The Brighter Sides of Gravity

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Abstract. This paper is an appendix to the article "From Bernoulli to Laplace and Beyond" (refenced below), and discusses different aspects of it: electromagnetism, field tensors, general relativity, and probability.

1. Short Review

In [1] it was shown that gravity of matter can be derived from the simple quadratic form

$$< j, \Box A >:= \sum_{\mu} \int_{\mathbb{R}^4} \bar{j}_\mu(x) \Box A_\mu(x) d^4x =$$

$$- 4\pi G < j, j >:= -4\pi G \sum_{\mu} \int_{\mathbb{R}^4} \left| j_\mu(x) \right|^2 d^4x (= -4\pi G E_{tot}^2), \quad (1.1)$$

where $j = (j_0, \ldots, j_3)$ is the $\mathbb{C}^4$-valued space-time square root desity of smooth energy-momentum-distributions with compact support in space-time, that approximate an N-particle system, and $< j, j >:= \int \sum_\mu \left| j_\mu(x) \right|^2 d^4x$ is the total energy square E_{tot}^2 of j.
That equation is nothing but the so-called "equivalence condition" in terms of absolute squares: it says that the total square energy of gravitational interaction is proportional to the square of the total energy of the system j.
Then, it was shown that $< j, A >=< S^* j, Sj >:=< j, S^2 j >= 0$, where $j \mapsto Sj$ and $j \mapsto S^2$ are operators that maps j to functionals Sj and $S^2 j$, resp.. So, $(\sum_\mu \gamma_\mu \partial_\mu) Sj = (-4\pi G)^{1/2} j$, which we can formally write as:

$$j(x) = e^{i(4\pi G)^{1/2} \left(\sum_\mu \gamma_\mu x_\mu\right) S} j_{free}. \quad (1.2)$$

Now, we do know that if j splits into the disjoint sum $j = j_1 + j_2$ we have $|j|^2 = |j_1|^2 + |j_2|^2 = \left| j_{free} \right|^2$, but j_{free} is not be equal to the disoint sum $j_{1,free} + j_{2,free}$, unless the interaction between j_1 and j_2 is equal to zero!
To get at the interaction, the trick is to substitute $j = j_{free} + iSj_{free}$. If I was to forget the factor i before Sj_{free}, which easily is overlooked, so, if I were to

put $j = j_{free} + Sj_{free}$, then I won't get to the desired gravitational attraction in the non-relativistic limit ($c \to \infty$ or $T \to 0$), due to additional mixed terms $< j_{1,free}, Sj_{2,free} >$ and $< j_{2,free}, Sj_{1,free} >$. And that resulting attractive force is so, because make it so by convention: We take the absolute squares of the energy-momentum distributions, because rest-energy and kinetic energy "have to be" positive, and we insist that the rest energies of j_1 and j_2 will both and independently be positive.
That in mind, how do we get to electrodynamics and the Coulomb force from this?

2. Electrodynamics

Up to the factor $-4\pi G$, equation 1.1 is equivalent to Maxwell's equations (written in Gaussian units): we get them by dropping the bra-vectors and replacing the positive mass values with signed charges. And the sign of the right hand side is plus, because of the convention to base the sign on the positive charge and the observation that positive charges repel. We can deliberately make the sign negative again, by basing the equations on the negative electron charge. Then the very same procedure as with the masses leads to the attractive Coulomb force for positive and negative charges. Now remember, that in the non-relativistic limit, as it comes to the square root of $\left|j_{free}\right|^2 + V$, we need to take $\left|j_{free}\right|$ as a factor out of the root, and the positive sign of that absolute value depends on the sign of the charge of j_{free}. So, we get Coulomb attraction and repulsion, depending on the sign of the charges in $j_{1,free}$ and $j_{2,free}$. There is no more to it than the PCT-theorem says: a negatively charged particle is a PT-inverted positively charged particle.

3. Field Tensor

Given a vector field $A = (A_0, \ldots, A_3)$, we can associate it with the 1-form $A = A_0 dx_0 + \cdots + A_3 dx_3$ and take its exterior derivative

$$dA = \sum_{0 \leq \mu < \nu \leq 3} \partial_\mu A_\nu dx_\mu \wedge dx_\nu.$$

That external derivative is represented by the matrix

$$(T_{\mu\nu}) := (\partial_\mu A_\nu - \partial_\nu A_\mu)_{0 \leq \mu, \nu \leq 3},$$

which is called "field tensor", and is physically interpreted to hold the energy flow, pressure, and strain of the field.

The question is to the validity of that physical interpretation:
No doubt, A can be interpreted as a 1-form, because it holds the interaction between a free system j_1, the "source" of $A = A(j_1)$, and a disjoint and also free target system j_2, in the sense that $2Re < j_2, A(j_1) > +|j_1|^2 + |j_2|^2$ is the square of total energy of the resulting, interacting system, so that I could loosely write $\int A_0(j_{2,0}, j_1)dx_0 + \cdots + A_3(j_{2,3}, j_1)dx_3 :=< j_2, A(j_1) >$.

In particular, that interaction is identical to zero, when only j_2 vanishes. The sheer statement $A(x) \neq 0$ for some $x \in \mathbb{R}^4$, be it in a point or in a region is completely meaningless, unless there is a particle of mass or charge unequal zero in that point or region, and in this case, that particle becomes part of the source system with which it interacts! So, there is no derivative unequal zero, when there is no targeting source j_2, and it does not seem to make sense to speak of A as "living" as physical quantity in space-time of its own.
This is likewise the reason, why a particle can radiate its electromagnetic field without loosing any energy, if only there is no other particle around it. Its only when two distant particles receive eachother's field, that the interaction sets in.

4. General Relativity

The fact that general relativity is fundamentally based on the field tensor, namely by its basic equation

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

where $T_{\mu\nu}$ is the field tensor, makes this theory problematic: Not only $T_{\mu\nu}$ is to undergo complicated transformations so that the whole equation becomes covariant, it should also lead to a convex metric in the location coordinates. But, in view of the (formal) equation 1.2, what needs to be bent out is complex; in a truely phase symmetric model, space and time will become complex along with energy and momentum.

5. Annotation on Baryons and Leptons

In section 2 we saw that charges are up to parity of either sign of energy: positive or negative. We may associate this with the restriction of S to be real-valued, such that 1.2 becomes a sinus modulation of j_{free}. But in a complex world, there are also positive and negative imaginary values. Now, if iS was to be negative and decrease from some $x \to y$, say, then 1.2 would become confinement. And the opposite was exclusion. And there are three coordinates, the location coordinates, that have the same negative signature, namely the location coordinates. The time axis, however has a positive signature (due to the Minkowski metrics), and it would enforce a parity flip upon $t \mapsto it$, which cannot be balanced out by a mere permutation of the location coordinates.
Certainly, a phase symmetric world always allows to turn an imaginary value into a positive one, but if that was all to it, the positive mass would need to suffice to explain gravitation in terms of positive energies, negative and positive charges would be a quirk of nature, and the standard model would be without mass, but with a highly confined huge energy that would not be visible to the outside. Only that it should weigh a lot. I know, and I perfectly believe that the Higgs experiment really gave evidences for the existing Higgs

bosons. My point is that sufficiency and necessity of a condition appear to be mixed up: I would expect that, given that the subparticles have a mass, then in the deep collision experiment, one would see the scattering result that was reported. But I am, as probably many others, not capable to deduce a non-trivial mass of the subparticles given the result of the Higgs experiment!

6. (Daniel) Bernoulli, Boltzmann, and Probability

When I talked about Bernoulli, it was Daniel Bernoulli, and not his uncle Jacob. Jacob became famous as pioneer of probability theory, wheras Daniel, the co-worker with Euler in Saint Petersburg, was the originator of hydrodynamics. Certainly, Daniel knew of his uncle's doings, and the fundamentals of hydrodynamics do have quite some of probabilistic shining. But they are not the same:
A smooth distribution of particles with compact support in space or space-time is an approximation of an N-particle system. A particle confined in there can at best be to have an average location, energy, and momentum. As such, that particle is described statistically. But, we can partition that smooth distribution by sequences of ever refining sums of smooth distributions (called partition of unity) with decreasing diameter of support, each. So, assuming that no spontaneous particle exchange happens and that each particle stays individually apart of the others, we could track that individual particle in the infinitesimal limit. That is, what makes hydromechanics different from a pure, global, statistical theory. Now, given a two part container with two separated gases at different initial temperatures, sure the two gases will finally take a state, in which both parts have the same average kinetic energy. Both parts then have reached an equilibrium state, in which a particle with a certain kinetic energy is to be found with equal probability in both parts of the container. Does that imply that every system should decay at short or long terms into an overall statistical means situation? As to the amount of failed efforts to prove this over the last decades, one could say: No. Are there any counter examples? Yes, at least it appears to be: It's the KAM theorem, (named after Kolmogorov, Moser, and Arnold) [2]: Given a Hamiltonian function that has a periodic motion as its solution, then there might be exception points in any "neighbourhood" of that motion, that may lead to an unstable behaviour. But astonishingly, there are "far more" stabilizing points in there too, that would force the motion into a quasi-periodic motion. To close the path back from Daniel to Jacob Bernoulli, "all it would need" was a proper topology on the set of Hamitonian solutions, which would give a Borel measure (of probability), such that the KAM will perhaps become expressible as "the probability of a quasi periodic motion around a periodic solution to evolve into a non-quasi-periodic solution is zero".

References

[1] H. D. Hüttenbach, *From Bernoulli to Laplace and Beyond*, `http://vixra.org/abs/1801.0025`, 2018.

[2] C. E. Wayne, *Introduction to KAM Theory* `http://math.bu.edu/people/cew/preprints/introkam.pdf`, 2008.

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