Refutation of the halting problem: not a problem

Taken from: en.wikipedia.org/wiki/Halting_problem

Given:

There is at least one n such that N(n) is equal to the statement H(a, i) meaning a halts on input i.

What follows is that:

Either there is an n such that N(n) = H(a,i), (1.1)
or there is an n' such that N(n') =~H(a,i). (2.1)

This means that this gives an algorithm to decide the halting problem [as Eq. 1.1 or Eq. 2.1 is a proof].

[There is an n such that N(n)=H(a,i)] Or [There is an n' such that N(n')=~H(a,i)] = proof (3.1)

"Since we know that there cannot be such an algorithm, it follows that the assumption that there is a consistent and complete axiomatization of all true first-order logic statements about natural numbers must be false." (4.1)

We assume the apparatus and method of Meth8 implementing variant logic system VŁ4.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: p q r s N n n' H(a,i); % possibility, for some (one); # necessity, for all;
~ Not; & And; + Or; > Imply; = Equivalent; @ Not equivalent
(p=p) 11, Tautology; (p@p) 00, Contradiction

The designated *proof* value is T.
The 16-valued truth tables are presented horizontally as row-major.

Eq. 1.1 is mapped as

(%q>((p&q)>s)) (1.2)

Eq. 2.1 is mapped as

(%r>((p&r)>~s)) (2.2)

Eq. 3.1 is mapped as

((%q>((p&q)>s)) + (%r>((p&r)>~s))) = (p=p) ; TTTT TTTT TTTT TTTT (3.2)

Because the truth table of Eq. 3.2 is tautologous (all T), this means the halting problem is in fact a theorem and not a problem. In other words:

> The assumption that there is a consistent and complete axiomatization of all true first-order logic statements about natural numbers must be *tautologous*. (4.2)

However, if Eq. 3.1 is written to replace the ">s" (implies s) in the antecedent parts with "=s" (equivalent to s), then Eq. 3.1 maps as

((%q>((p&q)=s)) + (%r>((p&r)=~s))) = (p=p) ; TTNT TTTT TTTT TNTT (3.3)

Because the truth table of Eq. 3.3 is not tautologous (not all T, but with some N as the non-contingent value of truth), this means the halting problem is not a problem of contradiction but rather an expression with values close to but not quite tautologous.

If the universal quantifier is applied to Eq. 3.3 on both main segments of the antecedent and consequent, then Eq. 3.3 maps as

#((%q>((p&q)=s)) + (%r>((p&r)=~s))) = #(p=p) ; TTTT TTTT TTTT TTTT (3.3)

and the halting problem becomes tautologous with the same status of theorem and result as in Eq. 3.2.

We conclude that Alan Turing's difficulty was in expressing the halting problem in the format of a two-valued logic which was not as expressive as in a four-valued logic to show nuances of what exactly the equation stated.

In comparison to Gödel's incompleteness theorems, Turing's halting problem has no superficial similarities other than being refuted as not a problem. Hence in contrast, both expressions are disparate and ultimately unrelated as to content meaning.