International Mathematical Forum, 1, 2006, no. 24, 1161 - 1166

Smarandache hyper $(\cap, \in)$-ideals on Smarandache Hyper K-algebras

Kyung Ho Kim

Department of Mathematics, Chungju National University
Chungju 380-702, Korea
ghkim@Chungju.ac.kr

Young Bae Jun

Department of Mathematics Education
Gyeongsang National University
Jinju 660-701, Korea
ybjun@gsnu.ac.kr

Eun Hwan Roh[1]

Department of Mathematics Education
Chinju National University of Education
Jinju 660-756, Korea
ehroh@cue.ac.kr

Habib Harizavi

Department of Mathematics
Sistan and Baluchestan University, Zhedan, Iran
harizavi@hamoon.usb.ac.ir

Abstract. We introduce the notion of a Smarandache hyper $(\cap, \in)$-ideal and Ω-reflexive in hyper K-algebra, and some related properties are given.

Mathematics Subject Classification: 06F35, 03G25

Keywords: Smarandache hyper $K(BCK)$-algebra, Smarandache hyper $(<, \in)$-ideal, Smarandache hyper $(\subseteq, \in)$-ideal Smarandache hyper $(\cap, \in)$-ideal, Ω-reflexive

[1]Corresponding author: Tel. + 82 55 740 1232.

1. Introduction

Generally, in any human field, a *Smarandache Structure* on a set A means a weak structure **W** on A such that there exists a proper subset B of A which is embedded with a strong structure **S**. In [6], W. B. Vasantha Kandasamy studied the concept of Smarandache groupoids, subgroupoids, ideal of groupoids, semi-normal subgroupoids, Smarandache Bol groupoids and strong Bol groupoids and obtained many interesting results about them. Smarandache semigroups are very important for the study of congruences, and it was studied by R. Padilla [5].

In this paper, we introduce the notion of a Smarandache hyper $(\cap, \in)$-ideal and Ω-reflexive in hyper K-algebra, and investigate its properties.

2. Preliminaries

We include some elementary aspects of hyper K-algebras that are necessary for this paper, and for more details we refer to [1] and [7]. Let H be a non-empty set endowed with a hyper operation "$\circ$", that is, $\circ$ is a function from $H \times H$ to $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$. For two subsets A and B of H, denote by $A \circ B$ the set $\bigcup\limits_{a \in A, b \in B} a \circ b$.

By a *hyper BCK-algebra* we mean a non-empty set H endowed with a hyperoperation "$\circ$" and a constant 0 satisfying the following axioms:

(HK1) $(x \circ z) \circ (y \circ z) \ll x \circ y$,
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK3) $x \circ H \ll \{x\}$,
(HK4) $x \ll y$ and $y \ll x$ imply $x = y$,

for all $x, y, z \in H$, where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined by $\forall a \in A$, $\exists b \in B$ such that $a \ll b$.

By a *hyper I-algebra* we mean a non-empty set H endowed with a hyper operation "$\circ$" and a constant 0 satisfying the following axioms:

(H1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(H2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(H3) $x < x$,
(H4) $x < y$ and $y < x$ imply $x = y$

for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a < b$. If a hyper I-algebra $(H, \circ, 0)$ satisfies an additional condition:

(H5) $0 < x$ for all $x \in H$,

then $(H, \circ, 0)$ is called a *hyper K-algebra* (see [1]).

Note that every hyper BCK-algebra is a hyper K-algebra.

In a hyper I-algebra H, the following hold (see [1, Proposition 3.4]):

(a1) $(A \circ B) \circ C = (A \circ C) \circ B$,
(a2) $A \circ B < C \Leftrightarrow A \circ C < B$,

(a3) $A \subseteq B$ implies $A < B$

for all nonempty subsets A, B and C of H.

In a hyper K-algebra H, the following hold (see [1, Proposition 3.6]):

(a4) $x \in x \circ 0$ for all $x \in H$.

Definition 2.1. [3, Definition 3.4] A *Smarandache hyper K-algebra* is defined to be a hyper K-algebra $(H, \circ, 0)$ in which there exists a proper subset Ω of H such that $(\Omega, \circ, 0)$ is a non-trivial hyper BCK-algebra.

Example 2.2. [3, Example 3.5] Let $H = \{0, a, b, c\}$ and define an hyper operation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{0\}$	$\{0\}$
b	$\{b\}$	$\{a\}$	$\{0, a\}$	$\{0, a\}$
c	$\{c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{0, b, c\}$

Then $(H, \circ, 0)$ is a Smarandache hyper K-algebra because $(\Omega = \{0, a, b\}, \circ, 0)$ is a hyper BCK-algebra.

Example 2.3. [3, Example 3.6] Let $H = \{0, a, b\}$ and define an hyper operation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a, b\}$	$\{0, a, b\}$	$\{0, a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Then $(H, \circ, 0)$ is not a Smarandache hyper K-algebra since $(\Omega_1 = \{0, a\}, \circ, 0)$ and $(\Omega_2 = \{0, b\}, \circ, 0)$ are not hyper BCK-algebras.

Definition 2.4. [3, Definition 3.9] A nonempty subset I of H is called a *Smarandache hyper* $(<, \in)$*-ideal* of H related to Ω (or briefly, Ω-*Smarandache hyper* $(<, \in)$*-ideal* of H) if it satisfies:

(c1) $0 \in I$,
(c2) $(\forall x \in \Omega)\ (\forall y \in I)\ (x \circ y < I \Rightarrow x \in I)$.

If I is a Smarandache hyper $(<, \in)$-ideal of H related to every hyper BCK-algebra contained in H, we simply say that I is a *Smarandache hyper* $(<, \in)$*-ideal* of H.

Definition 2.5. [3, Definition 3.14] A nonempty subset I of H is called a *Smarandache hyper* $(\subseteq, \in)$*-ideal* of H related to Ω (or briefly, Ω-*Smarandache hyper* $(\subseteq, \in)$*-ideal* of H) if it satisfies:

(c1) $0 \in I$,
(cw) $(\forall x \in \Omega)\ (\forall y \in I)\ (x \circ y \subseteq I \Rightarrow x \in I)$.

If I is a Smarandache hyper $(\subseteq, \in)$-ideal of H related to every hyper BCK-algebra contained in H, we simply say that I is a *Smarandache hyper* $(\subseteq, \in)$-*ideal* of H.

In what follows, let H and Ω denote a Smarandache hyper K-algebra and a non-trivial hyper BCK-algebra which is properly contained in H, respectively.

3. Main Results

Definition 3.1. A non-empty subset I of H is called a *Smarandache hyper* $(\cap, \in)$-*ideal* of H related to Ω (briefly, Ω-Smarandache hyper $(\cap, \in)$-ideal of H) if it satisfies:

(c1) $0 \in I$,
(ss) $(\forall x, y \in \Omega)((x \circ y) \cap I \neq \emptyset, y \in I \Rightarrow x \in I)$.

Example 3.2. Let $H = \{0, a, b, c\}$ and define the hyperoperation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0, c\}$
a	$\{a\}$	$\{0\}$	$\{a\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$	$\{b\}$
c	$\{c\}$	$\{c\}$	$\{c\}$	$\{0\}$

Then $(H, \circ, 0)$ is a Smarandache hyper K-algebra because $(H, \circ, 0)$ is a hyper K-algebra and $(\Omega = \{0, a, b\}, \circ, 0)$ is a hyper BCK-algebra. Moreover, $\{0, a\}$ and $\{0, b\}$ are an Ω-Smarandache hyper $(\cap, \in)$-ideal of H.

Theorem 3.3. *Let I be an Ω-Smarandache hyper $(\cap, \in)$-ideal of H. Then*

(i) *I is an Ω-Smarandache hyper $(\subseteq, \in)$-ideal of H,*
(ii) *I is an Ω-Smarandache hyper $(<, \in)$-ideal of H.*

Proof. (i) Let $x \in \Omega$ and $y \in I$ be such that $x \circ y \subseteq I$. Then $(x \circ y) \cap I \neq \emptyset$ and so $x \in I$. Therefore I is an Ω-Smarandache hyper $(\subseteq, \in)$-ideal of H.

(ii) Let $x \in \Omega$ and $y \in I$ be such that $x \circ y \ll I$. Then for each $a \in x \circ y$ there exists $b \in I$ such that $a \ll b$, i. e., $0 \in a \circ b$. It follows that $(a \circ b) \cap I \neq \emptyset$ so from (ss) that $a \in I$. Thus $x \circ y \subseteq I$ and so $(x \circ y) \cap I \neq \emptyset$, and we get $x \in I$. Hence I is an Ω-Smarandache hyper $(<, \in)$-ideal of H. □

The following example show that the converse of Theorem 3.3 is not true in general.

Example 3.4. Let $H = \{0, a, b, c\}$ and define the hyperoperation "$\circ$" on H by the following Cayley table:

$\circ$	0	a	b	c
0	$\{0\}$	$\{0, a\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$	$\{0, b\}$
c	$\{c\}$	$\{b\}$	$\{b, c\}$	$\{0, b, c\}$

Then $(H, \circ, 0)$ is a Smarandache hyper K-algebra because $(H, \circ, 0)$ is a hyper K-algebra and $(\Omega = \{0, a, b\}, \circ, 0)$ is a hyper BCK-algebra. Note that $I = \{0, b\}$ is an Ω-Smarandache hyper $(<, \in)$-ideal of H and an Ω-Smarandache hyper $(\subseteq, \in)$-ideal of H. But it is not an Ω-Smarandache hyper $(\cap, \in)$-ideal of H since $(c \circ b) \cap I = \{b\} \neq \emptyset$ and $b \in I$, but $c \notin I$.

Definition 3.5. An Ω-Smarandache hyper $(<, \in)$-ideal I of H is said to be Ω-*reflexive* if $(\forall x \in \Omega)(x \circ x \subseteq I)$.

Example 3.6. Let H be an Ω-Smarandache hyper K-algebra in Example 3.2, where $\Omega = \{0, a, b\}$.Then $I = \{0, b\}$ is an Ω-Smarandache hyper $(<, \in)$-ideal of H. Moreover, noticing that $x \circ x \subseteq I$ for all $x \in \Omega$, we know that I is Ω-reflexive. But $\{0, a\}$ is a not Ω-reflexive since $b \circ b \nsubseteq \{0, a\}$.

Lemma 3.7. *Let A, B, C and I be subsets of H. Then we have the following property:*

(i) *If $A \subseteq B \ll C$, then $A \ll C$,*
(ii) *If $A \circ x \ll I$ for all $x \in \Omega$, then $a \circ x \ll I$ for all $a \in A$,*
(iii) *If I is an Ω-Smarandache hyper $(<, \in)$-ideal of H and if $A \circ x \ll I$ for all $x \in I$, then $A \ll I$.*

Proof. (i) Straightforward.

(ii) Let $x \in \Omega$ be such that $A \circ x \ll I$. Note that $a \circ x \subseteq A \circ x \ll I$ for all $a \in A$. It follows from (i) that $a \circ x \ll I$ for all $a \in A$.

(iii) Assume that I is an Ω-Smarandache hyper $(<, \in)$-ideal of H and let $x \in I$ be such that $A \circ x \ll I$. For any $a \in A$, we have $a \circ x \subseteq A \circ x \ll I$ and so $a \circ x \ll I$. It follows (c2) that $a \in I$ which proves that $A \subseteq I$, we have $A \ll I$. □

Theorem 3.8. *Let I be an Ω-Smarandache hyper $(<, \in)$-ideal of H and Ω-reflexive. Then*

$$(\forall x, y \in \Omega)((x \circ y) \cap I \neq \emptyset \Rightarrow x \circ y \ll I).$$

Proof. Let $x, y \in \Omega$ be such that $(x \circ y) \cap I \neq \emptyset$. Then there exists $a \in (x \circ y) \cap I$, and so

$$(x \circ y) \circ a \subseteq (x \circ y) \circ (x \circ y) \ll x \circ x \subseteq I;$$

whence $(x \circ y) \circ a \ll I$ by Lemma 3.7 (i). It follows from Lemma 3.7 (iii) that $x \circ y \ll I$. □

Theorem 3.9. *Let I be an an Ω-Smarandache hyper $(<, \in)$-ideal of H. If I is Ω-reflexive, then I is an Ω-Smarandache hyper $(\cap, \in)$-ideal of H.*

Proof. Let $x, y \in \Omega$ be such that $(x \circ y) \cap I \neq \emptyset$ and $y \in I$. Then $x \circ y \ll I$ by Theorem 3.8. It follows from (c2) that $x \in I$. Hence I is an Ω-Smarandache hyper $(\cap, \in)$-ideal of H. □

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Received: October 29, 2005