The world constructed as a simple partition function

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We propose a simple partition function that unifies a surprisingly large amount of physical laws. The partition function is constructed from two conjugate-pairs: 1) an entropic-force conjugated to a thermal-length and 2) an entropic-power conjugated to a thermal-time. From its equation of state, we derive the Schrödinger equation, the Dirac equation, special relativity, general relativity, dark energy, Newton's law of gravitation, and Newton's law of inertia and show that its Lagrange multipliers are the Planck units. We also propose a solution to the problem of the arrow of time as a natural consequence of the construction.

Contents

1 Introduction

As the Planck units are allegedly constant throughout the universe, we had the idea to construct a partition function of statistical physics such that the Lagrange multipliers are the Planck units. Furthermore, we injected as thermodynamics conjugate-pairs the two quantities that we felt were the most fundamental: time and space. To recover the units of energy, time must be multiplied by a power and length (space) must be multiplied by a force; thus, the partition function describes arbitrary micro-states in terms of both space and time. Due to the simplicity and generality of the construction, it is perhaps reassuring that special relativity, general relativity, and dark energy are provable solutions of its equation of state. Furthermore, thermal fluctuations along the time and space quantities produce the Schrödinger and Dirac equations as thermo-statistical extensions to classical analogues. Thus, the construction suggests that both general relativity and the quantum world emerge from a more fundamental thermo-statistical world.

1.1 Statistical physics

We will provide a brief recap of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy,

$$S = -k_B \sum_{x \in X} p(x) \ln p(x) \tag{1.1}$$

subject to the fixed macroscopic quantities. The solution for this is the Gibbs ensemble. Typical thermodynamic quantities are:

quantity	*name*	*units*	*type*	
$T = 1/(k_B\beta)$	temperature	K	intensive	(1.2)
E	energy	J	extensive	(1.3)
$p = \gamma/\beta$	pressure	J/m^3	intensive	(1.4)
V	volume	m^3	extensive	(1.5)
$\mu = \delta/\beta$	chemical potential	J/kg	intensive	(1.6)
N	number of particles	kg	extensive	(1.7)

Taking these quantities as examples, the partition function becomes:

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{1.8}$$

The probability of occupation of a micro-state is:

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{1.9}$$

The average values and their variance for the quantities are:

$$\overline{E} = \sum_{x \in X} p(x) E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{1.10}$$

$$\overline{V} = \sum_{x \in X} p(x) V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{1.11}$$

$$\overline{N} = \sum_{x \in X} p(x) N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{1.12}$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{E,V} = -\frac{\mu}{T} \tag{1.13}$$

, which can be summarized as

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{1.14}$$

This is known as the equation of state of the thermodynamic system. The entropy can be recovered from the partition function and is given by:

$$S = k_B \left(\ln Z + \beta \overline{E} + \gamma \overline{V} + \delta \overline{N} \right) \tag{1.15}$$

2 *First proposed partition function: Time and Space*

We propose the following partition function, constructed as a Gibbs ensemble:

$$Z(\beta, \mathcal{S}, F) = \sum_{q \in \mathcal{Q}} e^{-\beta [E(q) - \mathcal{S} f(q) - F x(q)]} \tag{2.1}$$

where

quantity	*name*	*units*	*type*	
$T = 1/(k_B\beta)$	temperature	K	intensive	(2.2)
$\mathcal{S}$	entropic action	Js	intensive	(2.3)
F	entropic force	J/m	intensive	(2.4)
$E(q)$	energy	J	extensive	(2.5)
$f(q)$	thermal frequency	$1/s$	extensive	(2.6)
$x(q)$	thermal space	m	extensive	(2.7)

The partition function includes the familiar entropic force and the unfamiliar entropic action. Its equation of state is:

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{x} \tag{2.8}$$

We can convert it to a more intuitive representation by converting the frequency to a time and the action to a power. Let us do that now.

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{x} \tag{2.9}$$

$$TdS = d\overline{E} + \mathcal{S}d(\overline{t}^{-1}) + Fd\overline{x} \qquad [f := 1/t] \tag{2.10}$$

$$TdS = d\overline{E} - \mathcal{S}\overline{t}^{-2}d\overline{t} + Fd\overline{x} \qquad \left[d(t^{-1}) = -t^{-2}dt\right] \tag{2.11}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \qquad \left[P := \mathcal{S}t^{-2}\right] \tag{2.12}$$

This representation introduces two new quantities, defined as:

quantity	*name*	*units*	*type*	
P	entropic power	J/s	intensive	(2.13)
$t(q)$	thermal time	s	extensive	(2.14)

Thus, the equation of state admits these two formulations:

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{x} \qquad \text{action-frequency formulation} \tag{2.15}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \qquad \text{power-time formulation} \tag{2.16}$$

which we will refer to throughout the paper.

2.1 *Regimes and cycles*

We will derive the familiar laws of physics by studying the equation of state in terms of its regimes. To do so, we will fix some derivatives (e.g. $dS = 0$) and analyse what happens when we let the others vary.

2.2 *Special relativity*

Here, we use the power-time formulation and pose $dS = 0$ and $dE = 0$. We obtain the fundamental relation of special relativity linking space to time.

$$0 = -Pd\bar{t} + Fd\bar{x} \tag{2.17}$$

$$Fd\bar{x} = Pd\bar{t} \tag{2.18}$$

$$d\bar{x} = \frac{P}{F}d\bar{t} \tag{2.19}$$

As the power P and the force F are Lagrange multipliers of the partition function, they are constant throughout the system. Therefore, their quotient is also a constant.

$$c := \frac{P}{F} \tag{2.20}$$

Therefore,

$$d\bar{x} = cd\bar{t} \tag{2.21}$$

As the units of P/F are meters per second, c will be our working definition of the speed of light.

Remark: When P is the Planck power and F is the Planck force, we do indeed recover the speed of light:

$$P\left(\frac{1}{F}\right) = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{2.22}$$

2.3 *Light cones as thermodynamic cycles*

In this section, we look at the thermodynamic cycle of the system transiting through time and space starting at O to A to B and back to O, as illustrated on Figure 1. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles, such as those involving pressure and volume. Interestingly, the cycles can also be interpreted as light cones.

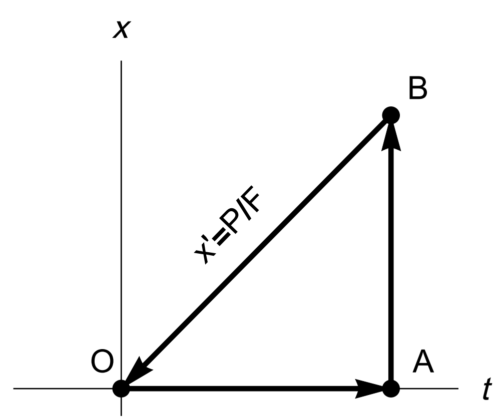

Figure 1: A "thermodynamic cycle" through space and time.

O to A: As O is translated forward in time to A while keeping the distance constant ($d\bar{x} = 0$), the entropy decreases over time.

$$\left(TdS = Fd\bar{x} - Pd\bar{t}\right|_{d\bar{x}=0} \tag{2.23}$$

$$\implies \frac{dS}{dt} = -\frac{P}{T} \tag{2.24}$$

A to B: As A is translated forward in space to B while keeping the time constant ($d\bar{t} = 0$), the entropy increases over the distance.

$$\left(TdS = Fd\bar{x} - Pd\bar{t}\right|_{d\bar{t}=0} \tag{2.25}$$
$$\implies \frac{dS}{dx} = \frac{F}{T} \tag{2.26}$$

O to B: As O is translated forward both in time and in space to B while keeping the entropy constant ($dS = 0$), the system has a speed of c.

$$\left(TdS = Fd\bar{x} - Pd\bar{t}\right|_{dS=0} \tag{2.27}$$
$$\implies \frac{dx}{dt} = \frac{P}{F} = c \tag{2.28}$$

We conclude that an object travelling at speed c is neither encouraged nor discouraged by entropy. The speed of light represents an inflexion point in the rate of entropy production over time. We will return to that notion in the section on the arrow of time.

2.4 *Lorentz's transformation*

To recover the Lorentz's factor γ, let us consider figure 2. Two observers start at the origin S and travel in space-time respectively to O and O'. We regard O' as traveling at speed $|v|$ in the reference frame of O. From standard trigonometry, we derive the following values for the length of the segment;

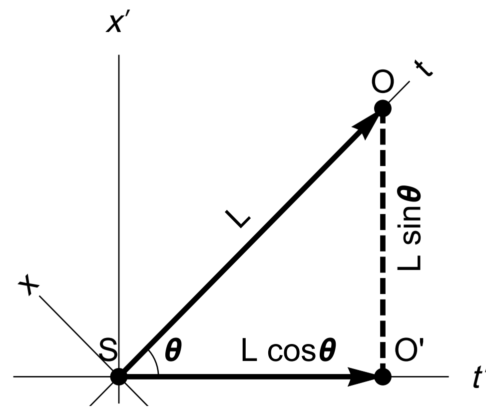

Figure 2: The spacetime intervals between two observers. Here O' travels at speed $|v|$ in O's reference frame.

Segment	Length	
$\lvert\overline{SO}\rvert$	L	(2.29)
$\lvert\overline{SO'}\rvert$	$L\cos\theta$	(2.30)
$\lvert\overline{O'O}\rvert$	$L\sin\theta$	(2.31)

From the Pythagorean theorem and solving for $\cos\theta$, we obtain:

$$|\overline{SO}|^2 = |\overline{SO'}|^2 + |\overline{O'O}|^2 \tag{2.32}$$
$$L^2 = (L\cos\theta)^2 + (L\sin\theta)^2 \tag{2.33}$$
$$1 = (\cos\theta)^2 + (\sin\theta)^2 \tag{2.34}$$
$$\sqrt{1-(\sin\theta)^2} = \cos\theta \tag{2.35}$$

We consider that the distance between two observers moving at constant speed is simply vt. Hence, $|\overline{O'O}| = vt$. Solving for $\sin\theta$, we obtain:

$$|\overline{O'O}| = vt = L \sin\theta \tag{2.36}$$

$$\implies \sin\theta = \frac{vt}{L} \tag{2.37}$$

From equation (2.35) and (2.37), we get the reciprocal of the Lorentz factor:

$$\sqrt{1 - \frac{v^2 t^2}{L^2}} = \cos\theta = \gamma^{-1} \tag{2.38}$$

$$\implies \gamma = \frac{1}{\sqrt{1 - \frac{v^2 t^2}{L^2}}} \tag{2.39}$$

Finally, we consider that L is the distance travelled by O in the reference frame of O' such that the entropy of O is constant over time. According to the relation $dx = cdt$, for this to be the case, the speed of O must be c. Thus, the distance travelled by O during time t is $L = ct$. We obtain:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.40}$$

which is the well-known Lorentz factor and is the multiplication constant connecting $|\overline{SO}|$ to $|\overline{SO'}|$.

2.5 *Inertial mass*

In this section, we will need to use the Unruh temperature[1]. As can be reviewed in the citations provided, the Unruh temperature is an exact result obtained from special relativity. The Unruh effect is the prediction that an accelerating observer will observe blackbody radiation (at the Unruh temperature), whereas an inertial observer would observe none. The Unruh temperature is:

$$T = \frac{\hbar a}{2\pi c k_B} \tag{2.41}$$

The Unruh temperature connects acceleration to the temperature. We will use it here to convert an entropic force expressed in terms of a temperature to an entropic force expressed in terms of acceleration.

Furthermore, we start from the power-time formulation and pose $d\overline{t} = 0$ and $d\overline{E} = 0$. As originally done by Eric Verldinde[2], from these starting points, we can derive $F = ma$ as follows:

[1] Stephen A. Fulling. Nonuniqueness of canonical field quantization in riemannian space-time. Phys. Rev. D, 7:2850–2862, May 1973. DOI: 10.1103/PhysRevD.7.2850. URL https://link.aps.org/doi/10.1103/PhysRevD.7.2850; P C W Davies. Scalar production in schwarzschild and rindler metrics. Journal of Physics A: Mathematical and General, 8(4): 609, 1975. URL http://stacks.iop.org/0305-4470/8/i=4/a=022; W. G. Unruh. Notes on black-hole evaporation. Phys. Rev. D, 14:870–892, Aug 1976. DOI: 10.1103/PhysRevD.14.870. URL https://link.aps.org/doi/10.1103/PhysRevD.14.870; and Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

[2] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

$$TdS = Fd\overline{x} \tag{2.42}$$

$$F = T\frac{dS}{dx} \tag{2.43}$$

$$F = \left(\frac{\hbar a}{2\pi c k_B}\right)\frac{dS}{dx} \tag{2.44}$$

$$F = \left(\frac{\hbar}{2\pi c k_B}\frac{dS}{dx}\right)a \tag{2.45}$$

This equation corresponds to $F = ma$ provided that $\left(\frac{\hbar}{2\pi c k_B}\frac{dS}{dx}\right) = m$. How reasonable is that? Well, for it to be the mass, it suffices that dS/dx is the inverse of the reduced Compton wavelength multiplied by a constant. Recall that the reduced Compton wavelength is $\hbar/(mc)$. Let us investigate:

$$\frac{\hbar}{2\pi c k_B}\frac{dS}{dx} = m \implies \frac{dS}{dx} = 2\pi k_B\left(\frac{mc}{\hbar}\right) \tag{2.46}$$

We obtain a relation between entropy and x. What could this mean? It means two things.

1. The further away an object is from the origin, the higher its positional entropy.

2. The more massive an object is, the higher its positional entropy.

Why then the factor 2π? The presence of π suggest a connection between a line and a circle. Therefore, a possible interpretation is that the entropy associated with positional entropy is scaled proportionally to the curvature of a circle (we can think of it as a one-dimensional case of the holographic principle). Then, as an object with a small Compton wavelength that can be more finely located, it requires more positional entropy to describe its position than an object with a large Compton wavelength. Why then the factor k_B? The factor k_B converts the reduced Compton wavelength to the units of entropy/length (joules per kelvin per meter).

3 *Second proposed partition function: Time and generalized length*

The first partition function we proposed was constructed with an entropic-force conjugated with a thermal-length. The length was, of course, linear and expressed by x. In this section, however, we extend the representation to consider a thermal-length described by an arbitrary function; after all, the mass of the universe is not linearly distributed with clockwork precision. To achieve this, we consider an arbitrary function $l(q) : q \to \mathbb{R}$ used to express the lengths of the micro-states. We will study such function via a Taylor expansion. A Taylor expansion requires that $\mathcal{Q}$ as in $q \in \mathcal{Q}$ be uncountable. As $l(q)$ is an arbitrary length with meter units, it will still be conjugated with the entropic-force. The Taylor expansion of $Fl(q)$ is:

$$Fl(q) = Fl(0) + Fl'(0)q + \frac{Fl''(0)}{2}q^2 + \frac{Fl'''(0)}{6}q^3 + O(q^4) \tag{3.1}$$

and its derivative with respect to q is:

$$Fdl(q) = Fl'(0)dq + Fl''(0)qdq + \frac{Fl'''(0)}{2}q^2dq + 4O(q^3)dq \tag{3.2}$$

As the micro-states $q \in \mathcal{Q}$ must be uncountable for the Taylor expansion of $l(q)$ to be well defined, the partition function must be continuous. Therefore, it becomes:

$$Z = \frac{1}{h}\int e^{-\beta[E(q)+\mathcal{S}f(q)+Fl(q)]}dq \tag{3.3}$$

and is integrated over $\mathcal{Q}$. Likewise, its equation of state is

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{l} \qquad \text{action-frequency formulation} \tag{3.4}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{l} \qquad \text{power-time formulation} \tag{3.5}$$

3.1 *Taylor expansion of $d\bar{l}$*

We convert the term $d\bar{l}$ of the power-time formulation into its Taylor expansion. The first change we will do is rename $q := x$. The multiplication term 4 in $4O(x^3)$ can be absorbed in to $O(x^3)$.

$$Fdl(x) = Fl'(0)dx + Fl''(0)xdx + \frac{Fl'''(0)}{2}x^2dx + O(x^3)dx \tag{3.6}$$

Then, injecting it into the power-time formulation, we obtain:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{l} \tag{3.7}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fl'(0)d\overline{x} + Fl''(0)\overline{x}d\overline{x} + \frac{Fl'''(0)}{2}\overline{x}^2d\overline{x} + O(\overline{x}^3)d\overline{x} \tag{3.8}$$

Something interesting appends with the units of the Taylor expansion. Let us investigate:

Taylor term	*quantity*	*units*	
$Fl'(0)d\overline{x}$	F	N	(3.9)
$''$	$l'(0)$	$\nexists$	(3.10)
$''$	$d\overline{x}$	m	(3.11)
$Fl''(0)xd\overline{x}$	F	N	(3.12)
$''$	$l''(0)$	$1/m$	(3.13)
$''$	$xd\overline{x}$	m^2	(3.14)
$Fl'''(0)x^2d\overline{x}$	F	N	(3.15)
$''$	$l'''(0)$	$1/m^2$	(3.16)
$''$	$x^2d\overline{x}$	m^3	(3.17)
$\vdots$	$\vdots$	$\vdots$	

Since $xd\overline{x}$ has units m^2 and $x^2d\overline{x}$ has units m^3, we pose $\gamma d\overline{A} := xd\overline{x}$ and $\alpha d\overline{V} := x^2d\overline{x}$. Furthermore, as $l'(0)$ has no units, we define it as the baseline $l'(0) := 1$ and we define $l''(0) := l_A/L$ and $l'''(0) := l_V/A$ as they respectively have units m^{-1} and m^{-2}. For empirical reasons (e.g., the observable universe is a sphere), we consider that $\gamma d\overline{A}$ describes the surface of a sphere and that $\alpha d\overline{V}$ describes the volume of a sphere. Therefore, to properly link $\gamma d\overline{A}$ to $xd\overline{x}$, the factor γ must be $1/(4\pi)$ and the factor α must be $3/(4\pi)$. Introducing these replacements, the equation of state becomes:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + l_A\frac{F}{4\pi L}d\overline{A} + l_V\frac{3F}{8\pi A}d\overline{V} + O(\overline{x}^3)d\overline{x} \tag{3.18}$$

where l_A and l_V are leftovers of the Taylor coefficients. We can recover three relations by varying the intensity of the Taylor approximation.

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + O(\overline{x})d\overline{x} \tag{3.19}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + l_A\frac{F}{4\pi L}d\overline{A} + O(\overline{x}^2)d\overline{x} \tag{3.20}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + l_A\frac{F}{4\pi L}d\overline{A} + l_V\frac{3F}{8\pi A}d\overline{V} + O(\overline{x}^3)d\overline{x} \tag{3.21}$$

With the first relation, and by posing $O(\overline{x})d\overline{x} \to 0$, we recover the first proposed partition function:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \tag{3.22}$$

Thus, the results derived with the previous partition function are importable into this more general equation of state.

3.2 Gravitational constant

To find a suitable definition for G, we must derive Newton's law of gravitation from the equation of states. A derivation of Newton's law of gravitation from the entropic perspective has been done before[3]. To obtain it, we start from the power-time formulation expanded with two Taylor terms:

[3] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + O(\overline{x}^2)d\overline{x} \tag{3.23}$$

Then, we pose $dE = 0$, $d\overline{t} = 0$ and $O(\overline{x}^2)d\overline{x} \to 0$. We obtain:

$$TdS = Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} \tag{3.24}$$

We notice that the term $d\overline{x}$ grows linearly as the term $d\overline{A}$ grows quadratically. Thus, as $\overline{x}$ is increased, there will be a point where $d\overline{A} \gg d\overline{x}$ (recall that $d\overline{A} = \overline{x}d\overline{x}$). The approximation yields:

$$TdS = l_A \frac{F}{4\pi L} d\overline{A} \tag{3.25}$$

This regime contains the holographic principle and, as a result, the entropy of the system grows proportional to $\overline{x}^2$, an area law. To recover Newton's law of gravity, and consistent with the holographic principle, we further pose the assumption that an entropy is associated to this area law and is given by bits occupying a small area L^2 on the surface of a sphere. In this case, the total number of bits on the surface is given by:

$$N = \frac{4\pi x^2}{L^2} \qquad \text{holographic assumption} \tag{3.26}$$

The term $\overline{x}d\overline{x}$ of the equation of state is associated to $x^2/2$ in the partition function. As a result of the equipartition theorem, which applies to quadratic energy terms, the average energy will be $\overline{E} = k_B T/2$. Multiplying $\overline{E}$ by N, we get the total energy associated with $\overline{x}d\overline{x}$:

$$E = \frac{1}{2}\left(\frac{4\pi x^2}{L^2}\right) k_B T \tag{3.27}$$

$$\implies T = \frac{L^2}{2\pi k_B}\frac{E}{x^2} \tag{3.28}$$

Consistent with thermodynamic equilibrium, we obtain a temperature T. As our goal is to recover the gravitational force, we inject this temperature in the entropic force relation.

$$F d\overline{x} = T dS \qquad \text{entropic force} \tag{3.29}$$

$$F d\overline{x} = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right) dS \qquad \text{derived temperature} \tag{3.30}$$

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right)\frac{dS}{d\overline{x}} \tag{3.31}$$

What then is $dS/d\overline{x}$? Recall equation 2.46; the connection between the reduced Compton wavelength and the distance entropy.

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right)\left(2\pi k_B \frac{mc}{\hbar}\right) \qquad \text{Compton wavelength} \tag{3.32}$$

$$F = \left(\frac{L^2 c}{\hbar}\right)\frac{Em}{x^2} \qquad \text{clean up} \tag{3.33}$$

We then convert E to its rest mass via $E = mc^2$.

$$F = \left(\frac{L^2 c^3}{\hbar}\right)\frac{Mm}{x^2} \tag{3.34}$$

We obtain the Newton's law of gravitation along with a definition for G.

$$F = G\frac{Mm}{x^2} \tag{3.35}$$

$$\implies G := \frac{L^2 c^3}{\hbar} \tag{3.36}$$

which further implies that

$$L = \sqrt{\frac{\hbar G}{c^3}} \qquad \text{Planck's length} \tag{3.37}$$

3.3 *Wave energy equation*

Here, we use the action-frequency formulation to derive a relation between frequency and energy. We pose $d\overline{x} = 0$ and $dS = 0$. We obtain:

$$d\overline{E} = \mathcal{S} d\overline{f} \tag{3.38}$$

Integrating each side, we obtain:

$$\int d\overline{E} = \int \mathcal{S} d\overline{f} \tag{3.39}$$

$$\overline{E} = \mathcal{S}\overline{f} + C \tag{3.40}$$

Posing $C = 0$ and $\mathcal{S} = h$, we obtain the photon frequency to energy relation $\overline{E} = h\overline{f}$.

3.4 Planck units

We have now obtained a definition for three of the fundamental constants.

$$h := \mathcal{S} \qquad c := \frac{P}{F} \qquad G := \frac{L^2c^3}{\hbar} \tag{3.41}$$

Thus, we can now prove that the Lagrange multipliers of the equation of states P and F are indeed the Planck units.

$$\begin{aligned}
& & \textit{expression} & \qquad \textit{quantity} \\
G = \frac{L^2c^3}{\hbar} & \implies & L = \sqrt{\frac{\hbar G}{c^3}} & \qquad \text{Planck's length} && (3.42) \\
& & t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}} & \qquad \text{Planck's time} && (3.43) \\
& & P = t^{-2}\mathcal{S} = 2\pi\frac{c^5}{G} & \qquad \text{Planck's power*} && (3.44) \\
\frac{P}{F} = c & \implies & F = 2\pi\frac{c^4}{G} & \qquad \text{Planck's force*} && (3.45) \\
& & \vdots & \qquad \vdots
\end{aligned}$$

*The reader will notice that we have obtained the definitions of P and F with an added multiplication constant 2π; whereas in the literature these quantities are defined without it. The definitions we have here are actually the correct ones. Indeed, in the literature, the Planck time is connected to the Planck angular frequency via $\omega_P = 1/t_P$. In reality, however $\omega = 2\pi/t$. Thus, for our equations to balance out, we cannot ignore the factor 2π and must use the corrected value for the Planck units which are $P = 2\pi c^5/G$ and $F = 2\pi c^4/G$.

3.5 General relativity

In this section, we will show how the term $d\overline{A}$ suggests that general relativity is entropic and emergent. Our goal is to derive the Einstein field equation of general relativity, starting from the $d\overline{A}$ regime. First, we start from the power-time formulation expanded with two Taylor terms:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + l_A\frac{F}{4\pi L}d\overline{A} + O(\overline{x}^2)d\overline{x} \tag{3.46}$$

Then, we pose $dS = 0$, $d\overline{t} = 0$ and $O(\overline{x}^2)d\overline{x} \to 0$. We obtain:

$$d\overline{E} = Fd\overline{x} + l_A\frac{F}{4\pi L}d\overline{A} \tag{3.47}$$

We notice that the term $d\overline{x}$ grows linearly and the term $d\overline{A}$ grows quadratically. Thus, as $\overline{x}$ is increased, there will be a point where $d\overline{A} \gg d\overline{x}$. The approximation yields:

$$d\overline{E} = l_A \frac{F}{4\pi L} d\overline{A} \tag{3.48}$$

Deriving general relativity from $dE = l_A \frac{F}{4\pi L} dA$ has indeed been done before in the literature, notably by Ted Jacobson, then later (and differently) by Erik Verlinde[4]. Furthermore, key insights were provided by Christoph Schiller[5]. Here, we will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

First, the entropic force F is constant throughout the system as a result of being a Lagrange multiplier. We have already shown that F is the Planck force. This has allowed us to derive special relativity and the speed of light; therefore, we must continue to use F as the Planck force here.

What then is L? Recall that earlier we used the Unruh temperature to link T to an acceleration and derive $F = ma$. Here and likewise, we will use special relativity to derive a relation between length and acceleration and use it to replace L. As per Schiller's paper, we select L as the maximum length that an accelerated object can have under special relativity[6].

$$L = \frac{c^2}{2a} \tag{3.49}$$

L is perhaps better understood as the acceleration of circular motion ($r = v^2/a$) at the speed of light ($v = c$). In the present context, L is the length associated with the maximum force, the Planck force. In the context of maximums, the force cannot accelerate the object beyond the speed of light, and therefore is best defined for a circular motion produced by a force perpendicular to the direction of motion. The maximum acceleration changes the direction of the motion, but does not increases the speed beyond the speed of light.

With $F = 2\pi c^4/G$, we obtain:

$$d\overline{E} = l_A \frac{c^2}{G} a d\overline{A} \tag{3.50}$$

With this result, Jacobson's proof directly follows. Starting from $dE = TdS$, he first connects dE to an arbitrary coordinate system and energy flow rates:

$$d\overline{E} = \int T_{ab} k^a d\Sigma^b \tag{3.51}$$

[4] Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75:1260–1263, Aug 1995. DOI: 10.1103/PhysRevLett.75.1260. URL https://link.aps.org/doi/10.1103/PhysRevLett.75.1260; and Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

[5] Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2

[6] Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2; Wolfgang Rindler. Relativity: special, general, and cosmological, 2003; and Ray A D'Inverno. Introducing einstein's relativity. 1992

Here T_{ab} is an energy-momentum tensor, k is a killing vector field, and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that the area part can be rewritten as follows:

$$a d\overline{A} = c^2 \int R_{ab} k^a d\Sigma^b \tag{3.52}$$

where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation, giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab} k^a d\Sigma^b = l_A \frac{c^2}{G} \int R_{ab} k^a d\Sigma^b \tag{3.53}$$

, which can only be satisfied if

$$T_{ab} = l_A \frac{c^2}{G} \left[R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right] \tag{3.54}$$

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant). Only the numerical value of l_A remains. The exact formulation of the field equation is obtained by posing the numerical value to $g_A := 1/(8\pi)$.

Remark: Had we not used the corrected Planck force ($F = 2\pi c^4/G$), we would have a 2π term dividing T_{ab} and l_A would have been $1/4$. Thus, the difference would have been absorbable. However, using the corrected Planck force has the consequence that all dimensionless numerical multipliers are attributed to the Taylor coefficient, making the derivation more aesthetically pleasing.

3.6 Dark energy

Connecting dark energy to a volumetric entropy has been suggested and discussed by other authors before[7]. First, we start from the power-time formulation expanded with three Taylor terms:

$$TdS = d\overline{E} - P d\overline{t} + F d\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + l_V \frac{3F}{8\pi A} d\overline{V} + O(\overline{x}^3) d\overline{x} \tag{3.55}$$

Them we pose $dE = 0$, $d\overline{t} = 0$ and $O(\overline{x}^3) d\overline{x} \to 0$. We obtain:

$$TdS = F d\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + l_V \frac{3F}{8\pi A} d\overline{V} \tag{3.56}$$

[7] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048; and Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic inflation. International Journal of Modern Physics A, 27(12):1250066, 2012. DOI: 10.1142/S0217751X12500662. URL http://www.worldscientific.com/doi/abs/10.1142/S0217751X12500662

We notice that as $d\overline{x}$ grows linearly, $d\overline{A}$ grows as the square and $d\overline{V}$ as the cube. Thus, there will be a point where $d\overline{V} \gg d\overline{A} \gg d\overline{x}$. The approximation yields:

$$TdS = l_V \frac{3F}{8\pi A} d\overline{V} \tag{3.57}$$

We notice that the factor F/A has the units of pressure. Hence, our goal will be to derive a value of the pressure p associated with volumetric entropy. As suggested by the factor F/A and in line with our earlier derivations, we will select F to be the corrected Planck force ($F = 2\pi c^4/G$) and will take A as the area of a sphere. In this case, the pressure relates to the force as

$$F = -pA \tag{3.58}$$

$$\implies p = -\frac{F}{A} = -\frac{F}{4\pi\overline{x}^2} \tag{3.59}$$

$$p = -\frac{c^4}{2G\overline{x}^2} \qquad \text{entropic pressure} \tag{3.60}$$

The sign of the force is negative because the force points in the direction of increased entropy, which is oriented outward of the enclosing area. Physically and as argued by Easson et al., it makes sense to connect the size of the sphere to the Hubble horizon. Therefore, we take the radius of the sphere to be the Hubble radius $x := c/H$. Finalizing our derivation, we obtain:

$$p = -\frac{c^2H^2}{2G} \tag{3.61}$$

This is close to the current measured value for the negative pressure associated with dark energy[8]. As we can see, the suggested entropic derivation of dark energy applies to the third term of the Taylor expansion.

[8] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048

4 *Discussion - Arrow of time*

Adding a time variable to a partition function adds a whole new dynamic to a thermal system. The system now becomes aware of future, past, and present configurations and can translate from time to space and from space to time for an entropic cost (provided that various limits are respected). By studying thermodynamic cycles involving space and time, we investigated what happens to the entropy when a system is translated forward or backward in time and draw conclusions that pertain to the arrow of time. In the model presented, space serves as an entropy sink for time; whose role is to deplete future alternatives to power *change* in the universe.

4.1 *Negative power*

In the power-time formulation, increasing $\bar{t}$, while keeping the other variables constant, decreases the entropy. Indeed, starting with the power-time formulation and posing $d\bar{x} = 0$, we obtain:

$$TdS = d\bar{E} - Pd\bar{t} \tag{4.1}$$

$$\implies T\frac{dS}{d\bar{t}} = \frac{d\bar{E}}{d\bar{t}} - P \tag{4.2}$$

As the law of conservation of energy requires that $d\bar{E}/d\bar{t} = 0$, we obtain the negative power:

$$T\frac{dS}{d\bar{t}} = -P \tag{4.3}$$

This result is expected for the following reason: to obtain the relation $d\bar{x} = cd\bar{t}$ with the correct signs, the power P must have a different sign than the force F in the equation of states. Thus, a positive force implies a negative power and vice versa. As we require a positive force to recover $F = ma$ (and not $F = -ma$), the sign of the force is already chosen for us. Therefore, the power must be negative.

We will now discuss this result in more detail.

Question: What is a negative power?

Let's take an example. Consider the case of an electric car; whose engine is powered by a battery. To propel the car, the battery supplies power to the engine. If the driver hits the breaks, such that regenerative breaking kicks in, the flow of power will reverse and the engine will supply power to the battery. Thus, the power is now considered to be negative and occurs when the engine depletes the energy of the system (e.g. the car slows down) to supply power to the battery.

Question: Why does time have a negative power?

Power is associated with time because it powers all *changes* that occur in the universe. To understand why it is negative, it helps to understand negative power in the context of thermodynamics. To do so, let's first recall its more familiar cousin: the negative temperature. If we understand temperature as the random movements of molecules, then a temperature is always equal to or above zero. However, statistical physics admits a generalized definition of temperature as the trade-off between energy and entropy. Most systems cannot admit a negative temperature because their entropy will always increase at higher energies; however, for some systems, e.g. the population inversion in a laser, the entropy saturates at higher energies. Thus, a negative temperature is possible.

In regards to time, the negative power has essentially the same interpretation; increasing time, while keeping the other variables constant, decreases the entropy. A decrease in entropy over time requires the application of a power.

4.2 *The second law of thermodynamics as an opposition to negative power*

Question: How does this result reconcile with the second law of thermodynamics, which states that entropy increases with time (or in some ideal cases stays constant)?

The power-time formulation admits other terms: $d\overline{x}$, $d\overline{A}$, and $d\overline{V}$. The term $-Pd\overline{t}$ encourages a reduction in the entropy over time, but the other variables, as their signs are positive, work in the other direction. Thus, the entropy of the system as a whole need not necessarily decrease over time. It is more accurate to say that increasing $\overline{t}$, while keeping the other variables constant, decreases the entropy. We will now study this into more detail.

To offset the decrease in entropy caused by the negative power, we suggest a proportional increase in the quantities $\overline{x}$, $\overline{A}$, and $\overline{V}$.

To simplify the power-time formulation, let us rename $\kappa := \frac{F}{16\pi L}$ and $p := \frac{3g_V F}{4\pi A}$ and pose $O(\overline{x}^3)d\overline{x} \to 0$. We obtain:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + \kappa d\overline{A} + pd\overline{V} \tag{4.4}$$

Dividing both sides by $d\overline{t}$, we obtain:

$$T\frac{dS}{d\overline{t}} = \frac{d\overline{E}}{d\overline{t}} - P + F\frac{d\overline{x}}{d\overline{t}} + \kappa\frac{d\overline{A}}{d\overline{t}} + p\frac{d\overline{V}}{d\overline{t}} \tag{4.5}$$

As per the law of conservation of energy, posing $d\overline{E}/d\overline{t} = 0$, we obtain:

$$T\frac{dS}{d\overline{t}} = -P + F\frac{d\overline{x}}{d\overline{t}} + \kappa\frac{d\overline{A}}{d\overline{t}} + p\frac{d\overline{V}}{d\overline{t}} \tag{4.6}$$

This result puts in opposition the change of entropy caused by a change of $\overline{t}$ to the change in entropy caused by a change of $\overline{x}$, $\overline{A}$ and $\overline{V}$. To investigate this result, let us look at these three cases:

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} < P \quad \Longrightarrow \quad \frac{dS}{dt} < 0 \quad \text{decreasing entropy} \tag{4.7}$$

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} = P \quad \Longrightarrow \quad \frac{dS}{dt} = 0 \quad \text{constant entropy} \tag{4.8}$$

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} > P \quad \Longrightarrow \quad \frac{dS}{dt} > 0 \quad \text{increasing entropy} \tag{4.9}$$

At (4.8), we have an inflexion point and a shift occurs in the direction of the production of entropy over time. It is the point at which the production of entropy caused by the space quantities overtake and exceed the reduction in entropy caused by the time quantity. The second law of thermodynamics states that $dS/dt \geq 0$ and will hold for (4.8) and (4.9), but will be violated for (4.7).

4.3 *Arrow of time*

In this section, we will explain why these results provide us with an understanding of the arrow of time. Indeed, it links the arrow of time to three concepts: 1) a reduction in entropy over time caused by the negative power, 2) an increase in entropy over time caused by the space quantities, and 3) a closed system's inability to reduce its own entropy. We will see how it corresponds to an observer's perception of time.

1. *At the beginning of time* all possible future alternatives are compatible with the present. Thus, the pool of entropy accessible to $\bar{t}$ is maximal. In contrast, the entropy associated with the space quantities is zero. Thus, the occupied micro-states have to be located at the same point in space. This matches our current empirical data regarding the Big Bang for which the entropy of space was very low and the entropy of time, as the future was as of yet undetermined, was very high.

2. *During the evolution* the future becomes past, thus the possible future alternatives are rarefied. This reduction in entropy caused by a growth in $\bar{t}$ produces a negative entropic power fuelled by the growth of entropy in the space quantities.

3. *At the "end of time"* there is no future alternatives. The full history of the system is now "set in stone". The system can no longer produce an entropic power to fuel changes and the entropy associated with the space quantities is at its maximum.

Question: The conventional wisdom is that the arrow of time is connected to an increase in entropy with time. Are you suggesting something else?

A partition function constructed without the use of a time quantity will follow the second law of thermodynamics. This statistical effect is partially explained by the H-theorem of Boltzmann; however, this changes when time is inserted as a thermodynamic quantity. Such a partition function then becomes aware of past, present, and future configurations. The rarefaction of futures configurations as time is increased is associable to a time which moves forward by closing future alternatives as it creates a past. Thus, an increase in the time quantity, while keeping other quantities constant, must be followed by a decrease in entropy.

To help fixate the idea, let us look at an example:

4.4 *The physics of future alternatives*

Here, we give a simple system which follows the requirements of the equation of states.

Suppose a system with n open binary future alternatives. At $\bar{t} = 0$, there are 2^n possible futures each equally compatible with the present macroscopic state. Thus, the entropy of the system (which includes a description of its possible futures) is equal to $S = k_B n \ln 2$. As time is increased, *events* occurs and future alternatives are closed. Say, at $\bar{t} = 1$, one event occurs: Thus, one future alternative becomes fixed to a specific value and the entropy of the system is reduced to $S = k_B(n-1)\ln 2$.

For instance, we might have:

$\bar{t}$	*event*	*future alternatives*	*entropy*	
0	Big Bang	$\{b_1, b_2, b_3, b_4, ..., b_n\}$	$k_B n \ln 2$	(4.10)
1	$b_3 \to 0$	$\{b_1, b_2, b_3 := 0, b_4, ..., b_n\}$	$k_B(n-1)\ln 2$	(4.11)
2	$b_1 \to 1$	$\{b_1 := 1, b_2, b_3 := 0, b_4, ..., b_n\}$	$k_B(n-2)\ln 2$	(4.12)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

As events occurs over time, an entropic power is generated. Furthermore, the second law of thermodynamics imposes that the space quantity ($Fd\bar{x}$) must grow proportionally. To maintain $dS/d\bar{t} = 0$, the growth must correspond to $d\bar{x} = cd\bar{t}$; special relativity. Extending this example to the continuous partition function, we also recover general relativity and dark energy as per the earlier derived equations of states. In the continuous case, we would use the natural bit (the nat, in base e) to express future possibilities. A continuous event would consume a non-integer quantity of future possibilities.

Question: What is an example of an event?

To bear entropy, an event must be intrinsically random. Thus, quantum measurements are the primary candidates. For example, let's consider the quantum measurement of a spin in the z direction. Before measurement, as the spin can either be up or down, its entropy is $\propto \ln 2$. After measurement, the entropy of the spin falls to 0 and the entropy of the measurement system increases proportionally. Thus, the event would be the measurement itself.

Question: But a system cannot decrease its entropy over time without violating the second law of thermodynamics!

A system can decrease its entropy if it is connected to an entropy sink. For example, biological life can reduce its entropy but only at the cost of severely increasing it in its environment. This requires excess energy and, in the case of Earth, the Sun supplies it. Thus, the power-time conjugate can decrease the entropy as long as it is connected to a sink.

Question: So, there should be a sink in the universe available to offset the decrease in entropy caused by increasing $\bar{t}$?

In the case of time, the sink is the universe itself. The laws of physics that we have derived are in fact the limits required to produce an entropy sink of sufficient size to accommodate a forward direction of time for an observer (we will discuss this more rigorously in a moment in the section on limiting relations).

Question: Can we calculate the exact future before it occurs?

An observer cannot pre-calculate his exact future before it occurs without increasing the size of the entropy sink. Here we make a distinction between calculating a probable future versus the exact future. Calculating a probable future does not necessarily imply a reduction of entropy within the system, but calculating the exact future requires consuming the entropy of all possible alternative futures. Therefore, an entropy sink is required to offset the reduction. Calculating an exact future is equivalent to causing it.

Question: Does the second law of thermodynamics need to be corrected for the wider system, which includes future states?

Yes. Time is usually considered to be an independent background to statistical physics and, to our knowledge, statistical physics has not been used with a time quantity before. When we do add time as a thermodynamic quantity to a partition function, a new behaviour emerges. Indeed, an observer cannot move into the future unless all alternative futures are 'closed'. Thus, its time-entropy must decrease when he does. The second law of thermodynamics is a consequence of the system increasing its space-entropy to offset the reduction in future alternatives as time moves along. Thus, the second law is a subset of a more general conservation of entropy law.

4.5 Limiting relations

With our new interpretation of space as an entropy sink for time, let us immediately prove three limits from first principle: the speed of light, a limiting stiffness, and a limiting volumetric flow rate applicable to the universe. To prove that these are limits, we will consider the assumption that an observer who evolves forward in time must see a growth in the size of its available entropy sink to offset the reduction in future alternatives. The limit occurs when the sink exactly offsets the reduction in entropy attributable to time (in which case $dS/dt = 0$). First, let us see how the power-time formulation implies a limiting speed.

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \tag{4.13}$$

$$\frac{T}{F}\frac{dS}{d\overline{t}} = \frac{1}{F}\frac{d\overline{E}}{d\overline{t}} - \frac{P}{F} + \frac{d\overline{x}}{d\overline{t}} \tag{4.14}$$

As always $d\overline{E}/d\overline{t} = 0$

$$\frac{T}{F}\frac{dS}{d\overline{t}} = -\frac{P}{F} + \frac{d\overline{x}}{d\overline{t}} \tag{4.15}$$

To see why this implies a limiting speed, first consider that the units of this equation are *length*/*time* and hence are indeed describing a speed. Second, consider the following three cases:

$$\frac{d\overline{x}}{d\overline{t}} = \frac{P}{F} \implies \frac{dS}{d\overline{t}} = 0 \tag{4.16}$$

$$\frac{d\overline{x}}{d\overline{t}} < \frac{P}{F} \implies \frac{dS}{d\overline{t}} < 0 \tag{4.17}$$

$$\frac{d\overline{x}}{d\overline{t}} > \frac{P}{F} \implies \frac{dS}{d\overline{t}} > 0 \tag{4.18}$$

We notice a reversal in the production of entropy at the inflection point where $dS/d\overline{t} = 0$. Therefore, for an observer at rest to evolve forward in time, it must see its entropy sink grow at the speed of $c := P/F$. Therefore, the entropy sink of an observer moving forward in time must grow at the speed of light.

The following relations each characterize a limiting quantity.

limited quantity	*units*	*limiting relation*	
Power	J/s	$\frac{T}{F}\frac{dS}{d\overline{t}} = -P$	(4.19)
Speed	m/s	$\frac{T}{F}\frac{dS}{d\overline{t}} = \frac{d\overline{x}}{d\overline{t}} - \frac{P}{F}$	(4.20)
Stiffness	m^2/s	$\frac{T}{\kappa}\frac{dS}{d\overline{t}} = \frac{d\overline{A}}{d\overline{t}} - \frac{P}{\kappa}$	(4.21)
Volumetric flow rate	m^3/s	$\frac{T}{p}\frac{dS}{d\overline{t}} = \frac{d\overline{V}}{d\overline{t}} - \frac{P}{p}$	(4.22)

Each relation can easily be obtained from the power-time formulation by posing the other quantities as 0. To show that the quantities are inflection limits, it suffices to notice that they each correspond to a growth of the entropy sink that an observer at rest must see to fuel its forward translation in time.

It is well known that a limiting speed implies special relativity, but what about the other two limits? It is less known, but a maximum stiffness does imply general relativity. In this context, we can interpret space as being very stiff but nonetheless compressible. The maximum volumetric flow rate is associated with dark energy and is responsible for the Hubble horizon - beyond which the flow rate would be exceeded. These are in fact the approaches (in disguise) that we took to derive general relativity and dark energy earlier.

5 *Thermal space-time*

What is thermal time and thermal space? Consider the thermodynamic quantities t and x of the power-time formulation. Their average value is given by the standard relations (from 1.10):

	quantity	average	
thermal-time	t	$\bar{t} = \frac{-\partial \ln Z}{\partial P}$	(5.1)
thermal-space	x	$\bar{x} = \frac{-\partial \ln Z}{\partial F}$	(5.2)

Furthermore, as thermal-time and thermal-space are thermodynamic averages, they will undergo fluctuations (from 1.10):

	quantity	fluctuation	
thermal-time	t	$\overline{(\Delta t)^2} = \frac{\partial^2 \ln Z}{\partial P^2}$	(5.3)
thermal-space	x	$\overline{(\Delta x)^2} = \frac{\partial^2 \ln Z}{\partial F^2}$	(5.4)

Using the original argument made by Einstein in 1905, which led to the derivation of Brownian motion, we argue here that fluctuations of the t and x variables produce a universal Brownian motion along the axis themselves. What does a thermal spacetime with fluctuations look like? The consequences of such are nothing to be feared; indeed, we will shortly show that Brownian motion over $\bar{x}$ will produce the Schrödinger equation and that Brownian motion over both $\bar{x}$ and $\bar{t}$ will produce the Dirac equation.

Question: Are we suggesting a pilot-wave interpretation where particles undergo Brownian motion until a measurement is made?

Not at all. Rather, we are suggesting that any positional or time information undergoes a "Dirac equation-like diffusion" so as to make positional or time information perishable over time. To illustrate, we can imagine placing a mark at a position in space. After a certain time, Brownian motion will diffuse the position of the marker at any number of possible locations until its actual position is measured again. Instead of being punctual, the marker could be continuous and weighted and the same diffusion-like behaviour will be observed. This Brownian motion would universally apply to the axis itself. This is not a claim that a particle is punctual.

5.1 *Schrödinger equation*

The derivation of the Schrödinger and Dirac equations as a result of universal Brownian motion has already been done by other authors. Therefore, we can import their proofs into our derivation. Here, we will offer a sketch and refer to their respective authors for the more rigorous treatment. The derivation of the Schrödinger equation from Brownian motion was done by Nelson[9] and reviewed by the same author some 46 years later[10]. The field is stochastic mechanics and it connects very nicely to our thermodynamic description of the world.

[9] Edward Nelson. Derivation of the schrodinger equation from newtonian mechanics. Phys. Rev., 150:1079–1085, Oct 1966. DOI: 10.1103/PhysRev.150.1079. URL https://link.aps.org/doi/10.1103/PhysRev.150.1079

[10] Edward Nelson. Review of stochastic mechanics. In Journal of Physics: Conference Series, volume 361, page 012011. IOP Publishing, 2012

Nelson first considers the Langevin equation,

$$d\left[x(t)\right] = v(t)dt \tag{5.5}$$

$$d\left[v(t)\right] = -\frac{\gamma}{m}v(t)dt + \frac{1}{m}W(t)dt \tag{5.6}$$

, which describes a particle in a fluid undergoing a Brownian motion as a result of the random collisions with the water molecules. Here $W(t)$ is a *noise* term responsible for the Brownian motion and $v(t)$ is a viscosity term specific to the properties of the fluid.

Nelson replaces the acceleration $d[v(t)]/dt$ by F/m (from $F = ma$). Then, he is able to show that the Langevin equation in gradient form becomes:

$$\nabla\left(\frac{1}{2}\vec{u}^2 + D\nabla\cdot\vec{u}\right) = \frac{1}{m}\nabla V \tag{5.7}$$

where $D := \hbar/(2m)$ is the diffusion coefficient, where $\vec{F} = -\nabla V$, where $\vec{u} = v\nabla\ln\rho$ and ρ is the probability density of $x(t)$. As this is a sketch, the proof of 5.7 is omitted here but can be reviewed in Nelson's paper. Eliminating the gradients on each side and simplifying the constants, Nelson obtains:

$$\frac{m}{2}\vec{u}^2 + \frac{\hbar}{2}\nabla \cdot \vec{u} = V - E \tag{5.8}$$

where E is the arbitrary integration constant. Nelson then converts this equation to a linear equation via a change of variable applied to the term $\vec{u}^2$. Posing,

$$\vec{u} = \frac{\hbar}{m}\frac{1}{\psi}\nabla\psi \tag{5.9}$$

Nelson obtains

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V - E\right]\psi = 0 \tag{5.10}$$

which is the time-independent Schrödinger's equation. The time-dependent Schrödinger's equation is recovered as per the usual replacement $\psi := e^{R+iS}$. Finally, Nelson obtains:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(x,t)\right]\psi(x,t) \tag{5.11}$$

, which is the time-dependent Schrödinger's equation.

5.2 *Dirac equation*

We recently used the entropic force $TdS = Fdx$ and the Unruh temperature to recover $F = ma$. Then, we used $0 = -Pdt + Fdx$ to recover special relativity. Finally, we showed that a Brownian motion resulting from the thermal fluctuations on x recovers the Schrödinger equation as a thermo-statistical analogue to $F = ma$. Of course, the natural question to ask is this: will the thermal fluctuations of both t and x be enough to recover the Dirac equation as a thermo-statistical analogue to special relativity? The answer is yes!

Similarly to the stochastic-mechanical derivation of the Schrödinger equation, other authors previously derived the Dirac equation from universal Brownian motion[11]. In the original stochastic-mechanical derivation, the origin of such universal Brownian motion is ambiguous and is, at best, imported as a hypothesis. Thus, the benefit of our construction is to provide a thermal source of such universal Brownian motion. Hence, the derivation of the Dirac equation and the Schrödinger equation by these authors can nicely be imported into our thermodynamic construction.

[11] D Mckeon and G N. Ord. Time reversal in stochastic processes and the dirac equation. Physical review letters, 69:3–4, 08 1992

The derivation of the Dirac equation was noticed by studying random walk effects that were applicable to telegraphic communication.

McKeon and Ord propose a random walk model in space and in time which, once applied to the telegraph equations, produces the Dirac equation. We provide a sketch of the proof here and refer to the authors' paper for the rigorous treatment. Starting from the equation for a random walk in space, the authors obtain:

$$P_{\pm}(x, t+\Delta t) = (1 - a\Delta t)P_{\pm}(x \mp \Delta x, t) + a\Delta t P_{\mp}(x \pm \Delta x, t) \tag{5.12}$$

Afterward, the authors extend this equation with a random walk in time and obtain:

$$\begin{aligned} F_{\pm}(x,t) = (1 - a_L\Delta t - a_R\Delta t)F_{\pm}(x \mp \Delta x, t - \Delta t) + \\ a_{L,R}\Delta t B_{\pm}(x \mp \Delta x, t + \Delta t) + a_{R,L}\Delta t F_{\mp}(x \pm \Delta x, t - \Delta t) \end{aligned} \tag{5.13}$$

where $F_{\pm}(x,t)$ is the probability distribution to go forward in time and $B_{\pm}(x,t)$ the probability distribution to go backward in time. They then introduce a causality condition such that $F_{\pm}(x,t)$ and $B_{\pm}(x,t)$ only depends on probabilities from the past.

$$F_{\pm}(x,t) = B_{\mp}(x \pm \Delta x, t + \Delta t) \tag{5.14}$$

From equation 5.13 and 5.14, they get

$$\begin{aligned} B_{\pm}(x,t) = (1 - a_L\Delta t - a_R\Delta t)B_{\pm}(x \mp \Delta x, t + \Delta t) + \\ a_{L,R}\Delta t B_{\mp}(x \pm \Delta x, t + \Delta t) + a_{R,L}\Delta t F_{\pm}(x \mp \Delta x, t - \Delta t) \end{aligned} \tag{5.15}$$

In the limit $\Delta x, \Delta t \to 0$ and with $\Delta x = v\Delta t$, they get

$$\pm v\frac{\partial F_{\pm}}{\partial x} + \frac{\partial F_{\pm}}{\partial t} = a_{L,R}(-F_{\pm} + B_{\pm}) + a_{R,L}(-F_{\pm} + F_{\mp}) \tag{5.16}$$

$$\pm v\frac{\partial B_{\mp}}{\partial x} + \frac{\partial B_{\mp}}{\partial t} = a_{L,R}(-B_{\mp} + F_{\mp}) + a_{R,L}(-B_{\mp} + B_{\pm}) \tag{5.17}$$

Posing these changes of variables,

$$A_{\pm} = (F_{\pm} - B_{\mp})\exp[(a_L + a_R)t] \tag{5.18}$$

$$\lambda := -a_L + a_R \tag{5.19}$$

then 5.17 becomes

$$v\frac{\partial A_{\pm}}{\partial x} \pm \frac{\partial A_{\pm}}{\partial t} = \lambda A_{\mp} \tag{5.20}$$

Finally, they pose $v = c$, $\lambda = mc^2/\hbar$ and $\psi = F(A_+, A_-)$, and they get

$$i\hbar \frac{\partial \psi}{\partial t} = mc^2 \sigma_y \psi - ic\hbar \sigma_z \frac{\partial \psi}{\partial x} \tag{5.21}$$

which is the Dirac equation in 1+1 spacetime.

6 *Conclusion*

Understanding the world from purely thermodynamic principles holds several conceptual advantages. The construction provides a possible means to explain the origins of the laws of physics as per John Wheeler's suggestion of law without law (or as order from disorder) - in this case thermo-statistical disorder. Indeed, the obscure origin of the Dirac and Schrödinger equations is now clearly shown to be a result of thermal fluctuations applicable to x and t. Second, the laws of inertia, general relativity, and dark energy are simply the result of taking the Taylor expansion of an arbitrary space-encoding function. Third, as these laws are derived from the general equation of state of the system, the laws of physics do not need to be invoked as a 'special case'. In the present construction, the laws of physics are a consequence of the mere fact that the world can be expressed as a statistical ensemble involving time and space; hence, the 'axiomatic-load' of the construction is minimal.

The construction allows a possible explanation of the arrow of time. Indeed, moving into the future requires a negative power. A possible cause of negative power is closing future alternatives, which works towards reducing the entropy over time. To preserve the second law of thermodynamics, an entropy sink must be grown as time moves forward to offset said entropy reduction. Thus, the passage of time is heavily connected to the size of the entropy sink. The minimal growth rate requirements of this entropy sink are precisely the limits required to derive special relativity, general relativity, and dark energy. Therefore, we conclude that the entropy sink spawns the observable universe. Its expansion is required for an observer to translate forward in time. The second law of thermodynamics, understood as an increase in entropy over time, is only half the truth. The second law is perceived in the entropy sink while the larger system, made to include future possibilities, has a constant entropy. In this system, future possibilities are consumed as time moves forward.

References

P C W Davies. Scalar production in schwarzschild and rindler metrics. Journal of Physics A: Mathematical and General, 8(4):609,

1975. URL http://stacks.iop.org/0305-4470/8/i=4/a=022.

Ray A D'Inverno. Introducing einstein's relativity. 1992.

Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3):273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048.

Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic inflation. International Journal of Modern Physics A, 27(12): 1250066, 2012. DOI: 10.1142/S0217751X12500662. URL http://www.worldscientific.com/doi/abs/10.1142/S0217751X12500662.

Stephen A. Fulling. Nonuniqueness of canonical field quantization in riemannian space-time. Phys. Rev. D, 7:2850–2862, May 1973. DOI: 10.1103/PhysRevD.7.2850. URL https://link.aps.org/doi/10.1103/PhysRevD.7.2850.

Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75:1260–1263, Aug 1995. DOI: 10.1103/PhysRevLett.75.1260. URL https://link.aps.org/doi/10.1103/PhysRevLett.75.1260.

D Mckeon and G N. Ord. Time reversal in stochastic processes and the dirac equation. Physical review letters, 69:3–4, 08 1992.

Edward Nelson. Derivation of the schrodinger equation from newtonian mechanics. Phys. Rev., 150:1079–1085, Oct 1966. DOI: 10.1103/PhysRev.150.1079. URL https://link.aps.org/doi/10.1103/PhysRev.150.1079.

Edward Nelson. Review of stochastic mechanics. In Journal of Physics: Conference Series, volume 361, page 012011. IOP Publishing, 2012.

Wolfgang Rindler. Relativity: special, general, and cosmological, 2003.

Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2.

W. G. Unruh. Notes on black-hole evaporation. Phys. Rev. D, 14: 870–892, Aug 1976. DOI: 10.1103/PhysRevD.14.870. URL https://link.aps.org/doi/10.1103/PhysRevD.14.870.

Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029.