Palindromic abundant numbers P for which P-q^2+1 is an abundant number for any q prime greater than 3

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Abstract. In this paper I make the following observation: there exist palindromic abundant numbers P such that n = P – q^2 + 1 is an abundant number for any q prime, q ≥ 5 (of course, for q^2 < P + 1). The first such P is the first palindromic abundant number 66 (with corresponding [q, n] = [5, 42], [7, 18]. Another such palindromic abundant numbers are 222, 252, 282, 414, 444, 474, 606, 636, 666. Up to 666, the palindromic abundant numbers 88, 272, 464, 616 don't have this property. Questions: are there infinite many such palindromic abundant numbers? What other sets of integers have this property beside palindromic abundant numbers?

Observation:

There exist palindromic abundant numbers P such that n = P – q^2 + 1 is an abundant number for any q prime, q ≥ 5 (of course, for q^2 < P + 1). The first such P is the first palindromic abundant number 66 (with corresponding [q, n] = [5, 42], [7, 18].

Note: see the sequence A005101 in OEIS for abundant numbers; see the sequence A098775 in OEIS for palindromic abundant numbers.

Questions: are there infinite many such palindromic abundant numbers? What other sets of integers have this property beside palindromic abundant numbers?

The first ten such palindromic abundant numbers:

: P = 66 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 42], [7, 18];

: P = 222 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 198], [7, 174], [11, 102], [13, 54];

: P = 252 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 228], [7, 204], [11, 132], [13, 84];

: P = 282 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 258], [7, 234], [11, 162], [13, 114];

: P = 414 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 390], [7, 366], [11, 294], [13, 246];

: P = 444 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 420], [7, 396], [11, 324], [13, 276], [17, 156], [19, 84];

: P = 474 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 450], [7, 426], [11, 354], [13, 306], [17, 186], [19, 114];

: P = 606 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 585], [7, 558], [11, 486], [13, 438], [17, 318], [19, 246], [23, 78];

: P = 636 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 612], [7, 588], [11, 516], [13, 468], [17, 348], [19, 276], [23, 108];

: P = 666 (indeed, P – q^2 + 1 = n, abundant, for [q, n] = [5, 642], [7, 618], [11, 546], [13, 498], [17, 378], [19, 306], [23, 138].

The other palindromic abundant numbers P up to 666, i.e. 88, 272, 464, 616 don't have the property showed: P – q^2 + 1 is deficient at least for [q, n] equal to, respectively, [5, 64], [5, 248], [11, 344], [5, 592].

From two randomly chosen larger palindromic abundant numbers, one doesn't have the property showed, i.e. 8888 (n = 8889 – 25 = 8864 being deficient) and one does it, i.e. 8448 (n = 8449 – q^2 is abundant for [q, n] equal to [5, 8424], [7, 8400], [11, 8328], [13, 8280], [17, 8160], [19, 8088], [23, 7920], [29, 7608], [31, 7488], [37, 7080], [41, 6768], [43, 6600], [47, 6240], [53, 5640], [59, 4968], [61, 4728], [67, 3960], [71, 3408], [73, 3120], [79, 2208], [83, 1560], [89, 528]).