TDBF: Two Dimensional Belief Function

Yangxue Li[a], Yong Deng[a,*]

[a] *Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China*

Abstract

How to efficiently handle uncertain information is still an open issue. In this paper, a new method to deal with uncertain information, named as two dimensional belief function (TDBF), is presented. A TDBF has two components, T=(m_A,m_B). The first component, m_A, is a classical belief function. The second component, m_B, also is a classical belief function, but it is a measure of reliability of the first component. The definition of TDBF and the discounting algorithm are proposed. Compared with the classical discounting model, the proposed TDBF is more flexible and reasonable. Numerical examples are used to show the efficiency of the proposed method.

Keywords: Two Dimensional Belief Function, Dempster-Shafer evidence theory, belief function, Z-numbers, conflict management.

---

1. Introduction

It is inevitable to deal with uncertain information in real word[1, 2]. To address this issue, many methods has been proposed, such as probability theory[3], Dempster-Shafer evidence theory[4, 5], fuzzy set[6–8], rough sets[9], Z-numbers[10], D numbers[11] and so on.

Among these methods, Dempster-Shafer evidence theory[4, 5] is one of the most widely used math tools[12–20]. Evidence theory has many advantages to handle un-

*Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China

Email address: dengentropy@uestc.edu.cn, prof.deng@hotmail.com (Yong Deng)

certain information[21–23]. For example, the belief function is more flexible to model uncertainty compared with probability distribution. In addition, Dempster's combination rule is used to combine evidences from different sources without prior information. However, illogical results may be obtained by classical Dempster combination rule when collected evidence highly conflicts each other[24–26].

Z-numbers is proposed by Zadeh[10]. A Z-number has two components, Z=(A,B). The first component, A, is a restriction (constraint) on the values which a real-valued uncertain variable, X, is allowed to take. The second component, B, is a measure of reliability (certainty) of the first component[10]. Similarly, in evidence theory, discounting coefficient is a measure of sensor report's reliability. e.g. discounting factors based on dissimilarity measure[27], dynamic discounting rates[28]. Nevertheless, the simple value is not enough and reasonable to express the experts evaluation. To solve this problem, a new math model named as two dimensional belief function, TDBF, is proposed in this paper.

A TDBF is an ordered pair of basic probability assignments denotes as T = (m_A,m_B). The first component m_A is a classical basic probability assignment (BPA), which usually is collected from the sensors. The second component m_B is a measure of reliability for the first component, which can be collected by the experts. The frame of discernment of m_B set $\Theta = \{Y, N\}$, where 'Y' denotes 'positive' and 'N' denotes 'negative'. The power set of $\Theta = \{Y, N\}$ consists of two singletons {Y} and {N}, an universal Θ and the empty set $\emptyset$. So, the TDBF can carry more information than not only classical BPA, but also the discounting BPA with single value discounting coefficient. In addition, the following research shows that the combination of TDBF also can partially address the issue of evidence conflicts.

The paper is organized as follows. The preliminaries of Dempster-Shafer theory[4, 5], Z-numbers[10] and classical discounting method[5] are briefly introduced in Section 2. In Section 3, the TDBF (Two Dimensional Belief Function) has been presented. In Section

4, we use some numerical examples to illustrate the application of the proposed method. Finally, this paper is concluded in Section 5.

2. Preliminaries

In this section, some preliminaries are briefly introduced.

2.1. D-S evidence theory

Some basic definitions of D-S theory are briefly introduced as following[4, 5]:

Definition 1. A set of hypotheses Θ is the exhaustive hypotheses of variable, X. The elements are mutually exclusive in Θ. Then Θ is called the frame of discernment, defined as follows[4, 5]:

$$\Theta = \{H_1, H_2, \cdots, H_i, \cdots, H_N\} \tag{1}$$

The power set of Θ is denoted by 2^Θ, and

$$2^\Theta = \{\emptyset, \{H_1\}, \cdots, \{H_N\}, \{H_1, H_2\}, \cdots, \{H_1, H_2, \cdots, H_i\}, \cdots, \Theta\} \tag{2}$$

where $\emptyset$ is an empty set.

Definition 2. A BPA function m is a mapping of 2^Θ to a probability interval $[0, 1]$, formally defined by[4, 5]:

$$m: \quad 2^\Theta \to [0, 1] \tag{3}$$

which satisfies the following conditions:

$$m(\emptyset) = 0 \quad \sum_{A \in 2^\Theta} m(A) = 1 \quad 0 \leq m(A) \leq 1 \quad A \in 2^\Theta \tag{4}$$

The mass m(A) represents how strongly the evidence supports A.

For the same evidence, the different BPAs come from the different evidence resources. The Dempster's combination rule can be used to obtain the combined evidence[4]:

$$\begin{cases} m(\emptyset) = 0 \\ m(A) = \frac{\sum\limits_{B \bigcap C = A} m_1(B) m_2(C)}{1 - K} \end{cases} \tag{5}$$

where $K = \sum\limits_{B \bigcap C = \emptyset} m_1(B) m_2(C)$.

2.2. *Z-numbers*

Z-numbers is a new tool to model uncertain information[10, 29].

Definition 3. A Z-number is an ordered pair of fuzzy numbers, $Z = (A, B)$. A Z-number is associated with a real-valued uncertain variable, X, with the first component, A, is a restriction (constraint) on X. The second component, B, is a measure of reliability (certainty) of the first component.

2.3. *Classical discounting method*

A discounting coefficient is between 0 and 1. So, let m_j be a belief mass given by the source S_j and let x_j be a coefficient which represents the confidence degree one has in source S_j. Denote $m_{x_j,j}$ the belief mass m_j discounted by a coefficient $(1 - x_j)$ and defined as follows[5, 30]:

$$
\begin{aligned}
m_{x_j,j}(A) &= x_j m_j(A) \qquad \forall A \subset \Theta, \\
m_{x_j,j}(\Theta) &= 1 - x_j + x_j m_j(\Theta).
\end{aligned}
\tag{6}
$$

3. TDBF: Two Dimension Belief Function

3.1. *The definition of TDBF*

Definition 4. A TDBF, T=(m_A,m_B), consists of two basic probability assignments. m_A is a classical BPA, and m_B also is a classical BPA and a measure of reliability (certainty) of m_A.

The frame of discernment of m_B set $\Theta = \{Y, N\}$, in which 'Y' denotes 'support' and 'N' denotes 'not-support'. The power set of $\Theta = \{Y, N\}$ consists of two singletons {Y} and {N}, a universal Θ and the empty set $\emptyset$. Then the $m(\{Y\})$ express how reliable m_A is, the $m(\{N\})$ express how unreliable m_A is, and the $m(\Theta)$ express that is no idea to measure the reliability of m_A. According to the definition of TDBF, if a body of evidence is close to real value, its $m(\{Y\})$ will be high and $m(\{N\})$ will be low. On the contrary, if a body of evidence is distant to real value, its $m(\{Y\})$ will be low and $m(\{N\})$ will be high. Compared with single value discounting coefficient, the proposed mode is more reasonable, flexible and comprehensive.

Example 3.1. *Assume the frame of discernment is* $\Theta = \{x_1, x_2, x_3\}$ *we given a BPA from a sensor as* $m_A(\{x_1\}) = 0.6, m_A(\{x_2\}) = 0.1, m_A(\{x_3, x_4\}) = 0.3$. *There are ten experts to measure this BPA. Seven experts think it is reliable, two experts think it is unreliable, and one has no idea and do not give any opinion. So, we can confirm the* m_B*:*

$$m_B(\{Y\}) = 0.7, m_B(\{N\}) = 0.2, m_B(\{\Theta\}) = 0.1. \tag{7}$$

3.2. The combination of TDBF

Given two TDBFs, the combination rule is defined as follows:

$$\begin{cases} m_Z(A) = m_A(\{x_i\}) \times m_B(Y) + m_B(N) \times (1 - m_A(\{x_i\})), & \forall x_i \subset \Theta \\ m_Z(A) = m_A(A_i) \times m_B(Y), & \forall A_i \subset \Theta \\ m_Z(\Theta) = m_A(\Theta) \times m_B(Y) + m_B(\Theta) \end{cases} \tag{8}$$

Where A_i is multi subset of Θ, and x_i is single subset of Θ, $i = 1, 2, 3, \cdots, n$.

The mass of $m(\{Y\})$ is distributed to the $m_A(A_i)$ proportionally. This value is front support of this subset. The mass of $m_B(\{N\})$ is distributed to the reverses of single subsets proportionally. This value is side support of this subset. Why don't distribute $m_B(\{N\})$ to reverses of multi subsets? The reason is that the masses of multi subsets will focus on single subsets in procedure of combination finally. And the mass of $m_B(\{\Theta\})$ is distributed to the $m_A(\{\Theta\})$ all. The addition of the front support and side support is the total support of $m_Z(A_i)$. The procedure is illustrated in Fig.1. We can get a new mass after normalization. When all $m_B(\{N\}) = 0$, the combination of TDBF degenerate the classical discounting method. When all $m_B(\{Y\}) = 1$, the combination of TDBF degenerate the Dempster's combination rule.

If there are n pieces of evidence, one can use the classical Dempster's rule to combine the new masses $n-1$ times.

4. Numerical Example

The real world, such as human sociality is very complex since the fact in complex systems are interact each other dynamically [31–33]. How to model this complexity is still an open issue. Some tools are used, for example, complex networks, to address this issue [34–37]. Among these tools, MADM is a common used tool to model complex system. In this section, some numerical examples on decision making are used to illustrate the application of our approach.

4.1. Target recognition

Example 4.1. *There are three known targets, x_1, x_2, x_3. So the frame of discernment is $\Theta = \{x_1, x_2, x_3\}$. A target appeared, three bodies of evidences are given by radars. The TDBFs of these evidences are given in Table 1 and Table 2. Then identify the target:*

First step, using Eq.8 to combine m_A and m_B of $T = (m_A, m_B)$:

$$\begin{aligned} m_{Z,1}(\{x_1\}) &= m_{A,1}(\{x_1\}) \times m_{B,1}(\{Y\}) + (1 - m_{A,1}(\{x_1\})) \times m_{B,1}(\{N\}) \\ &= 0.5 \times 0.8 + (1 - 0.5) \times 0.1 \\ &= 0.45 \end{aligned} \tag{9}$$

$$\begin{aligned} m_{Z,1}(\{x_2\}) &= m_{A,1}(\{x_2\}) \times m_{B,1}(\{Y\}) + (1 - m_{A,1}(\{x_2\})) \times m_{B,1}(\{N\}) \\ &= 0 \times 0.8 + (1 - 0) \times 0.1 \\ &= 0.1 \end{aligned} \tag{10}$$

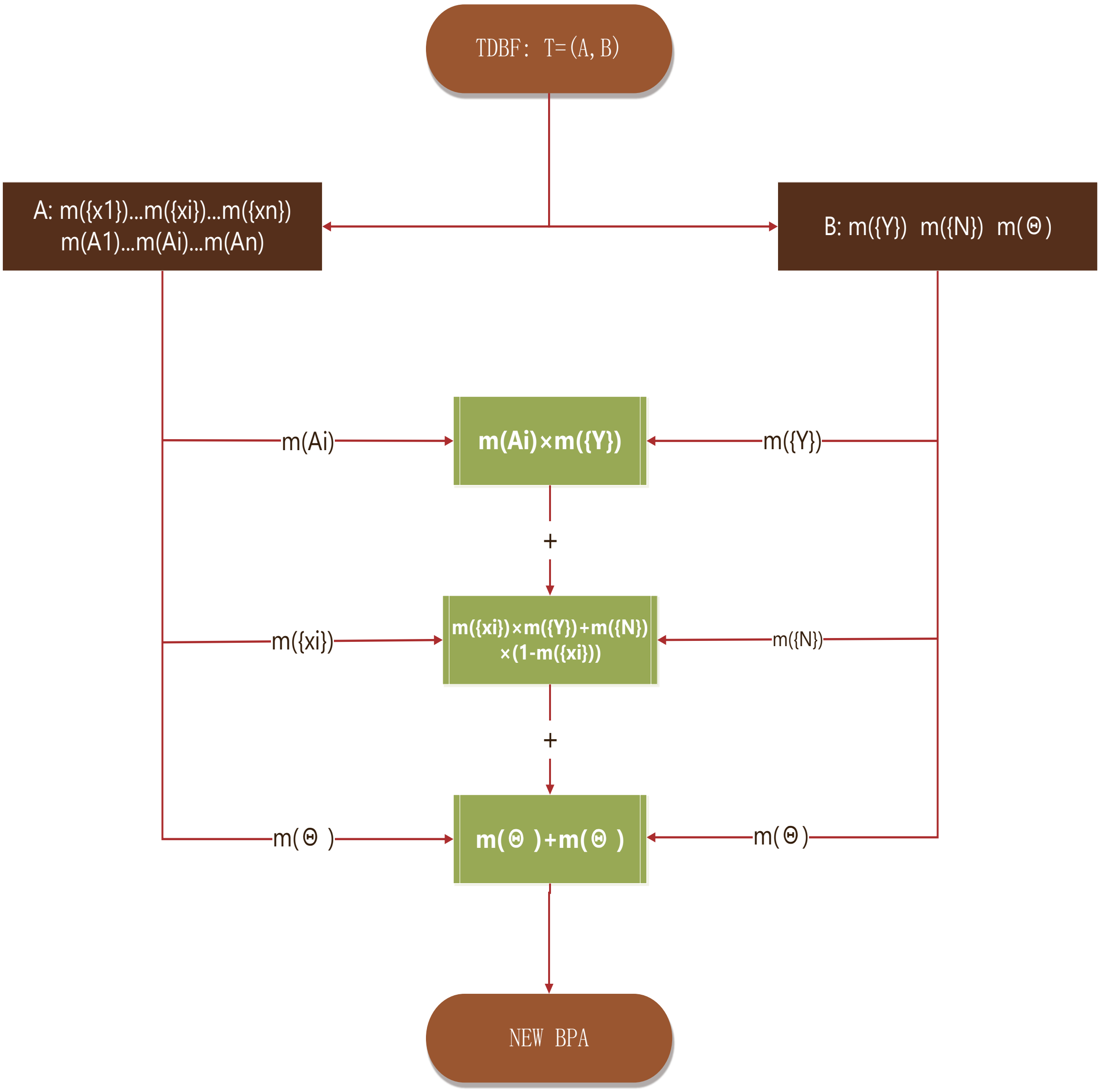

Figure 1: The procedure of TDBF's combination

$$\begin{aligned} m_{Z,1}(\{x_3\}) &= m_{A,1}(\{x_3\}) \times m_{B,1}(\{Y\}) + (1 - m_{A,1}(\{x_3\})) \times m_{B,1}(\{N\}) \\ &= 0 \times 0.8 + (1-0) \times 0.1 \\ &= 0.1 \end{aligned} \tag{11}$$

$$\begin{aligned} m_{Z,1}(\{x_1, x_2\}) &= m_{A,1}(\{x_1, x_2\}) \times m_{B,1}(\{Y\}) \\ &= 0.2 \times 0.8 \\ &= 0.16 \end{aligned} \tag{12}$$

$$\begin{aligned} m_{Z,1}(\Theta) &= m_{A,1}(\Theta) \times m_{B,1}(\{Y\}) + m_{Z,1}(\Theta) \\ &= 0.3 \times 0.8 + 0.1 \\ &= 0.34 \end{aligned} \tag{13}$$

We can get $m_{Z,2}$ and $m_{Z,3}$ using the same method:

$m_{Z,2} : m_{Z,2}(\{x_1\}) = 0.26, m_{Z,2}(\{x_2\}) = 0.2, m_{Z,2}(\{x_3\}) = 0.29,$
$m_{Z,2}(\{x_1, x_3\}) = 0.25, m_{Z,2}(\Theta) = 0.3$

$m_{Z,3} : m_{Z,3}(\{x_1\}) = 0.5, m_{Z,3}(\{x_2\}) = 0.25, m_{Z,3}(\{x_3\}) = 0.2,$
$m_{Z,3}(\{x_1, x_3\}) = 0.21, m_{Z,3}(\Theta) = 0.1$

After normalization:

$m_{Z,1} : m_{Z,1}(\{x_1\}) = 0.3913, m_{Z,1}(\{x_2\}) = 0.0870, m_{Z,1}(\{x_3\}) = 0.0870,$
$m_{Z,1}(\{x_1, x_2\}) = 0.1391, m_{Z,1}(\Theta) = 0.2956$

$m_{Z,2} : m_{Z,2}(\{x_1\}) = 0.2, m_{Z,2}(\{x_2\}) = 0.1538, m_{Z,2}(\{x_3\}) = 0.2231,$
$m_{Z,2}(\{x_1, x_3\}) = 0.1923, m_{Z,2}(\Theta) = 0.2308$

$m_{Z,3} : m_{Z,3}(\{x_1\}) = 0.3968, m_{Z,3}(\{x_2\}) = 0.1984, m_{Z,3}(\{x_3\}) = 0.1587,$
$m_{Z,3}(\{x_1, x_3\}) = 0.1667, m_{Z,3}(\Theta) = 0.0794$

Given result by using the classical Dempster's rule to combine these three masses two times:

$m : m(\{x_1\}) = 0.6643, m(\{x_2\}) = 0.1067, m(\{x_3\}) = 0.1547, m(\{x_1, x_2\}) = 0.0056,$
$m(\{x_1, x_3\}) = 0.0566, m(\Theta) = 0.0121$

If using the classical discounting method to combine these evidence, the result as follows:

$m : m(\{x_1\}) = 0.4843, m(\{x_2\}) = 0, m(\{x_3\}) = 0.0721, m(\{x_1, x_2\}) = 0.0873,$
$m(\{x_1, x_3\}) = 0.1201, m(\Theta) = 0.2402$

It can be seen from these results, both the classical discounting method and the propose TDBF can correctly recognize the target is x_1. However compared with classical discounting method, combination of TDBF is more reasonable since not only the support degree but also the disagree is taken into consideration.

Example 4.2. *The other example of target recognition is shown as Table 3 and Table 4. The result by using classical discounting method as follows:*

$m : m(\{x_1\}) = 0.3678, m(\{x_2\}) = 0.2194, m(\{x_3\}) = 0.0796, m(\{x_1, x_2\}) = 0.0606,$
$m(\{x_1, x_3\}) = 0.0130, m(\Theta) = 0.2596$

However, the result by using the combination of TDBF is:

$m : m(\{x_1\}) = 0.6145, m(\{x_2\}) = 0.2374, m(\{x_3\}) = 0.1437, m(\{x_1, x_2\}) = 0.0015,$
$m(\{x_1, x_3\}) = 0.0008, m(\Theta) = 0.0021$

Which are graphically shown in Fig.2.

As can be seen from Fig.2, when the reliability of one evidence source is small, although the classical discounting method can recognize the target, the value of $m(\{x_1\})$ is small and no more than 0.5. However, the proposed method not only can recognize the target, but also support the target strongly. In this case, the proposed method is superior to classical discounting method.

4.2. Conflict management

Example 4.3. *The combination of TDBF can partially address the issue of evidence conflict. From three different sensors, the system has collected three bodies of evidence are shown as Table 5, the information by experts is shown as Table 6.*

The results by combination rule of Dempster and TDBF are shown in Table 5.

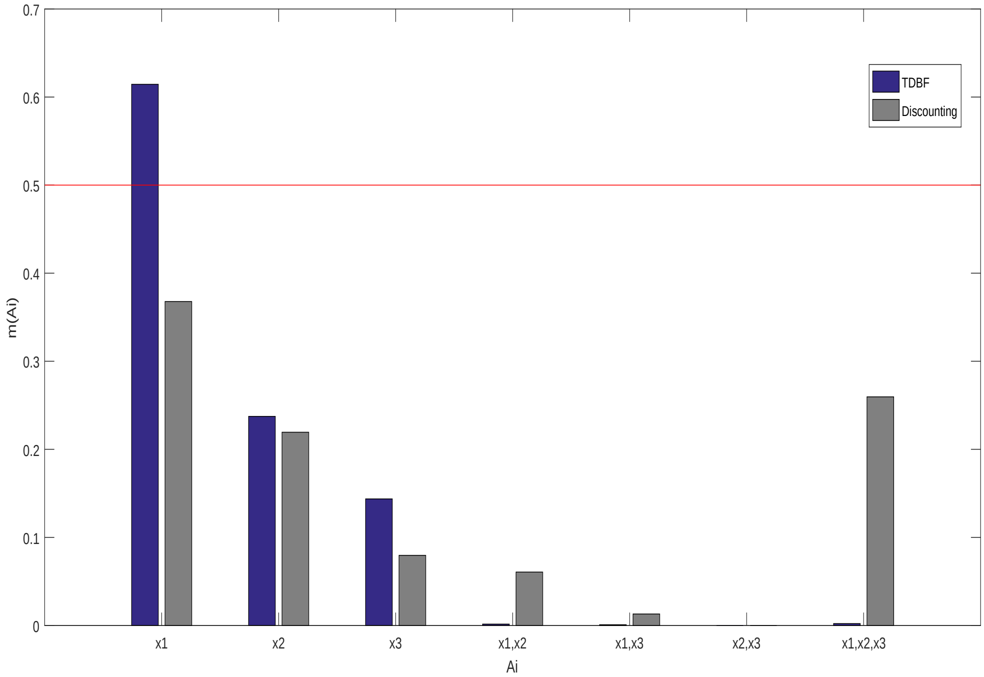

Figure 2: The results of TDBF's combination and discounting method

Sources			
$m_{A,1}$	$m_{A,1}(\{x_1\})$=0.5	$m_{A,1}(\{x_1,x_2\})$=0.2	$m_{A,1}(\Theta)$=0.3
$m_{A,2}$	$m_{A,2}(\{x_1\})$=0.2	$m_{A,2}(\{x_3\})$=0.3	$m_{A,2}(\{x_1,x_3\})$=0.5
$m_{A,3}$	$m_{A,3}(\{x_1\})$=0.6	$m_{A,3}(\{x_2\})$=0.1	$m_{A,3}(\{x_1,x_3\})$=0.3

Table 1: m_A of $T = (m_A, m_B)$ in Example4.1

Sources			
$m_{B,1}$	$m_{B,1}(\{Y\})$=0.8	$m_{B,1}(\{N\})$=0.1	$m_{B,1}(\Theta)$=0.1
$m_{B,2}$	$m_{B,2}(\{Y\})$=0.5	$m_{B,2}(\{N\})$=0.2	$m_{B,2}(\Theta)$=0.3
$m_{B,3}$	$m_{B,3}(\{Y\})$=0.7	$m_{B,3}(\{N\})$=0.2	$m_{B,3}(\Theta)$=0.1

Table 2: m_B of $T = (m_A, m_B)$ in Example4.1

Sources				
$m_{A,1}$	$m_{A,1}(\{x_1\})=0.5$	$m_{A,1}(\{x_2\})=0.3$	$m_{A,1}(\{x_3\})=0.1$	$m_{A,1}(\{x_1,x_2\})=0.1$
$m_{A,2}$	$m_{A,2}(\{x_1\})=0.3$	$m_{A,2}(\{x_2\})=0.3$	$m_{A,2}(\{x_3\})=0.2$	$m_{A,2}(\{x_1,x_3\})=0.2$
$m_{A,3}$	$m_{A,3}(\{x_1\})=0.7$	$m_{A,3}(\{x_2\})=0.2$	$m_{A,3}(\{x_3\})=0.1$	

Table 3: m_A of $T = (m_A, m_B)$ in Example4.2

Sources			
$m_{B,1}$	$m_{B,1}(\{Y\})=0.7$	$m_{B,1}(\{N\})=0.2$	$m_{B,1}(\Theta)=0.1$
$m_{B,2}$	$m_{B,2}(\{Y\})=0.2$	$m_{B,2}(\{N\})=0.7$	$m_{B,2}(\Theta)=0.1$
$m_{B,3}$	$m_{B,3}(\{Y\})=0.8$	$m_{B,3}(\{N\})=0.1$	$m_{B,3}(\Theta)=0.1$

Table 4: m_B of $T = (m_A, m_B)$ in Example4.2

Sources			
$m_{A,1}$	$m_{A,1}(\{x_1\})=0.9$	$m_{A,1}(\{x_2\})=0.1$	$m_{A,1}(\{x_3\})=0$
$m_{A,2}$	$m_{A,2}(\{x_1\})=0$	$m_{A,2}(\{x_2\})=0.9$	$m_{A,2}(\{x_3\})=0.1$
$m_{A,3}$	$m_{A,3}(\{x_1\})=0.6$	$m_{A,3}(\{x_2\})=0.1$	$m_{A,3}(\{x_1,x_3\})=0.3$

Table 5: m_A of $T = (m_A, m_B)$ in Example4.3

Sources			
$m_{B,1}$	$m_{B,1}(\{Y\})=0.8$	$m_{B,1}(\{N\})=0.1$	$m_{B,1}(\Theta)=0.1$
$m_{B,2}$	$m_{B,2}(\{Y\})=0.4$	$m_{B,2}(\{N\})=0.2$	$m_{B,2}(\Theta)=0.4$
$m_{B,3}$	$m_{B,3}(\{Y\})=0.7$	$m_{B,3}(\{N\})=0.2$	$m_{B,3}(\Theta)=0.1$

Table 6: m_B of $T = (m_A, m_B)$ in Example4.3

combination rule	m_1, m_2	m_1, m_2, m_3
Dempster	$m(\{x_1\}) = 0$, $m(\{x_2\}) = 1$, $m(\{x_3\}) = 0$	$m(\{x_1\} = 0$, $m(\{x_2\}) = 1$, $m(\{x_1, x_3\}) = 0$, $m(\{x_3\}) = 0$
TDBF	$m(\{x_1\}) = 0.6085$, $m(\{x_2\}) = 0.2267$, $m(\{x_3\}) = 0.1116$, $m(\{\Theta\}) = 0.0532$	$m(\{x_1\}) = 0.7462$, $m(\{x_2\}) = 0.1331$, $m(\{x_3\}) = 0.0970$, $m(\{x_1, x_3\}) = 0.0161$, $m(\Theta) = 0.0076$

Table 7: The results of combination by different rules

As can be seen in Table 5, when combining conflicting evidences, the classical Dempster's rule products illogical results, Dempster's combination results show that, though more bodies of evidence collected later support target x_1, only a body of evidence don't support target x_1 at all, the result is zero after combining. On the contrary, TDBF's combination provides reasonable results due to the consideration of evidence reliability.

5. Discussions and Conclusions

Dempster-Shafer evidence theory is an effective method to handle uncertain information, but it lacks the reliability of evidence, it assumes every evidence is factual. But the issue of reliability of information is of pivotal importance in planning, decision-making, formulation of algorithms and management of information[38, 39]. In TDBF, it uses a BPA to express the reliability of evidence, not only the support degree but also the disagree is taken into consideration. Compared with classical BPA and discounting BPA with single value discounting coefficient, TDBF is more complete, reasonable and flexible to handle uncertain information. In addition, combination of TDBF can deal with the evidence conflict.

Acknowledgment

The work is partially supported by National Natural Science Foundation of China (Grant Nos. 61573290, 61503237).

[1] G. Klir, M. Wierman, Uncertainty-based information: elements of generalized information theory, Vol. 15, Springer Science & Business Media, 1999.

[2] R. K. Bhatnagar, L. N. Kanal, Handling uncertain information: a review of numeric and non-numeric methods, in: Machine Intelligence and Pattern Recognition, Vol. 4, Elsevier, 1986, pp. 3–26.

[3] W. Feller, An introduction to probability theory and its applications, Vol. 2, John Wiley & Sons, 2008.

[4] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, The annals of mathematical statistics (1967) 325–339.

[5] G. Shafer, et al., A mathematical theory of evidence, Vol. 1, Princeton university press Princeton, 1976.

[6] L. A. Zadeh, Fuzzy sets, in: Fuzzy Sets, Fuzzy Logic, And Fuzzy Systems: Selected Papers by Lotfi A Zadeh, World Scientific, 1996, pp. 394–432.

[7] R. Zhang, B. Ashuri, Y. Deng, A novel method for forecasting time series based on fuzzy logic and visibility graph, Advances in Data Analysis and Classification 11 (4) (2018) 759–783. doi:DOI: 10.1007/s11634-017-0300-3.

[8] L. Fei, H. Wang, L. Chen, Y. Deng, A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators, Iranian Journal of Fuzzy Systems (2017) accepted.

[9] Z. Pawlak, J. Grzymala-Busse, R. Slowinski, W. Ziarko, Rough sets, Communications of the ACM 38 (11) (1995) 88–95.

[10] L. A. Zadeh, A note on z-numbers, Information Sciences 181 (14) (2011) 2923–2932.

[11] Y. Deng, D numbers: Theory and applications, JOURNAL OF INFORMATION &COMPUTATIONAL SCIENCE 9 (9) (2012) 2421–2428.

[12] S. Le Hegarat-Mascle, I. Bloch, D. Vidal-Madjar, Application of dempster-shafer evidence theory to unsupervised classification in multisource remote sensing, IEEE transactions on geoscience and remote sensing 35 (4) (1997) 1018–1031.

[13] O. Basir, X. Yuan, Engine fault diagnosis based on multi-sensor information fusion using dempster–shafer evidence theory, Information Fusion 8 (4) (2007) 379–386.

[14] H.-R. Bae, R. V. Grandhi, R. A. Canfield, An approximation approach for uncertainty quantification using evidence theory, Reliability Engineering & System Safety 86 (3) (2004) 215–225.

[15] H. Xu, Y. Deng, Dependent evidence combination based on shearman coefficient and pearson coefficient, IEEE Access (2018) 10.1109/ACCESS.2017.2783320.

[16] H. Zheng, Y. Deng, Evaluation method based on fuzzy relations between Dempster-Shafer belief structure, International Journal of Intelligent Systemsdoi:10.1002/int.21956.

[17] T. Liu, Y. Deng, F. Chan, Evidential supplier selection based on DEMATEL and game theory, International Journal of Fuzzy Systems (2017) DOI: 10.1007/s40815–017–0400–4.

[18] R. Sun, H.-Z. Huang, Q. Miao, Improved information fusion approach based on ds evidence theory, Journal of mechanical science and technology 22 (12) (2008) 2417–2425.

[19] J.-W. Zhuge, D.-W. Wang, Y. Chen, Z.-Y. Ye, W. Zou, A network anomaly detector based on the ds evidence theory, Journal of software 17 (3) (2006) 463–471.

[20] D. Qingdong, X. Lingyu, Z. Hai, Ds evidence theory applied to fault diagnosis of generator based on embedded sensors, in: Information Fusion, 2000. FUSION 2000. Proceedings of the Third International Conference on, Vol. 1, IEEE, 2000, pp. TUD5–3.

[21] P. Limbourg, E. De Rocquigny, Uncertainty analysis using evidence theory–confronting level-1 and level-2 approaches with data availability and computational constraints, Reliability Engineering & System Safety 95 (5) (2010) 550–564.

[22] M. Lalmas, Dempster-shafer's theory of evidence applied to structured documents: modelling uncertainty, in: ACM SIGIR Forum, Vol. 31, ACM, 1997, pp. 110–118.

[23] V. Kaftandjian, O. Dupuis, D. Babot, Y. M. Zhu, Uncertainty modelling using dempster–shafer theory for improving detection of weld defects, Pattern recognition letters 24 (1) (2003) 547–564.

[24] L. A. Zadeh, A simple view of the dempster-shafer theory of evidence and its implication for the rule of combination, Ai Magazine 7 (2) (1986) 85–90.

[25] Y. Deng, Generalized evidence theory, Applied Intelligence 43 (3) (2015) 530–543.

[26] H. Liu, Z. Zhao, X. Liu, Combination of conflict evidences in ds theory, Journal of University of Electronic Science and Technology of China 37 (5) (2008) 701–704.

[27] Z.-g. Liu, J. Dezert, Q. Pan, G. Mercier, Combination of sources of evidence with different discounting factors based on a new dissimilarity measure, Decision Support Systems 52 (1) (2011) 133–141.

[28] M. C. Florea, A.-L. Jousselme, É. Bossé, Dynamic estimation of evidence discounting rates based on information credibility, RAIRO-Operations Research 44 (4) (2010) 285–306.

[29] B. Kang, G. Chhipi-Shrestha, Y. Deng, K. Hewage, R. Sadiq, Stable strategies analysis based

on the utility of z-number in the evolutionary games, Applied Mathematics and Computation-doi:10.1016/j.amc.2017.12.006.

[30] E. Lefevre, O. Colot, P. Vannoorenberghe, Belief function combination and conflict management, Information fusion 3 (2) (2002) 149–162.

[31] W. Jiang, B. Wei, J. Zhan, C. Xie, D. Zhou, A visibility graph power averaging aggregation operator: A methodology based on network analysis, Computers & Industrial Engineering 101 (2016) 260–268. doi:10.1016/j.cie.2016.09.009.

[32] Z. Wang, C. T. Bauch, S. Bhattacharyya, A. d'Onofrio, P. Manfredi, M. Perc, N. Perra, M. Salathé, D. Zhao, Statistical physics of vaccination, Physics Reports 664 (2016) 1–113.

[33] Y. Yu, G. Xiao, J. Zhou, Y. Wang, Z. Wang, J. Kurths, H. J. Schellnhuber, System crash as dynamics of complex networks, Proceedings of the National Academy of Sciences 113 (42) (2016) 11726–11731.

[34] W. Jiang, B. Wei, Y. Tang, D. Zhou, Ordered visibility graph average aggregation operator: An application in produced water management, Chaos: An Interdisciplinary Journal of Nonlinear Science 27 (2) (2017) Article ID 023117. doi:10.1063/1.4977186.

[35] X. Li, M. Jusup, Z. Wang, H. Li, L. Shi, B. Podobnik, H. E. Stanley, S. Havlin, S. Boccaletti, Punishment diminishes the benefits of network reciprocity in social dilemma experiments, Proceedings of the National Academy of Sciences (2017) Published online, doi: 10.1073/pnas.1707505115.

[36] Q. Zhang, M. Li, Y. Deng, Measure the structure similarity of nodes in complex networks based on relative entropy, Physica A: Statistical Mechanics and its Applications 491 (2018) 749–763.

[37] Z. Wang, M. Jusup, R. W. Wang, L. Shi, Y. Iwasa, Y. Moreno, J. Kurths, Onymity promotes cooperation in social dilemma experiments, Science Advances 3 (3) (2017) e1601444.

[38] C. K. Murphy, Combining belief functions when evidence conflicts, Decision support systems 29 (1) (2000) 1–9.

[39] X. Zheng, Y. Deng, Dependence assessment in human reliability analysis based on evidence credibility decay model and iowa operator, Annals of Nuclear Energy 112 (2018) 673–684.