A thermal construction of the world

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Published: January 13, 2018. Last revision: January 13, 2018

We propose a simple partition function that unifies a surprisingly large amount of physical laws. The partition function is constructed from two conjugate-pairs: 1) an entropic-force conjugated to a thermal-length and 2) an entropic-power conjugated to a thermal-time. From its equation of state, we derive the Schrödinger equation, the Dirac equation, special relativity, general relativity, dark energy, Newton's law of gravitation, Newton's law of inertia and show that its Lagrange multipliers are the Planck units. We also propose a solution to the problem of the arrow of time as a natural consequence of the construction.

Contents

1 Introduction

As the Planck units are allegedly constant throughout the universe, we had the idea to construct a partition function of statistical physics such that the Lagrange multiplier are the Planck units. Furthermore, the two quantities which we felt were most fundamental, time and space, we injected as thermodynamic conjugate-pairs. To recover the units of energy, time must be multiplied by a power and space (length) must be multiplied by a force. Thus, the partition function describes both space and time; space-time. It is reassuring (although perhaps surprising) that special relativity, general relativity and dark energy are provable solutions of the equation of state. Furthermore, thermal fluctuations along the time and length quantities produces the Schrödinger equation and the Dirac equation. Thus, the construction suggests that both general relativity and the quantum world are emergent from a more fundamental thermo-statistical world.

1.1 Statistical physics

We will provide a brief recap of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy

$$S = -k_B \sum_{x \in X} p(x) \ln p(x) \tag{1.1}$$

, where

$$S \in \mathbb{R}_{\geq 0} \quad \text{entropy} \tag{1.2}$$

$$k_B \approx 1.38 \times 10^{-23} \frac{m^2 kg}{s^2 K} \quad \text{Boltzmann constant} \tag{1.3}$$

$$X \quad \text{ensemble of micro-states} \tag{1.4}$$

$$x \in X \quad \text{micro-state} \tag{1.5}$$

$$p(x) \in \mathbb{R} \cap [0,1] \quad \text{probability of the system being in micro-state } x \tag{1.6}$$

Observable	Conjugate	Relation
Energy E	Temperature T	$\beta = 1/(k_b T)$
Volume V	Pressure p	$\gamma = p/(k_b T)$
Number of particles N	Chemical potential μ	$\delta = -\mu/(k_b T)$

Table 1: Typical observables of statistical mechanics.

subject to the fixed macroscopic observables. The solution for this is the Gibbs ensemble. Taking the observables listed in Table 1 as examples, the partition function becomes

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{1.7}$$

, where

$$Z \in \mathbb{R}_{>0} \qquad \text{normalization constant} \tag{1.8}$$

The probability of occupation of a micro-state is;

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{1.9}$$

The average values and their variance for the observables are;

$$\overline{E} = \sum_{x \in X} p(x) E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{1.10}$$

$$\overline{V} = \sum_{x \in X} p(x) V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{1.11}$$

$$\overline{N} = \sum_{x \in X} p(x) N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{1.12}$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{E,V} = -\frac{\mu}{T} \tag{1.13}$$

which can be summarized as

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{1.14}$$

This is known as the equation of state of the thermodynamic system. The entropy can be recovered from the partition function. It is given by

$$S = k_B \left(\ln Z + \beta \overline{E} + \gamma \overline{V} + \delta \overline{N} \right) \tag{1.15}$$

2 *First proposed partition function: Time and Length*

We propose the following partition function constructed as a Gibbs ensemble.

$$Z(\beta, \mathcal{S}, F) = \sum_{q \in \mathcal{Q}} e^{-\beta[E(q) - \mathcal{S} f(q) - F x(q)]} \tag{2.1}$$

where,

type	*quantity*	*name*	*units*	
intensive	$T = 1/(k_B \beta)$	temperature	K	(2.2)
intensive	$\mathcal{S}$	entropic action	Js	(2.3)
intensive	F	entropic force	J/m	(2.4)
extensive	$E(q)$	energy	J	(2.5)
extensive	$f(q)$	thermal frequency	$1/s$	(2.6)
extensive	$x(q)$	thermal space	m	(2.7)

The partition function includes the familiar entropic force and the unfamiliar entropic action. Its equation of state is

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{x} \tag{2.8}$$

We can convert it to a more intuitive representation by converting the frequency to a time and the action to a power. Let us do that now.

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{x} \tag{2.9}$$

$$TdS = d\overline{E} + \mathcal{S}d(\overline{t}^{-1}) + Fd\overline{x} \qquad [f := 1/t] \tag{2.10}$$

$$TdS = d\overline{E} - \mathcal{S}\overline{t}^{-2}d\overline{t} + Fd\overline{x} \qquad \left[d(t^{-1}) = -t^{-2}dt\right] \tag{2.11}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \qquad \left[P := \mathcal{S}t^{-2}\right] \tag{2.12}$$

This representation introduces two new quantities, defined as:

type	*quantity*	*name*	*units*	
intensive	P	entropic power	J/s	(2.13)
extensive	$t(q)$	thermal time	s	(2.14)

The equation of state admits these two formulations:

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{x} \qquad \text{action-frequency formulation} \tag{2.15}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \qquad \text{power-time formulation} \tag{2.16}$$

2.1 *Regimes and cycles*

We will derive the familiar laws of physics by studying the equation of state in terms of its regimes. To do so, we will fix some derivatives (e.g. $dS = 0$) and analyse what happens when we let the other varies.

2.2 *Special relativity*

Here, we use the power-time formulation and pose $dS = 0$ and $dE = 0$. We obtain the fundamental relation of special relativity linking space to time.

$$0 = -Pd\bar{t} + Fd\bar{x} \tag{2.17}$$

$$Fd\bar{x} = Pd\bar{t} \tag{2.18}$$

$$d\bar{x} = \frac{P}{F}d\bar{t} \tag{2.19}$$

As the power P and the force F are Lagrange multiplier of the partition function, they are constant throughout the system. Thus, their quotient is also a constant.

$$c := \frac{P}{F} \tag{2.20}$$

Thus,

$$d\bar{x} = cd\bar{t} \tag{2.21}$$

As the units of P/F are meters per second, c will be our working definition of the speed of light.

Remark 2.22. *Indeed, in the case where P is the Planck power and F is the Planck force, we do recover the speed of light.*

$$P\frac{1}{F} = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{2.23}$$

2.3 *Inertial mass*

In this section we will need to use the Unruh temperature[1]. As can be reviewed in the citations provided, the Unruh temperature is an exact result obtained from special relativity. The Unruh effect is the prediction that an accelerating observer will observe blackbody radiation (at the Unruh temperature) where an inertial observer would observe none. The Unruh temperature is:

$$T = \frac{\hbar a}{2\pi c k_B} \qquad \text{Unruh temperature} \tag{2.24}$$

The Unruh temperature connects acceleration to temperature. We will use it here to convert an entropic force expressed in terms of a temperature to an entropic force expressed in terms of acceleration.

[1] Stephen A. Fulling. Nonuniqueness of canonical field quantization in riemannian space-time. Phys. Rev. D, 7:2850–2862, May 1973. DOI: 10.1103/PhysRevD.7.2850. URL https://link.aps.org/doi/10.1103/PhysRevD.7.2850; P C W Davies. Scalar production in schwarzschild and rindler metrics. Journal of Physics A: Mathematical and General, 8(4): 609, 1975. URL http://stacks.iop.org/0305-4470/8/i=4/a=022; W. G. Unruh. Notes on black-hole evaporation. Phys. Rev. D, 14:870–892, Aug 1976. DOI: 10.1103/PhysRevD.14.870. URL https://link.aps.org/doi/10.1103/PhysRevD.14.870; and Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

Furthermore, we start from the power-time formulation and pose $d\bar{t} = 0$ and $d\bar{E} = 0$. From these starting points, we can derive $F = ma$ (as per Eric Verldinde's original derivation[2]):

$$TdS = Fd\bar{x} \tag{2.25}$$

$$F = T\frac{dS}{dx} \tag{2.26}$$

$$F = \left(\frac{\hbar a}{2\pi c k_B}\right)\frac{dS}{dx} \tag{2.27}$$

$$F = \left(\frac{\hbar}{2\pi c k_B}\frac{dS}{dx}\right) a \tag{2.28}$$

[2] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

This equation corresponds to $F = ma$ provided that $\left(\frac{\hbar}{2\pi c k_B}\frac{dS}{dx}\right) = m$. How reasonable is that? Well, for it to be the mass, it suffices that dS/dx is the inverse of reduced Compton wavelength multiplied by a constant. Recall that the reduced Compton wavelength is $\frac{\hbar}{mc}$. Let us investigate,

$$\frac{\hbar}{2\pi c k_B}\frac{dS}{dx} = m \implies \frac{dS}{dx} = 2\pi k_B\left(\frac{mc}{\hbar}\right) \tag{2.29}$$

We obtain a relation between entropy and distance. What could this mean? It means two things.

1. The further away an object is from the origin, the higher is its space-entropy.

2. The more massive an object is, the higher is its space-entropy.

Why then the factor 2π? The presence of π suggest a connection between a line and a circle. Hence, a possible interpretation is that the entropy associated with linear space-entropy is distributed over the curvature of a circle (we can think of it as a one dimensional case of the holographic principle). Then, as an object with a small Compton wavelength can be more finely located, it requires more entropy to describe its position than an object with a large Compton wavelength. Why then the factor k_B? The factor k_B converts the reduced Compton wavelength to the units of entropy/length (Joules per Kelvin per meter).

2.4 *Thermal space-time*

What is time and space? Consider the thermodynamic quantities t and x of the power-time formulation. Their average value is given by the standard relations (from 1.10).

	quantity	average	
thermal-time	t	$\overline{t} = \frac{-\partial \ln Z}{\partial P}$	(2.30)
thermal-space	x	$\overline{x} = \frac{-\partial \ln Z}{\partial F}$	(2.31)

Furthermore, as thermal-time and thermal-space are thermodynamic averages, they will undergo fluctuations (from 1.10).

	quantity	fluctuation	
thermal-time	t	$\overline{(\Delta t)^2} = \frac{\partial^2 \ln Z}{\partial P^2}$	(2.32)
thermal-space	x	$\overline{(\Delta x)^2} = \frac{\partial^2 \ln Z}{\partial F^2}$	(2.33)

Using the original argument made by Einstein in 1905 which lead to the derivation of Brownian motion, we argue here that fluctuations of the t and x variables produce a universal Brownian motion along the axis themselves. What does a thermal spacetime with fluctuations look like? The consequences of such are nothing to be feared. Indeed, we will shortly show that Brownian motion over $\overline{x}$ will produce the Schrödinger equation and that Brownian motion over both $\overline{x}$ and $\overline{t}$ will produce the Dirac equation.

Remark 2.34. *Are we suggesting a pilot-wave interpretation where particles undergo Brownian motion until a measurement is made?*

Not at all. Rather, we are suggesting that any positional or time information undergoes a "Dirac equation-like diffusion" so as to make positional or time information perishable over time. To illustrate, we can imagine placing a mark at a position in space. After a certain time, Brownian motion will diffuse the position of the marker at any number of possible locations until its actual position is measured again. Instead of being punctual, the marker could be continuous and weighted and the same diffusion-like behaviour will be observed. This Brownian motion would universally apply to the axis themselves. This is not a claim that a particle is punctual.

2.5 *Schrödinger equation*

The derivation of the Schrödinger equation and the Dirac equation have already been produced by others as a general result of Brownian motion applicable to the time and space axis. Thus, we can import their proofs into our derivation. As they have already been published, we will only offer a sketch and refer to their respective authors for a more rigorous treatment. The derivation of the

Schrödinger equation from Brownian motion has been done by Nelson[3] and reviewed by the same author some 46 years later[4]. The field is stochastic mechanics and it connects very nicely to our thermodynamic description of the world.

[3] Edward Nelson. Derivation of the schrodinger equation from newtonian mechanics. Phys. Rev., 150:1079–1085, Oct 1966. DOI: 10.1103/PhysRev.150.1079. URL https://link.aps.org/doi/10.1103/PhysRev.150.1079

[4] Edward Nelson. Review of stochastic mechanics. In Journal of Physics: Conference Series, volume 361, page 012011. IOP Publishing, 2012

Nelson first considers the Langevin equation,

$$d\left[x(t)\right] = v(t)dt \tag{2.35}$$

$$d\left[v(t)\right] = -\frac{\gamma}{m}v(t)dt + \frac{1}{m}W(t)dt \tag{2.36}$$

which describes a particle in a fluid undergoing a Brownian motion as a result of the random collisions with the water molecules. Here $W(t)$ is a *noise* term responsible for the Brownian motion and $v(t)$ is a viscosity term specific to the properties of the fluid.

Nelson replaces the acceleration $d[v(t)]/dt$ by F/m (from $F = ma$). Then, he is able to show that the Langevin equation in gradient form becomes:

$$\nabla\left(\frac{1}{2}\vec{u}^2 + D\nabla\cdot\vec{u}\right) = \frac{1}{m}\nabla V \tag{2.37}$$

where $D := \hbar/(2m)$ is the diffusion coefficient, where $\vec{F} = -\nabla V$, where $\vec{u} = v\nabla \ln\rho$ and ρ is the probability density of $x(t)$. As this is a sketch, the proof of 2.37 is omitted here but can be reviewed in Nelson's paper. Eliminating the gradients on each side and simplifying the constants, Nelson obtains:

$$\frac{m}{2}\vec{u}^2 + \frac{\hbar}{2}\nabla\cdot\vec{u} = V - E \tag{2.38}$$

where E is the arbitrary integration constant. Nelson then converts this equation to a linear equation via a change of variable applied to the term $\vec{u}^2$. Posing,

$$\vec{u} = \frac{\hbar}{m}\frac{1}{\psi}\nabla\psi \tag{2.39}$$

Nelson obtains

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V - E\right]\psi = 0 \tag{2.40}$$

which is the time-independent Schrödinger's equation. The time-dependent Schrödinger's equation is recovered as per the usual replacement $\psi := e^{R+iS}$. Finally, Nelson obtains:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(x,t)\right]\psi(x,t) \tag{2.41}$$

which is the time-dependent Schrödinger's equation.

2.6 *Dirac equation*

We have recently used the entropic force $TdS = Fdx$ and the Unruh temperature to recover $F = ma$. Then, we have used $0 = -Pdt + Fdx$ to recover special relativity. Finally, we showed that a Brownian motion resulting from the thermal fluctuations on x recovers the Schrödinger equation as a thermal analogue to $F = ma$. Of course, the natural question to ask is, will the thermal fluctuations of both t and x be enough to recover the Dirac equation as a thermal analogue to special relativity? The answer is yes!

Similarly to the stochastic-mechanical derivation of the Schrödinger equation, the derivation of the Dirac equation from Brownian motion has been done before by other authors[5]. For such derivation to apply universally, the Brownian motion must be universal. But, the origin of such universal Brownian motion is left ambiguous and at best is imported as an hypothesis. Thus, the benefit of our construction is to provide a thermal source of such universal Brownian motion. Hence, the derivation of the Dirac equation and the Schrödinger equation by these authors is nicely importable into our thermal construction of the world.

[5] D Mckeon and G N. Ord. Time reversal in stochastic processes and the dirac equation. Physical review letters, 69:3–4, 08 1992

The derivation of the Dirac equation was noticed by studying random walk effects applicable to telegraphic communication. McKeon and Ord proposes a random walk model in space and in time which once applied to the telegraph equations produces the Dirac equation. We provide a sketch of the proof here and refer to the authors' paper for the rigorous treatment. Starting from the equation for a random walk in space, the authors obtain:

$$P_{\pm}(x,t+\Delta t) = (1-a\Delta t)P_{\pm}(x \mp \Delta x,t) + a\Delta t P_{\mp}(x \pm \Delta x,t) \tag{2.42}$$

then, the authors extend this equation with a random walk in time and obtain:

$$F_{\pm}(x,t) = (1-a_L\Delta t - a_R\Delta t)F_{\pm}(x \mp \Delta x, t-\Delta t) + a_{L,R}\Delta t B_{\pm}(x \mp \Delta x, t+\Delta t) + a_{R,L}\Delta t F_{\mp}(x \pm \Delta x, t-\Delta t) \tag{2.43}$$

where $F_{\pm}(x,t)$ is the probability distribution to go forward in time and $B_{\pm}(x,t)$ to go backward in time. They then introduce a causality condition such that $F_{\pm}(x,t)$ and $B_{\pm}(x,t)$ only depends on probabilities from the past.

$$F_{\pm}(x,t) = B_{\mp}(x \pm \Delta x, t + \Delta t) \tag{2.44}$$

From equation 2.6 and 2.44, they get

$$B_{\pm}(x,t) = (1 - a_L \Delta t - a_R \Delta t) B_{\pm}(x \mp \Delta x, t + \Delta t) + a_{L,R} \Delta t B_{\mp}(x \pm \Delta x, t + \Delta t) + a_{R,L} \Delta t F_{\pm}(x \mp \Delta x, t - \Delta t) \tag{2.45}$$

In the limit $\Delta x, \Delta t \to 0$ and with $\Delta x = v\Delta t$, they get,

$$\pm v \frac{\partial F_{\pm}}{\partial x} + \frac{\partial F_{\pm}}{\partial t} = a_{L,R}(-F_{\pm} + B_{\pm}) + a_{R,L}(-F_{\pm} + F_{\mp}) \tag{2.46}$$

$$\pm v \frac{\partial B_{\mp}}{\partial x} + \frac{\partial B_{\mp}}{\partial t} = a_{L,R}(-B_{\mp} + F_{\mp}) + a_{R,L}(-B_{\mp} + B_{\pm}) \tag{2.47}$$

Posing these changes of variables,

$$A_{\pm} = (F_{\pm} - B_{\mp}) \exp[(a_L + a_R)t] \tag{2.48}$$

$$\lambda := -a_L + a_R \tag{2.49}$$

then 2.47 becomes

$$v \frac{\partial A_{\pm}}{\partial x} \pm \frac{\partial A_{\pm}}{\partial t} = \lambda A_{\mp} \tag{2.50}$$

Finally, they pose $v = c$, $\lambda = mc^2/\hbar$ and $\psi = F(A_+, A_-)$, they get

$$i\hbar \frac{\partial \psi}{\partial t} = mc^2 \sigma_y \psi - ic\hbar \sigma_z \frac{\partial \psi}{\partial x} \tag{2.51}$$

which is the Dirac equation in 1+1 spacetime.

3 *Second proposed partition function: Time and generalized length*

The first partition function we proposed was constructed with an entropic-force conjugated with a thermal-length. The length was of course linear and expressed by x. In this section however, we extend the representation to consider a thermal-length described by an arbitrary function. After all, the mass of the universe is not linearly distributed with clockwork precision. To achieve this, we consider an arbitrary function $l(q) : q \to \mathbb{R}$ used to express the lengths of the micro-states. We will study such function via a Taylor expansion. A Taylor expansion requires that $\mathcal{Q}$ as in $q \in \mathcal{Q}$ be uncountable. As $l(q)$ is an arbitrary length with meter units, it will still be conjugated with the entropic-force. The Taylor expansion of $Fl(q)$ is:

$$Fl(q) = Fl(0) + Fl'(0)q + \frac{Fl''(0)}{2}q^2 + \frac{Fl'''(0)}{6}q^3 + ... \tag{3.1}$$

and its derivative with respect to q is:

$$Fdl(q) = Fl'(0)dq + Fl''(0)qdq + \frac{Fl'''(0)}{2}q^2dq + ... \tag{3.2}$$

As the micro-states $q \in \mathcal{Q}$ must be uncountable for the Taylor expansion of $l(q)$ to be well defined, the partition function must be continuous. Thus, it becomes:

$$Z = \frac{1}{k}\int e^{-\beta[E(q)+\mathcal{S}f(q)+Fl(q)]}dq \tag{3.3}$$

and is integrated over $\mathcal{Q}$. Likewise, its equation of state is

$$TdS = d\overline{E} + \mathcal{S}d\overline{f} + Fd\overline{l} \qquad \text{action-frequency formulation} \tag{3.4}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{l} \qquad \text{power-time formulation} \tag{3.5}$$

3.1 *Taylor expansion of $d\bar{l}$*

We convert the term $d\bar{l}$ of the power-time formulation into its Taylor expansion. The first change we will do is rename $q := x$.

$$Fdl(x) = Fl'(0)dx + Fl''(0)xdx + \frac{Fl'''(0)}{2}x^2dx + ... \tag{3.6}$$

The injecting it into the power-time formulation, we obtain:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{l} \tag{3.7}$$

$$TdS = d\overline{E} - Pd\overline{t} + Fl'(0)d\overline{x} + Fl''(0)\overline{x}d\overline{x} + \frac{Fl'''(0)}{2}\overline{x}^2d\overline{x} + ... \tag{3.8}$$

Something interesting appends with the units of the Taylor expansion. Let us investigate:

Taylor term	quantity	units	
$Fl'(0)d\overline{x}$	F	N	(3.9)
$''$	$l'(0)$	$\nexists$	(3.10)
$''$	$d\overline{x}$	m	(3.11)
$Fl''(0)xd\overline{x}$	F	N	(3.12)
$''$	$l''(0)$	$1/m$	(3.13)
$''$	$xd\overline{x}$	m^2	(3.14)
$Fl'''(0)x^2d\overline{x}$	F	N	(3.15)
$''$	$l'''(0)$	$1/m^2$	(3.16)
$''$	$x^2d\overline{x}$	m^3	(3.17)
$\vdots$	$\vdots$	$\vdots$	(3.18)

Since $xd\overline{x}$ has units m^2 and $x^2d\overline{x}$ has units m^3, we pose $\gamma d\overline{A} := xd\overline{x}$ and $\alpha d\overline{V} := x^2d\overline{x}$. Furthermore, as $l'(0)$ has no units we define it as the baseline $l'(0) := 1$ and we define $l''(0) := l_A/L$ and $l'''(0) := l_V/A$ as they respectively have units m^{-1} and m^{-2}. For observational reasons, we consider that $\gamma d\overline{A}$ describes the surface of a sphere and that $\alpha d\overline{V}$ describes the volume of of a sphere. Thus, to properly link $\gamma d\overline{A}$ to $xd\overline{x}$, the factor γ must be $1/(4\pi)$, and the factor α must be $3/(4\pi)$. Introducing these replacements, the equation of state becomes:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} + l_A\frac{F}{4\pi L}d\overline{A} + l_V\frac{3F}{8\pi A}d\overline{V} + ... \qquad (3.19)$$

where g_A and g_V are the leftovers of the Taylor coefficient. We can recover the first proposed partition function by posing $d\overline{A} = 0$ and $d\overline{V} = 0$:

$$TdS = d\overline{E} - Pd\overline{t} + Fd\overline{x} \qquad (3.20)$$

Thus, all results derived in the previous section are importable into this more general equation of state.

3.2 General relativity

In this section, we will show how the term $d\overline{A}$ suggests that general relativity is entropic and emergent. Our goal is of course to derive the Einstein field equation of general relativity starting from the $d\overline{A}$ regime. First we pose $dS = 0$, $d\overline{t} = 0$, $d\overline{x} = 0$ and $d\overline{V} = 0$. We obtain:

$$d\overline{E} = l_A\frac{F}{4\pi L}d\overline{A} \qquad (3.21)$$

Deriving general relativity from $dE = l_A \frac{F}{4\pi L} dA$ has indeed been done before in the literature, notably by Ted Jacobson, then later (and differently) by Erik Verlinde[6]. Furthermore, keys insights are provided by Christoph Schiller[7]. Here, we will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

[6] Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75:1260–1263, Aug 1995. DOI: 10.1103/PhysRevLett.75.1260. URL https://link.aps.org/doi/10.1103/PhysRevLett.75.1260; and Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

[7] Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2

First, the entropic force F is constant throughout the system as a result of being a Lagrange multiplier. We have already selected F to be the Planck force in order to derive special relativity and the speed of light. Thus, we must continue to use F as the Planck force here.

What then is L? Recall that we have used the Unruh temperature to link T to an acceleration and derive $F = ma$ earlier. Here and likewise, we will use special relativity to derive a relation between length and acceleration and use it to replace L. As par Schiller's paper, we select L as the maximum length that an accelerated object can have under special relativity - see Schiller, 2005, Rindler, 2003, D'Inverno, 1992. It is also the acceleration of circular motion ($r = v^2/a$) occurring at the speed of light ($v = c$). In the present context, L is the length associated with the maximum force; the Planck force. In the context of maximums, the force cannot accelerate the object beyond the speed of light, thus it can only be defined for a circular motion produced by a force perpendicular to the direction of motion. The maximum acceleration changes the direction of the motion, but does not increases the speed beyond the speed of light.

$$L = \frac{c^2}{2a} \tag{3.22}$$

We obtain:

$$d\overline{E} = l_A \frac{c^2}{2\pi G} a d\overline{A} \tag{3.23}$$

With this result, Jacobson's proof directly follows. starting from $dE = TdS$, he first connects dE to an arbitrary coordinate system and energy flow rates,

$$dE = \int T_{ab} k^a d\Sigma^b \tag{3.24}$$

Here T_{ab} is an energy-momentum tensor, k is a killing vector field and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that the area part can be rewritten as follows:

$$adA = c^2 \int R_{ab} k^a d\Sigma^b \tag{3.25}$$

where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab}k^a d\Sigma^b = l_A \frac{c^2}{2\pi G} \int R_{ab}k^a d\Sigma^b \tag{3.26}$$

which can only be satisfied if

$$T_{ab} = l_A \frac{c^2}{2\pi G} \left[R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right] \tag{3.27}$$

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant). Only the numerical value of l_A remains. The exact formulation of the field equation is obtained when the value of the Taylor leftover g_A is $1/4$. Why $1/4$? We admit that we are unsure but as general relativity is a consequence of area entropy, it might be related to the factor $1/4$ of the Bekenstein-Hawking entropy.

3.3 *Dark energy*

Associating dark energy to a volumetric entropy has been suggested and discussed by other authors before[8]. We start from the power-time formulation and pose $d\overline{E} = 0$, $d\overline{t} = 0$, $d\overline{x} = 0$ and $d\overline{A} = 0$. We obtain:

$$TdS = l_V \frac{3F}{8\pi A} d\overline{V} \tag{3.28}$$

We notice that the factor F/A has the units of pressure. Hence, our goal will be to derive a value of the pressure p associated with a volumetric entropy. As suggested by the factor F/A and in line with our earlier derivations, we will select F to be the Planck force and will take A as the area of a sphere. In this case, the pressure relates to the force as

$$F = -pA \tag{3.29}$$

$$\implies p = -\frac{F}{A} = -\frac{F}{4\pi x^2} \tag{3.30}$$

$$p = -\frac{c^4}{4\pi G\overline{x}^2} \qquad \text{entropic pressure} \tag{3.31}$$

The sign of the force is negative because the force points in the direction of increased entropy, which is oriented outward the enclosing area. Physically and as argued by Easson et al., it makes sense to

[8] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048; and Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic inflation. International Journal of Modern Physics A, 27(12):1250066, 2012. DOI: 10.1142/S0217751X12500662. URL http://www.worldscientific.com/doi/abs/10.1142/S0217751X12500662

connect the size of the sphere to the Hubble horizon. Thus, we take the radius of the sphere to be the Hubble radius $x := c/H$. Finalizing our derivation, we obtain:

$$p = -\frac{c^2H^2}{4\pi G} \tag{3.32}$$

This is close to the current measured value for the negative pressure associated with dark energy[9]. As we can see, the suggested entropic derivation of dark energy applies to the third term of the Taylor expansion.

[9] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048

4 Discussion - Arrow of time

Adding a time variable to a partition function adds a whole new dynamic to a thermal system. The system now becomes aware of future, past and present entropy and can translate from time to space and from space to time for an entropic cost (provided that various limits are respected). By studying thermodynamic cycles involving space and time, we were able to investigate what happens to the entropy when a system is translated forward or backward in time and draw conclusions in regards to the arrow of time. In the model presented, space serves as an entropy sink that encourages a forward arrow of time.

4.1 Negative power

Theorem 4.1. *In the power-time formulation, increasing $\overline{t}$, while keeping the other variables constant, decreases the entropy.*

Proof. We start with the power-time formulation and pose $d\overline{E} = 0$, $d\overline{x} = 0$, $d\overline{A} = 0$ and $d\overline{V} = 0$.

$$TdS = -Pdt \tag{4.2}$$

$$\implies T\frac{dS}{dt} = -P \tag{4.3}$$

We obtain a negative power. □

Remark 4.4. *To obtain the relation $d\overline{x} = cd\overline{t}$ with the correct signs, the power P must have a different sign than the force F. Thus, a positive force implies a negative power and vice-versa. As we require a positive force to recover $F = ma$ (and not $F = -ma$), then the sign of the force is already chosen for us. Thus, the power must be negative.*

Definition 4.5 (Time-entropy)**.** *The time-entropy is the contribution to the entropy over time by the negative power.*

We will now discuss this result in more detail.

Remark 4.6. *To facilitate the discussion, we present this section in the form of a dialogue between the authors and the hypothetical character Alice.*

> Alice: - *Why does the entropy decreases with time?*

We have to be careful about the formulation of the question. The power-time formulation admits other terms: $d\overline{x}$, $d\overline{A}$ and $d\overline{V}$. The term $-Pd\overline{t}$, as it has a negative sign, works towards reducing the entropy over time, but the other variables, as their signs are positive, work in the other direction. Thus, the entropy of the system as a whole need not necessarily decrease. It is more accurate to say that increasing $\overline{t}$, while keeping the other variables constant, decreases the entropy.

In this situation, the time-entropy decreases because the system has a negative power. So a similar question would be what is a negative power? Before we answer, let us first recall its more familiar cousin: the negative temperature.

If we understand temperature as the random movements of molecules then a temperature is always equal to or above zero. However, statistical physics admits a generalized definition of temperature as the tradeoff between energy and entropy. Most systems cannot admit a negative temperature as their entropy will always increase at higher energies. But for some systems, for example the population inversion in a laser, the entropy saturate at higher energies. Thus, a negative temperature is possible.

A negative power has essentially the same interpretation. As $\overline{t}$ is increased, the entropy is decreased.

> Alice: - *You have explained macroscopically what happens, but how does this pans out in the partition function. Why is there less entropy at increased $\overline{t}$?*

To understand why, consider that at present time we do not know what the future will be. Thus, the entropy of the future is very high. In fact, multiple futures are compatible with the present. As time increases, possible futures are erased and replaced with a singular present. Thus, the entropy of the system decreases as time increases.

> Alice: - *How does this result reconcile with the second law of thermodynamics which states that the entropy increases with time (or in some ideal cases stays constant)?*

To answer that, we need to look at the other variables of the equation of state.

4.2 The second law as an offset to negative power

Theorem 4.7. *To offset the decrease in entropy caused by the negative power, we suggest a proportional increase in the quantities $\overline{E}$, $\overline{x}$, $\overline{A}$ and $\overline{V}$.*

Proof. To simplify the equation, let us rename $\kappa := \frac{F}{16\pi L}$ and $p := \frac{3g_V F}{4\pi A}$. We pose $dE = 0$.

$$TdS = -Pd\bar{t} + Fd\bar{x} + \kappa d\overline{A} + pd\overline{V} + ... \tag{4.8}$$

$$T\frac{dS}{d\bar{t}} + P = F\frac{d\bar{x}}{d\bar{t}} + \kappa\frac{d\overline{A}}{d\bar{t}} + p\frac{d\overline{V}}{d\bar{t}} + ... \tag{4.9}$$

□

Definition 4.10 (Space-entropy)**.** *The space-entropy is the contribution by the following term to the entropy over time.*

$$F\frac{d\bar{x}}{d\bar{t}} + \kappa\frac{d\overline{A}}{d\bar{t}} + p\frac{d\overline{V}}{d\bar{t}} + ... \tag{4.11}$$

Note that P is not present in the definition as it is already associated with the time-entropy.

To investigate this result, let us look at three cases;

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} < P \implies \frac{dS}{dt} < 0 \quad \text{decreasing entropy} \tag{4.12}$$

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} = P \implies \frac{dS}{dt} = 0 \quad \text{constant entropy} \tag{4.13}$$

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} > P \implies \frac{dS}{dt} > 0 \quad \text{increasing entropy} \tag{4.14}$$

At (4.13), a shift occurs in the direction of the production of entropy over time. It is the point at which the space-entropy overtakes and exceeds the reduction in time-entropy. The second law of thermodynamics, which states that $dS/dt \geq 0$ will hold for (4.13) and (4.14), but will be violated for (4.12).

4.3 Discussion

In this section, we will explain why these two theorems provide us with an understanding of time and its arrow. Indeed, it links the arrow of time to three concepts; 1) a reduction in time-entropy over time, 2) an increase in space-entropy over time and 3) a system's inability to reduce its own entropy. We will see that this derivation more closely matches the observer's understanding of time. Indeed,

1. at the beginning of time the number of possible future alternatives is maximal. To reflect this, the time-entropy is at its maximum at $t = 0$, and the space-entropy is equal to 0. This matches our current empirical data in that the space-entropy at the Big Bang is very low.

2. during the evolution the future becomes past which is immutable. As the past becomes immutable, the time-entropy is reduced to 0. This is because we "remember" or "observe" only one past. This reduction in time-entropy is offset by a growth in space-entropy, which is related to the size and complexity of the space produced by the other quantities. This growth in space-entropy obeys the observed second law of thermodynamic.

3. at the end of time there is no future. The full history of the system is now "set in stone". The time-entropy is at its mimimum and the space-entropy is at its maximum. This matches the hypothesis of the heat death.

> Alice: - *I am still not clear on this. The conventional wisdom in physics is that the arrow of time is connected to an increase in entropy with time. Now you seem to be saying the inverse of that.*

Yes, we are suggesting that time has an arrow principally because the time-entropy decreases with time, and yes, it contradicts the conventional wisdom. But nonetheless it is correct for the following reason:

A partition function constructed without the use of a time quantity will follow the second law of thermodynamic. This statistical effects is heuristically explained by the H-theorem of Boltzmann. However, the changes when time is inserted as a thermodynamic quantity. Such a partition function then becomes aware of the past, the present and the future. The occupancy of the micro-state varies with time and the past differs from the future. Thus, the time symmetry is broken.

> Alice: - *But why is time specifically associated with a decrease in entropy, and not an increase?*

The partition function with the time encodes all past, present and future states as micro-states. Thus, the system will carry an entropy attributable to the undecidability of the future. The role of increasing time is to consume this entropy by closing possible futures. In other words, time collapses future undecidability into a singular past.

> Alice: - *But, a system cannot decrease its entropy without violating the second law of thermodynamics.*

A system can decrease its entropy if it is connected to an entropy sink. For example, biological life can reduce its entropy but only at the cost of severely increasing it in its environment. This requires excess energy and in the case of Earth, the Sun supplies it. Thus, the power-time conjugate can decrease the entropy as long as it is connected to a sink.

Alice: - *So, there should be a sink in the universe available to offset the decrease in time-entropy over time?*

In the case of time, the sink is the universe itself. The laws of physics that we have derived are in fact the limits required to produce a entropy sink of sufficient size to accommodate a forward direction of time for an observer.

Alice: - *Why do we remember the past but not the future?*

An observer cannot pre-calculate his exact future before it occurs without increasing the size of the sink. Here we make a distinction between calculating a probable future versus the exact future. Calculating a probable future does not necessarily imply a reduction of entropy within the system but calculating the exact future requires consuming the entropy of all possible alternative futures. Thus, an entropy sink is required.

Alice: - *Does the second law of thermodynamic need to be corrected for the wider system which includes future states?*

Yes. To our knowledge, statistical physics has not been used with a time quantity before. Time is usually considered to be an independent background to statistical physics. But, when we add a time as a thermodynamic quantity to a partition function, new physics emerge.

Indeed, an observer cannot move into the future unless all alternative futures are closed. Thus, its time-entropy must decrease. The second law of thermodynamics is a consequence of the system increasing its space-entropy to offset the reduction in future alternatives as time moves along.

4.4 *Limiting relations*

With our new interpretation of space as an entropy sink for time, let us immediately prove three limits from first principle; the speed of light, a limiting viscosity and a limiting volumetric flow rate applicable to the universe. To prove that these are limits, we will consider the assumption that an observer who evolves forward in time must see a growth in the size of its available entropy sink to offset the reduction in future alternatives. The limit occurs when the sink exactly offsets the reduction in time-entropy - in which case $dS/dt = 0$.

Theorem 4.15. *The power-time formulation implies a limiting speed.*

Proof.

$$TdS = Fdx - Pdt \tag{4.16}$$

$$\frac{T}{F}\frac{dS}{d\bar{t}} = \frac{d\bar{x}}{d\bar{t}} - \frac{P}{F} \tag{4.17}$$

□

To see why this implies a limiting speed, first consider that the units of this equation are *length/time* hence are describing a speed. Second, consider the following three cases;

$$\frac{d\bar{x}}{d\bar{t}} = \frac{P}{F} \implies \frac{dS}{d\bar{t}} = 0 \tag{4.18}$$

$$\frac{d\bar{x}}{d\bar{t}} < \frac{P}{F} \implies \frac{dS}{d\bar{t}} < 0 \tag{4.19}$$

$$\frac{d\bar{x}}{d\bar{t}} > \frac{P}{F} \implies \frac{dS}{d\bar{t}} > 0 \tag{4.20}$$

We notice a reversal in the production of entropy at the inflexion point where $dS/d\bar{t} = 0$. Therefore, for an observer at rest to evolve forward in time, it must see its entropy sink grow at the speed of $c := P/F$. Thus, the entropy sink of an observer moving forward in time must grow at the speed of light.

Theorem 4.21. *The following relations each characterize a limiting quantity.*

limited quantity	*units*	*limiting relation*	
Power	J/s	$\frac{T}{F}\frac{dS}{d\bar{t}} = -P$	(4.22)
Speed	m/s	$\frac{T}{F}\frac{dS}{d\bar{t}} = \frac{d\bar{x}}{d\bar{t}} - \frac{P}{F}$	(4.23)
Viscosity	m^2/s	$\frac{T}{\kappa}\frac{dS}{d\bar{t}} = \frac{d\bar{A}}{d\bar{t}} - \frac{P}{\kappa}$	(4.24)
Volumetric flow rate	m^3/s	$\frac{T}{p}\frac{dS}{d\bar{t}} = \frac{d\bar{V}}{d\bar{t}} - \frac{P}{p}$	(4.25)

Proof. Each relation can easily be obtained from the power-time formulation by posing the other quantities to 0. To show that the quantities are inflexion limits, it suffices to notice that they each correspond to a growth of the entropy sink that an observer at rest must see to fuel its forward translation in time. □

It is well-known that a limiting speed implies special relativity, but what about to other two? It is less well known, but nonetheless, a maximum viscosity does implies general relativity. In this context, we can interpreted space as being encoded by bits moving very slowly (like molasses) on the surface of a sphere. The maximum volumetric flow rate is associated with dark energy and is responsible for the Hubble horizon - beyond which the flow rate would be exceeded. These are in fact the approaches that we have taken to derive general relativity and dark energy.

5 Conclusion

Understanding the world from a purely thermodynamic perspective holds several conceptual advantages. First, the obscure origin of the Dirac and the Schrödinger equation is now clearly shown to be a result of thermal fluctuations applicable to x and t. Second, the complicated laws of general relativity and dark energy simply result as a consequence of taking the Taylor expansion of an arbitrary function with meter units. Third, as these laws are derived from the general equation of state, the world does need to be a 'special' case or to impose a 'fine-tuning'. The laws of physics are consequence of the mere fact that the world can be expressed as a partition function.

The construction allows an explanation of the arrow of time. Indeed, moving into the future requires closing alternatives which reduces the entropy. Thus, the passage of time is heavily connected to the size of the universe which serves as an entropy sink. The second law of thermodynamics, understood as an increase in entropy over time, is only half the truth. The second law is perceived in the space variables while the larger system, made to include future possibilities, has a constant entropy over time. In this system future possibilities are consumed as time moves forward. The second law of thermodynamics is therefore corrected to a law of conservation of entropy for the larger system comprised both thermal time and thermal space.

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