A NEW BINOMIAL FORMULA FOR THE SUM OF TWO POWERS

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ABSTRACT. In this paper, we reveal a new binomial formula that expresses the sum of, or difference between two powers, $a^x \pm b^y$, as a binomial expansion of a single power, $z$. Like the standard binomial formula it includes the normal binomial coefficients, factors and indices, but includes an additional non-standard factor. The new formula (with an upper index $z$) mimics a standard binomial formula (to the power $z$) without the value of the binomial expansion changing even when $z$ itself changes. This has exciting implications for certain diophantine equations. This short paper simply highlights its existence.

Introduction The standard binomial formula expands a single power of a binomial expression. Here we reveal a new binomial formula that expands the sum/difference of two powers, $a^x \pm b^y$, as if it were a single power. The critical thing about this new formula is that it has an upper index, $z$, which can have any value without changing the overall value of the sum itself. It thus mimics a standard binomial formula (to the power $z$) without its own value changing even when $z$ itself changes.

1. THE BINOMIAL FORMULA FOR THE SUM/DIFFERENCE BETWEEN TWO POWERS

The new formula for the difference between two powers, for all $z \in \mathbb{Z}$, is:

$$a^x \pm b^y = \sum_{k=0}^{z} \binom{z}{k} (a + b)^{z-k} (-ab)^k (a^{x-z-k} \pm b^{y-z-k}).$$

The last factor, $(a^{x-z-k} \pm b^{y-z-k})$, is non-standard. By comparison, the standard binomial formula for a single power to $z$ (as a difference between two powers) is:

$$(p \pm q)^z = \sum_{k=0}^{z} \binom{z}{k} p^{z-k} (\pm q)^k.$$

So from our new formula, we say that $a^x \pm b^y$ is equal to:

$$(a+b)^{(a^x \pm b^y)} - ab(a^{x-2} \pm b^{y-2})$$

$$(a+b)^2(a^{x-2} \pm b^{y-2}) - 2ab(a+b)(a^{x-3} \pm b^{y-3}) + (ab)^2(a^{x-4} \pm b^{y-4})$$

$$(a+b)^3(a^{x-3} \pm b^{y-3}) - 3ab(a+b)^2(a^{x-4} \pm b^{y-4}) + 3(ab)^2(a+b)(a^{x-5} \pm b^{y-5}) - (ab)^3(a^{x-6} \pm b^{y-6})$$

And so on, ad infinitum.

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