# A new Cosmological Model.

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Abstract.

The purpose of this article is to present a new cosmological model, based on the union of the Wilhelm De Sitter Model applying the equations of the Russian physicist Anatoli Logunov in the Relativistic Theory of Gravitation (TRG).

The general ideas of the Relativistic Theory of Gravitation (GRT) are based on the ideas of Poincaré, Minkowski, Einstein and Hilbert, all within the framework of Special Relativity (SR).

## 1. Introduction.

The Cosmological Model developed by De Sitter is within the conceptual framework of General Relativity (RG) and Special Relativity (RE), the complete model was developed by De Sitter and Einstein as a whole, although Einstein was not comfortable with the idea of a Empty universe.

Logunov along with other Russian physicists like Metrivishvili and Petrov formulated a new Theory of the Gravity, because they thought that the General Relativity (GR) was incomplete, its formulation is more intimidating mathematically, but it can get to be more useful since it is based on a pseudoeuclidean space. This theory also does not lead to the formation of Black Holes, because the conditions of Hawking-Penrose are not fulfilled.

Therefore, in our Model there was no initial singularity at the beginning of the Universe, this will seem to evoke an instant of creation, but this is for the reader's consideration, since the Model is useful if you try to describe a Theory of Quantum Gravity or a Quantum Field Theory in Curved Space-Time.

## 2- Formulation of the Lagrangian.

In the original model of Wilhelm De Sitter, the Lagrangian is very simple, as are the equations of movement that come from it. The Lagrangian of De Sitter is:

$$L=\frac{1}{16\pi G}(R-2\Lambda) \qquad (1)$$

Our Lagrangian will be a simple modification of this Lagrangian applying the ideas of the Relativistic Theory of Gravitation (TRG).

The first modification is that we redefined the Ricci R Curvature Tensioner to the form:

$$\underline{R}= -\underline{g}^{\mu\nu}(\Gamma_{\mu\nu}^{\lambda}\Gamma_{\lambda\sigma}^{\sigma}-\Gamma_{\mu\sigma}^{\lambda}\Gamma_{\nu\lambda}^{\sigma})-\partial_{\nu}(\underline{g}^{\mu\nu}\Gamma_{\mu\sigma}^{\sigma}-\underline{g}^{\mu\sigma}\Gamma_{\mu\sigma}^{\nu})= -\underline{g}^{\mu\nu}(G_{\mu\nu}^{\lambda}G_{\lambda\sigma}^{\sigma}-G_{\mu\sigma}^{\lambda}G_{\nu\lambda}^{\sigma})-D_{\nu}(\underline{g}^{\mu\nu}G_{\mu\sigma}^{\sigma}-\underline{g}^{\mu\sigma}G_{\mu\sigma}^{\nu}) \quad (2)$$

With

$$G_{\mu\nu}^{\lambda}=\frac{1}{2}g^{\lambda\sigma}(D_{\mu}g_{\sigma\nu}+D_{\nu}g_{\sigma\mu}-D_{\sigma}g^{\mu\nu}) \quad (3)$$

Here $D_{\mu}$ represents the covariant derivative with respect to the metric $\gamma_{\mu\nu}$.

Applying this in the Lagrangian de De Sitter we write it in the form

$$L=\frac{1}{16\pi G}\{[-g\mu\nu(G\mu\nu\lambda G\lambda\sigma\sigma - G\mu\sigma\lambda G\nu\lambda\sigma) - D\nu(g\mu\nu G\mu\sigma\sigma - g\mu\sigma G\mu\sigma\nu)] - 2\Lambda\} \quad (4)$$

The Cosmological Constant in the Model depends on a tiny term that we will denote with the letter m and represents the mass of the graviton, the expression of the Cosmological Constant is

$$\Lambda=\frac{1}{2}(Gm/c^{2}) \quad (5)$$

In units G = h = c is reduced to $\frac{1}{2}m^{2}$.

Now applying the definition (5) to the Lagrangian is rewritten to the form

$$\frac{1}{16\pi G}\left\{[-g\mu\nu(G\mu\nu\lambda G\lambda\sigma\sigma - G\mu\sigma\lambda G\nu\lambda\sigma) - D\nu(g\mu\nu G\mu\sigma\sigma - g\mu\sigma G\mu\sigma\nu)] - \left(\frac{Gm}{c2}\right)\right\}$$

this expression is valid whenever m> 0 is taken, if m = 0 only the first terms are applied and the last one is eliminated, this does not affect the final equations of the Model.

Not this last modification the Lagrangian is finished.

### 3- Model Dynamics.

The dynamics of the movement Cosmological Model is more complicated than the one handled in General Relativity (RG) but it leads to the same final results.

The first equation that we are going to define is the metric, which will govern the distance between two events in the pseudoeuclidean time space.

$$d\sigma^{2} = \gamma_{\mu\nu}(x)dx^{\mu}dx^{\nu} \quad (7)$$

$\gamma_{\mu\nu}(x)$ is the metric of the pseudoeuclidean space, and that defines a large part of the equations of the system in which we work, with it we can also define the metric $g_{\mu\nu}$:

$$\underline{g}_{\mu\nu}= \underline{\gamma}_{\mu\nu}+\underline{\phi}_{\mu\nu} \quad (8)$$

with $\underline{\gamma}_{\mu\nu}=\sqrt{-\gamma}\gamma_{\mu\nu}$, $\underline{\phi}_{\mu\nu}=\sqrt{-\gamma}\phi_{\mu\nu}$, $\gamma=\det\gamma_{\mu\nu}$.

With the current definition of g we will now define one of the most important tensors the modified Riemann Tensor

$$\underline{R}^{\mu\nu\rho\sigma}=\frac{R}{12\pi}(\underline{g}^{\mu\nu}\underline{g}^{\mu\alpha}-\underline{g}^{\nu\beta}\underline{g}^{\alpha\beta}) \quad (9)$$

as we said the modified Riemann Tensor was defined from (8) and (2), if we take an equality between General Relativity (GR) and Relativistic Theory of Gravitation (TRG) there is the relationship

$$\underline{R}_{\mu\nu}=R^{\mu}{}_{\nu\rho\sigma} \quad (10)$$

With the relations that we have already defined it is possible to write the curvature equations, to describe the gravitational fields and the matter fields.

The first equation that we will give is the one that describes the curvature of Space-Time of a free gravitational field, taking the Tensor energy-impulse $t^{\mu\nu}_g$

$$R^{\mu\nu}-\frac{1}{2}m^2(g^{\mu\nu}g^{\mu\alpha}-g^{\nu\beta}g^{\alpha\beta}) \quad (11)$$

O well

$$D_\alpha D_\beta(\gamma^{\alpha\beta}g^{\mu\nu}+\gamma^{\mu\nu}g^{\alpha\beta}-\gamma^{\alpha\mu}g^{\alpha\nu}-\gamma^{\alpha\nu}g^{\beta\mu})+m^2(g^{\mu\nu}-\gamma^{\mu\nu})=16\pi t^{\mu\nu}_g \quad (11b)$$

Both forms are equivalent, there is no preference for any.

The $t^{\mu\nu}_g$ tensor is valid only for gravitational fields, its definition is equal to how Hilbert described it

$$t^{\mu\nu}_g = -2\frac{\delta L}{\delta\gamma\mu\nu} \quad (12)$$

L refers to the gravitational Lagrangian.

But since the $t^{\mu\nu}_g$ tensor only defines the gravitational fields, the tensor that corresponds to the fields of matter depends on the Lagrangian of matter:

$$t^{\mu\nu}_M = -2\frac{\delta L}{\delta\gamma\mu\nu} \quad (13)$$

With both tensors $t^{\mu\nu}_g$ and $t^{\mu\nu}_M$ we can define a more general-order first order tensor that we will denote $t_{\mu\nu}$

$$t^{\mu\nu}= t^{\mu\nu}_g+ t^{\mu\nu}_M \quad (14)$$

can also be obtained from a generalized Lagrangian of the type

$$L= L_g(\gamma^{ik},\phi)+L_M(g^{ik},\phi) \quad (15)$$

The most general form of (11) and (11b) will depend to a large extent on the $t_{\mu\nu}$ tensor.

Then we will define the curvature equation

$$R^{\mu\nu}-\frac{1}{2}m^2(g^{\mu\nu}g^{\mu\alpha}-g^{\nu\beta}\gamma^{\alpha\beta}) \quad (16)$$

which is very similar to the equations that emerge from General Relativity (GR).

The only problem in our Model is that a large part of the equations depend on m which is the mass of the gravitate, if m = 0 the equations will turn to the form:

$$D_\alpha D_\beta(\gamma^{\alpha\beta}g^{\mu\nu}+\gamma^{\mu\nu}g^{\alpha\beta}-\gamma^{\alpha\mu}g^{\alpha\nu}-\gamma^{\alpha\nu}g^{\beta\mu})= 16\pi t^{\mu\nu} \quad (17)$$

described in the usual way

$$\sqrt{-g}R=8\pi(T^{\mu\nu}-g^{\mu\nu}T) \quad (17b)$$

with equation (17b) the set of curvature equations of the Model is finished.

4- Friedmann equations.

As we know, Friedmann's equations describe a homogeneous and isotropic Universe in the context of General Relativity (GR).

The equations of the Model depend as well as the original equations of energy density (ρ) and pressure (p), but a new parameter ω is also introduced that implies corrections in the quantum scale of Space-Time.

The first Friedmann-type equations that we will describe have the typical form of the original equations of Birkhoff's Theorem:

$$\frac{1}{R}\frac{dR}{d\tau}= -\frac{4\pi G}{3}(\rho+\frac{3p}{c2})-2\omega(1-\frac{1}{R6})$$

$$\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2=\frac{8\pi G}{3}\rho-\frac{\omega}{R6}(1-\frac{3R4}{a}+2R^6),\ \omega=\frac{1}{12}(\frac{mc}{\hbar})^2 \quad (18)$$

As we said previously the parameter ω modifies the equations in the quantum scale, there will also be other corrections of this type.

The critical density of the Universe ($\rho_c$) is calculated from

$$\rho_c=\frac{3H2(\tau)}{8\pi G},\ H^2(\tau)=\frac{1}{R}\frac{dR}{d\tau}= 10^{-28}\frac{gr}{cm} \qquad (19)$$

The normal density according to the proper time is:

$$\rho(\tau)= \rho_c+\frac{1}{16\pi G}(\frac{mc2}{\hbar})^2 \qquad (20)$$

Here we find again a factor of the quantum order, but that does not greatly affect the final value of $\rho(\tau)$. The pressure p exerted by the energy on the Space-Time is

$$p=\frac{1}{3}\rho c^3 \quad (21)$$

The expansion parameter R is calculated from $R_{\mu\nu}$ with the equation

$$R_{\mu\nu}g_{\mu\nu}=-16\pi G\frac{\rho(\tau)}{c2} \quad (22)$$

The minimum value that you can take this expansion parameter is from the scale of

$$R_{\mu\nu}g_{\mu\nu}=\frac{2}{3}(\frac{mc}{\hbar})^2 \quad (23)$$

5-Conclusion.

The model that we presented has the value in that it can help us to know what happened in the first minutes of the Universe without being present the calculation failures that General Relativity (GR) presents due to the presence of the initial singularity, this is because The Hawking-Penrose conditions are met.

It is also a good model since it is space is pseudoeuclideo, which is wonderful if we try to write a Quantum Field Theory in a Curved Space-Time. This is thanks to the brilliant formulation of the Relativistic Theory of Gravitation (TRG).

6-Dedication

This article, my first published I dedicate it to one of the best people in my life, was my best friend, my confidant and I can hear myself talking about these equations all the time: Lupita (ricitos) I regret having done something that you I made him angry.

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