2 - (y^n)(w^n) + v^n = 0$.
This quadratic equation is satisfied by a unique set of positive integers for y.
In that $x = \frac{v}{y}$, a unique set of positive integral values of x is determined as the subset of the unique values of $\frac{v}{y}$. Thus, values of $\frac{xy}{z}$ determine (x, y, z), and values of (x, y, z) determine $\frac{xy}{z}$. So, values of $\frac{xy}{z}$ and of (x, y, z) correspond one-to-one. □

Since $\frac{xy}{z}$ corresponds one-to-one with (x, y, z), and $\frac{AB}{C} = \frac{xy}{z}$, thus, positive coprime values of $\frac{AB}{C}$ correspond one-to-one with positive primitive values of (x, y, z).
If positive primitive values of $\frac{AB}{C}$ should correspond one-to-one with positive primitive values of (A, B, C), then, it would follow that $A = x$; $B = y$; and, $C = z$.
With $\frac{AB}{C} = \frac{xy}{z}$, evidently, the subset of all positive coprime values for $\frac{AB}{C}$ is equal to the subset of all positive coprime values for $\frac{xy}{z}$. So, arguing similarly to proof of PROP. yields : For *some* values of n such that a positive integral value of (A, B, C) exists, any given positive coprime value of $\frac{AB}{C}$ (equal to a coprime of $\frac{xy}{z}$) has a primitive value of (A, B, C) that corresponds one-to-one with a coprime $\frac{AB}{C}$.
Consequently, (x, y, z) and (A, B, C) have equal positive primitive values. □

3. Results and Conclusion

For any given integral value of $n \geq 3$, the values of positive integral triple $((4g^n)^{\frac{1}{n}}, (h - 2g^n)^{\frac{1}{n}}, (h + 2g^n)^{\frac{1}{n}})$ do not satisfy (1) for the following reasons :
Expanding the terms of $(4g^n)^{\frac{1}{n}}$ yields $2^{\frac{2}{n}}g$. By inspection, it is evident that, with rational g and $n \geq 3$, the set of all positive rational values of $2^{\frac{2}{n}}g$ is null.
Therefore, when $n \geq 3$, the positive integral subset of all values for $2^{\frac{2}{n}}g$ is null.
Hence, when $n \geq 3$, no positive integral triple (A, B, C) satisfies equation (1).
Ergo, when $n \geq 3$, no positive integral triple (x, y, z) satisfies $x^n + y^n = z^n$.

Q.E.D.