Mass and ground state energy of the oscillator of a black body and the lowest radiation temperature

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According to the Planck's theory of black body radiation, a black body is a collection of oscillators which are responsible for its radiation. There are no results available in the literature about the mass and vibrational ground state energy of the oscillator. In the present research, ground state energy and mass of the oscillator are calculated from Planck's theory of black body radiation and de Broglie's wave particle duality relation. It is observed that the mass of the oscillator is $1.6 \times 10^{-39} kg$ and vibrational ground state energy is $3.587 \times 10^{-23} J$ subject to the constraint that the minimum temperature for radiation of a black body is $2.598K$.

keywords : Mass of the oscillator; Vibrational ground state energy; Lowest radiation temperature.

I. INTRODUCTION

Black body radiation [1–3] is one of the most important problems which are unable to explain properly with classical mechanics. Between 1984 and 1900, German physicist Max Planck worked on this problem which leads to the foundation of quantum theory. He assumes that a black body is a collection of numbers of oscillators. Different oscillator has a different frequency. An oscillator emits or absorbs energy proportional to its oscillation frequency. Thus, emission or absorption of energy of a black body is discrete rather continuous which he terms as quantization of energy. He used this concept of quantization of electromagnetic radiation to explain the energy distribution nature of a black body.

Louis de Broglie, a French physicist, proposed in 1925 that every object has a wave-particle dual character [4]. According to the de Broglie, wavelength (λ) of a particle having momentum p is $\frac{h}{p}$, where h is Planck's constant. So far, no one implies the de Broglie's duality condition, *ie.* $\lambda = \frac{h}{p}$, on the vibrating oscillator. In present research, it is observed that if we impose de Broglie's duality relation on the vibrating oscillator, we get different equations for the mass and the vibrational ground state energy of the oscillator. We also get an equation for the minimum temperature for radiation of a black body.

II. THEORY

According to Planck [5], monochromatic vibrations of resonators of a black body are responsible for its absorption or emission of electromagnetic radiation. Let the number of identical resonators having oscillation frequency ν is N at thermal equilibrium of temperature T. If U is the thermal energy of a single resonator then the total energy of the body will be NU. It is known that entropy S is related to its vibrational energy and temperature as -

$$\frac{dS}{dU} = \frac{1}{T} \tag{1}$$

Applying the equipartition principle, we get kinetic energy of one mode of vibration at a temperature T is $\frac{1}{2}kT$ where k is Boltzmann constant. Thus, total kinetic energy of a complete vibration is kT as every vibration has two modes. Now we get

$$U = kT$$

or,

$$dU = kdT \tag{2}$$

From equation 1 and equation 2 we get

$$dS = kd(lnT)$$

and

$$S = klnT + A \tag{3}$$

where A is integration constant. If we consider that at the critical temperature T_c, entropy of the oscillator is 0 then we get from Equation 3,

$$klnT_c + A = 0$$

or,

$$A = -klnT_c \tag{4}$$

Putting the value of A in Equation 3 we get

$$S = kln(\frac{T}{T_c}) \tag{5}$$

Now we may impose de Broglie's relation. Let, p is the momentum and m is the mass of a resonator. From the kinetic theory of gas it is known that at a temperature T kinetic energy of a gas particle is $\frac{1}{2}kT$. This is applicable for our oscillator. Thus, we get

$$\frac{1}{2}kT = \frac{p^2}{2m}$$

or,

$$kT = \frac{p^2}{m} \tag{6}$$

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From de Broglie's duality relation we get

$$p = \frac{h}{\lambda} = \frac{h\nu}{c} \tag{7}$$

where h is Planck's constant, λ is the de Broglie wavelength of the resonator, ν is corresponding frequency and c is the velocity of light in vacuum. From equation 6 and equation 7 we get

$$kT = \frac{h^2\nu^2}{mc^2}$$

or,

$$T = \frac{h^2\nu^2}{mkc^2} \tag{8}$$

Putting the value of T taken from Equation 8 in Equation 5 we get

$$S = kln\left(\frac{h^2\nu^2}{mkc^2T_c}\right) \tag{9}$$

Following Planck's derivation [5] of entropy of an oscillator we get

$$S = k\left\{\left(1+\frac{U}{h\nu}\right)ln\left(1+\frac{U}{h\nu}\right) - \frac{U}{h\nu}ln\frac{U}{h\nu}\right\} \tag{10}$$

where ν is the frequency of the oscillator. Equation 10 may be rearranged as

$$S = kln\left\{\left(1+\frac{U}{h\nu}\right)\left(1+\frac{h\nu}{U}\right)^{\frac{U}{h\nu}}\right\} \tag{11}$$

Comparing equation 9 and equation 11 we get

$$\left(\frac{h^2\nu^2}{mkc^2T_c}\right) = \left\{\left(1+\frac{U}{h\nu}\right)\left(1+\frac{h\nu}{U}\right)^{\frac{U}{h\nu}}\right\} \tag{12}$$

Now from equation 2 and equation 8 we get

$$\frac{U}{h\nu} = \frac{h\nu}{mc^2} \tag{13}$$

Putting the value of $\frac{U}{h\nu}$ in equation 12 we get

$$\left(\frac{h^2\nu^2}{mkc^2T_c}\right) = \left(1+\frac{h\nu}{mc^2}\right)\left(1+\frac{mc^2}{h\nu}\right)^{\frac{h\nu}{mc^2}} \tag{14}$$

Using the value of T from equation 8 we get from equation 14

$$\frac{T}{T_c} = \left(1+\frac{h\nu}{mc^2}\right)\left(1+\frac{mc^2}{h\nu}\right)^{\frac{h\nu}{mc^2}} \tag{15}$$

Considering our oscillator as harmonic oscillator, energy of its n^{th} vibrational state would be

$$E_n = (2n+1)\frac{h}{4\pi}\omega \tag{16}$$

where, ω is angular frequency of the oscillator. $\omega = 2\pi\nu$ where ν is linear frequency. At temperature T, energy of the oscillator is kT. Thus, from Equation 16 we get

$$kT = (2n+1)\frac{h\nu}{2} \tag{17}$$

Comparing Equation 8 and Equation 17 we get

$$(2n+1)\frac{h\nu}{2} = \frac{h^2\nu^2}{mc^2}$$

$$\frac{h\nu}{mc^2} = \frac{(2n+1)}{2} \tag{18}$$

Now replacing the value of $\frac{h\nu}{mc^2}$ by $\frac{(2n+1)}{2}$ in Equation 15 we get

$$\frac{T}{T_c} = \frac{1}{2}\sqrt{\frac{(2n+3)^{(2n+3)}}{(2n+1)^{(2n+1)}}} \tag{19}$$

From Equation 19 we get $\frac{T}{T_c}$ for different vibrational states. Variation of $\frac{T}{T_c}$ with vibrational states is presented in Figure 1.

FIG. 1: Variation of $\frac{T}{T_c}$ with vibrational states of the oscillator of a black body

III. DISCUSSIONS

The plot of $\frac{T}{T_c}$ against the vibrational states of the oscillator presented in Figure 1 is very interesting. The value of $\frac{T}{T_c}$ for $n = 0$ is 2.598 and T is greater than T_c for all n. As at the lowest vibrational state $T > T_c$, we can say that the oscillator has positive entropy at its ground state of vibration which is not obvious. At present it is not clear. But, we may assume that this entropy is not a thermal entropy. There may be another contribution to the total entropy of a system which may be very small in quantity. So far, there is no idea about the exact value of T_c. But, if T_c is very small, T would also be small. For example, for $T_c = 0.1K$, $T = 0.2598K$; $T_c = 0.0001K$, $T = 0.0002598K$; and so on. Thus, we can say, when $T_c \to 0$, $T \to 0$. This is valid if $A \neq 0$ in Equation 3. If we consider $A = 0$, we get $T = 2.598K$ for $n = 0$. Then we can say $2.598K$ is the lower limit of temperature of a black body to emit any radiation.

From Equation 17 and 18 we get

$$\frac{kT}{mc^2} = \left(\frac{2n+1}{2}\right)^2 \tag{20}$$

From Equation 20 we get for $n = 0$,

$$m = \frac{4kT}{c^2} \tag{21}$$

For, $n = 0$, $T = 2.598K$ (for $A = 0$). Thus, from Equation 21 we get, mass of the oscillator is $1.6 \times 10^{-39} kg$.

We can calculate the ground state vibrational energy of the oscillator which is kT. If we consider $A = 0$, then for the ground state (*ie.* $n = 0$) $T = 2.598K$. Thus, the ground state vibrational energy of the oscillator is $3.587 \times 10^{-23} J$.

IV. CONCLUSIONS

The wave particle duality relation is applied to Planck's black body theory to measure different properties of the oscillator. It is observed that if we consider the integration constant (A) = 0, then the lower limit of temperature of a black body to emit any energy is $2.598K$, the mass of the oscillator is $1.6 \times 10^{-39} kg$ and the ground state vibrational energy is $3.587 \times 10^{-23} J$.

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