Poulet numbers P for which P-q^2 is an abundant number for any q prime greater than 3

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Abstract. In this paper I make the following observation: there exist Poulet numbers P such that n = P – q^2 is an abundant number for any q prime, q ≥ 5 (of course, for q^2 < P). The first such P is 1105 (with corresponding [q, n] = [5, 1080], [7, 1056], [11, 984], [13, 936], [17, 816], [19, 744], [23, 576], [29, 264], [31, 144]). Another such Poulet numbers are 1387, 1729, 2047, 2701, 2821. Up to 2821, the Poulet numbers 341, 561, 645, 1905, 2465 don't have this property. Questions: are there infinite many such Poulet numbers? What other sets of integers have this property beside Poulet numbers?

Observation:

There exist Poulet numbers P such that P – q^2 is an abundant number for any q prime, q ≥ 5 (of course, for q^2 < P). The first such number is 1105 (indeed, up to 1105 there is no such other Poulet number: 341 – 5^2 = 316, deficient; 561 – 5^2 = 536, deficient; 645 – 7^2 = 596, deficient).

Note: see the sequence A005101 in OEIS for abundant numbers.

Questions: are there infinite many such Poulet numbers? What other sets of integers have this property beside Poulet numbers?

The first four such Poulet numbers:

: P = 1105 (indeed, P – q^2 = n, abundant, for [q, n] = [5, 1080], [7, 1056], [11, 984], [13, 936], [17, 816], [19, 744], [23, 576], [29, 264], [31, 144];

: P = 1387 (indeed, P – q^2 = n, abundant, for [q, n] = [5, 1362], [7, 1338], [11, 1266], [13, 1218], [17, 1098], [19, 1026], [23, 858], [29, 546], [31, 426], [37, 18];

: P = 1729 (indeed, P – q^2 = n, abundant, for [q, n] = [5, 1724], [7, 1680], [11, 1608], [13, 1560], [17, 1440], [19, 1368], [23, 1200], [29, 888], [31, 768], [37, 360], [41, 48];

: P = 2047 (indeed, P – q^2 = n, abundant, for [q, n] = [5, 2022], [7, 1998], [11, 1926], [13, 1878], [17, 1758], [19, 1686], [23, 1518], [29, 1206], [31, 1086], [37, 678], [41, 366]; [43, 198];

: P = 2701 (indeed, P – q^2 = n, abundant, for [q, n] = [5, 2676], [7, 2652], [11, 2580], [13, 2532], [17, 2412], [19, 2340], [23, 2172], [29, 1860], [31, 1740], [37, 1332], [41, 1020]; [43, 852], [47, 492];

: P = 2821 (indeed, P – q^2 = n, abundant, for [q, n] = [5, 2796], [7, 2772], [11, 2700], [13, 2652], [17, 2532], [19, 2460], [23, 2292], [29, 1980], [31, 1860], [37, 1452], [41, 1140]; [43, 972], [47, 612], [53, 12].

The other Poulet numbers P up to 2821, i.e. 341, 561, 645, 1905, 2465 don’t have the property showed: P – q^2 is deficient at least for [q, n] equal to, respectively, [5, 316], [5, 536], [7, 596], [11, 1784], [7, 2416].