Theoretical Lower Limit of Mass of Phonon and Critical Mass for Matter-Dark Matter conversion

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In this article, theoretical lower limit of mass of phonon and critical mass for matter-dark matter conversion is presented. Using Planck's equation for black body radiation and de Broglie's wave particle duality, we can get a relation between mass of phonon and frequency of emitted radiation which opens up several questions and possibilities. From this relation, with few assumptions, we may have a critical mass bellow which we have dark matter and above we get normal matter. With the help of this relation, maximum matter density and limit of string length may be reviewed. Calculated critical mass, considering the present value of Planck's constant and Boltzmann constant, is $7.367 \times 10^{-51} Kg$. It is also observed that if phonon obeys de Broglie's equation, generation of an electromagnetic radiation of frequency less than $56638721410 Hz$ is not possible by thermal heating.

I. INTRODUCTION

Between 1984 and 1900 German physicist Max Planck started to work on black body radiation [1–3] which leads to the formation of quantum theory. He developed and used the theory of electromagnetic radiation to explain the nature of radiation of a black body. So far, characteristics of black body radiation is well studied and satisfactorily explained by Planck's theory. By the time, in 1925, French physicist Louis de Broglie found the wave particle duality relation [4] which is another important discovery towards the formation of quantum mechanics. In this article, wave particle duality relation is applied on vibrations of oscillators of a black body which brings several interesting features of matter and radiation in light. We reached to an equation where frequency of the resonator is related to the mass of the resonator of the emitting body through Planck's constant, h, and Boltzmann constant, k. Using this equation, interaction between light and matter is explained in a different way. From this equation we get the lower limit of mass of the resonator i.e. phonon. Considering the present estimated value of mass density at the center of a black hole we can calculate the dimension of a resonator which is of the order of $10^{-24}m$, higher than the Planck length ($10^{-35}m$). Thus, at the Planck length, density of mass would be 10^{33} times higher than the present estimated value of mass density at the center of a black hole which is $1.5\times10^{20} kgm^{-3}$. We also found that if a particle has mass lower than a particular value (critical mass we can say) it would behave like a dark matter. Thus, matter-dark matter conversion may be explained with the help of the present hypothesis.

II. THEORY

According to Planck [5], electromagnetic radiation depends on the monochromatic vibrations of the resonators of the body which is absorbing or emitting electromagnetic radiation. Now let us consider a body which is emitting electromagnetic radiation has N number of identical resonators at thermal equilibrium of temperature T. If U be the energy of a single vibrating resonator then total energy of the body will be UN. Entropy S of a monochromatic vibrating resonator is related to its vibrational energy and temperature as -

$$\frac{dS}{dU} = \frac{1}{T} \tag{1}$$

According to the equipartition principle, kinetic energy of one mode of vibration at a temperature T is $\frac{1}{2}kT$ where k is Boltzmann constant. As one complete vibration has two modes, kinetic energy of one resonator at temperature T is kT. Thus, we get

$$U = kT$$

or,

$$dU = kdT \tag{2}$$

From equation 1 and equation 2 we get

$$dS = kd(lnT)$$

and

$$S = klnT + A \tag{3}$$

where A is integration constant. As A is a scaling factor, we may consider $A = 0$ for simplicity. Then we get from equation 3,

$$S = klnT \tag{4}$$

If p be the momentum and m be the mass of the resonator, we get

$$\frac{1}{2}kT = \frac{p^2}{2m}$$

or,

$$kT = \frac{p^2}{m} \tag{5}$$

Considering de Broglie's wave particle duality relation we can have

$$p = \frac{h}{\lambda} = \frac{h\nu}{c} \quad (6)$$

where h is Planck's constant, λ is wave length associated with that resonator due to its momentum, ν is frequency corresponds to λ and c is the velocity of light in vacuum. From equation 5 and equation 6 we get

$$kT = \frac{h^2\nu^2}{mc^2}$$

or,

$$T = \frac{h^2\nu^2}{mkc^2} \quad (7)$$

Replacing T in equation 4 by equation 7 we get

$$S = kln\left(\frac{h^2\nu^2}{mkc^2}\right) \quad (8)$$

Planck [5] derived an alternative expression for entropy of a resonator as follows

$$S = k\left\{\left(1+\frac{U}{h\nu}\right)ln\left(1+\frac{U}{h\nu}\right) - \frac{U}{h\nu}ln\frac{U}{h\nu}\right\} \quad (9)$$

Equation 9 may be written as

$$S = kln\left\{\left(1+\frac{U}{h\nu}\right)\left(1+\frac{h\nu}{U}\right)^{\frac{U}{h\nu}}\right\} \quad (10)$$

Comparing equation 8 and equation 10 we get

$$\left(\frac{h^2\nu^2}{mkc^2}\right) = \left\{\left(1+\frac{U}{h\nu}\right)\left(1+\frac{h\nu}{U}\right)^{\frac{U}{h\nu}}\right\} \quad (11)$$

From equation 2 and equation 5 we get

$$U = \frac{h^2\nu^2}{mc^2} \quad (12)$$

Thus,

$$\frac{U}{h\nu} = \frac{h\nu}{mc^2} \quad (13)$$

Putting this value of $\frac{U}{h\nu}$ in equation 11 we get

$$\left(\frac{h^2\nu^2}{mkc^2}\right) = \left(1+\frac{h\nu}{mc^2}\right)\left(1+\frac{mc^2}{h\nu}\right)^{\frac{h\nu}{mc^2}} \quad (14)$$

In equation 14, m is the mass of the resonator (which is considered as phonon in modern physics). Thus, mc^2 is the energy (E) equivalent to the mass of the resonator. Following Einstein's energy quantization relation we can write

$$mc^2 = E = h\nu_0$$

or,

$$\frac{mc^2}{h} = \nu_0 \quad (15)$$

Here it should be mentioned that ν and ν_0 are not same. ν is the vibrational frequency of the resonator and ν_0 is the frequency of mass equivalent of resonator. Replacing $\frac{mc^2}{h}$ by ν_0 in equation 14 we get

$$\frac{h\nu^2}{k\nu_0} = \left(1+\frac{\nu}{\nu_0}\right)\left(1+\frac{\nu_0}{\nu}\right)^{\frac{\nu}{\nu_0}} \quad (16)$$

III. NUMERICAL ANALYSIS AND DISCUSSIONS

Since h and k are constant in equation 16 we have very interesting relation between ν and ν_0. It shows that lowest energy vibration (i.e. lowest value of frequency, ν) of a resonator (phonon) depends on its mass (defined by ν_0) only. Planck [2] considered that a resonator emits electromagnetic radiation with frequency same as that of the resonator. Thus, it is possible that for any object which emits electromagnetic radiations, should have a critical temperature bellow which all vibrating resonators would be at its lowest energy level. Thus, emission spectrum of an object bellow its critical temperature should give information about the number of resonators with different mass. Not only that, as different frequencies correspond to resonator of different masses, we should be able to calculate the mass of every resonator through equation 16.

In equation 16, ν_0 is the frequency related to the mass of the resonator. ν_0 is a non zero positive quantity. To get an idea about the mass of a resonator, let us consider a small value of ν_0 as $\nu_0 = 1Hz$ then mass of this resonator would be $7.36 \times 10^{-51}kg$ which is 10^{21} times lighter than that of lightest quark (the down quark, mass $= 7.297 \times 10^{-30}kg$). Putting $\nu_0 = 1Hz$ in equation 16 we get $\nu = 56638721410Hz$. Thus, if lowest value of ν_0 is $1Hz$ and both Planck's theory for black body radiation and de Broglie's wave particle duality relation are true, we can conclude that, it is impossible to generate an electromagnetic radiation with frequency less than $56638721410Hz$ by thermal excitation only. Thus, $56638721410Hz$ or $56.63872141GHz$ frequency may be considered as *cut off frequency* for thermal emission. This explains why we never get any significant emission at very low frequency even at very low temperature. In Planck's equation there is no cut off frequency except an exponential decay which implies every object even at very low temperature would emit electromagnetic radiations spontaneously, though energy density may be very small which is not true.

The lowest value of ν may be calculated from equation 16 by numerical method. A plot of ν against ν_0 is presented in Figure 1. It is observed that for very small values of ν_0, lowest value of ν varies within

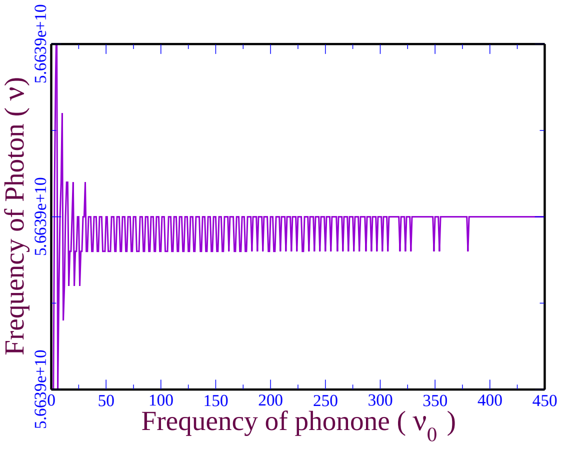

FIG. 1: Calculation of minimum frequency of resonator through numerical variation method

$5.663872 \times 10^{10} Hz$ and $5.663895 \times 10^{10} Hz$. After $\nu_0 = 380 Hz$, ν does not change with ν_0. We get a fixed value of $\nu = 5.663895 \times 10^{10} Hz$. This value of ν is marginally higher than that obtained by considering $\nu_0 = 1 Hz$. If we consider $\nu = \nu_0$ then we get $\nu = 83345207372 Hz$ i.e. $83.34 GHz$ which is significantly higher than the values obtained from other two methods. But, in all three methods we get ν values in the GHz region. Thus, lower limit of thermal emission is in GHz region considering present values of Planck's constant ($h = 6.62607 \times 10^{-34} JS$) and Boltzmann constant ($k = 1.3806 \times 10^{-23} JK^{-1}$).

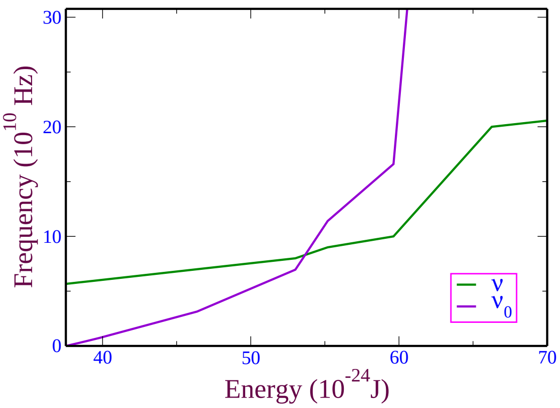

FIG. 2: Variation of ν and ν_0 at different energy region

To find ν_0 for higher energy emission another set of numerical analysis is done using fixed values of ν taken at different energy region like $x-ray$, $\gamma\ ray$, $Ultra\ Violate$, $Visible$, $Infra\ Red$, etc. Calculated results are presented in Table I. Mass of the corresponding resonator is also calculated and presented in the same Table. Previously we have found that for $83.34 GHz$ frequency, $\nu = \nu_0$. Above this value of frequency, ν_0 is always greater than ν and bellow this value ν_0 is less than ν. Thus, $83.34 GHz$ frequency may be considered as **inversion frequency**. Above inversion frequency ν_0 increases more rapidly than ν. The trend is just opposite bellow the inversion frequency. The change of ν and ν_0 with increase of emission energy are presented in Figure 2. Near cut off frequency, ν_0 exponentially drops to $1 Hz$. At very high energy region, ν_0 exponentially increases. This implies that probability of emission of very high energy electromagnetic radiation on thermal heating is very less. These two limits (upper limit and lower limit) explain physically why we always get a peak instead of any exponential increase or decrease in energy density plot against temperature of a black body. The existence of cut off frequency is very important else we should have lower energy emission from every object even at very low temperature.

TABLE I: Frequency and mass of the resonator for higher energy emission

Region of emission	ν (Hz)	ν_0 (Hz)	Mass of the resonator (kg)
$\gamma - ray$	3×10^{21}	4.32×10^{32}	3.18×10^{-18}
$X - ray$	3×10^{19}	4.32×10^{28}	3.18×10^{-22}
$X - ray$	3×10^{17}	5.00×10^{24}	3.68×10^{-26}
UV	7.5×10^{14}	2.99×10^{19}	2.20×10^{-31}
$Visible$	4.3×10^{14}	8.99×10^{18}	6.62×10^{-32}
IR	3×10^{12}	5.00×10^{14}	3.68×10^{-36}
IR	3×10^{11}	3.12×10^{12}	2.30×10^{-38}
$Microwave$	2×10^{11}	1.18×10^{12}	8.69×10^{-39}
$Microwave$	1×10^{11}	1.66×10^{11}	1.22×10^{-39}
$Microwave$	9×10^{10}	1.14×10^{11}	8.40×10^{-40}
$Microwave$	8.33×10^{10}	8.33×10^{10}	6.14×10^{-40}
$Microwave$	8.00×10^{10}	6.96×10^{10}	5.13×10^{-40}
$Microwave$	7.00×10^{10}	3.41×10^{11}	2.51×10^{-40}
$Microwave$	6.00×10^{10}	7.19×10^{9}	5.30×10^{-41}
$Microwave$	5.90×10^{10}	4.95×10^{9}	3.65×10^{-41}
$Microwave$	5.80×10^{10}	2.80×10^{9}	2.08×10^{-41}
$Microwave$	5.70×10^{10}	7.28×10^{8}	5.36×10^{-42}
$Microwave$	5.67×10^{10}	1.22×10^{8}	8.99×10^{-43}
$Microwave$	5.665×10^{10}	2.21×10^{7}	1.63×10^{-43}
$Microwave$	5.664×10^{10}	2.12×10^{6}	1.56×10^{-44}
$Microwave$	5.6639×10^{10}	1.2×10^{5}	8.84×10^{-45}
$Microwave$	5.66389×10^{10}	2.0	1.47×10^{-50}
$Microwave$	$5.663872141 \times 10^{10}$	1.0	7.37×10^{-51}

From Table I we get mass of the resonator for emission in the UV-Visible reason is nearly equal to the mass of an electron ($9.11 \times 10^{-31} kg$). It is well known that electromagnetic radiation in this region is related to the electronic transition. This fact supports infavour of this work. Mass of the resonator in the $X - ray$ and $\gamma - ray$ region is near to the mass of proton and neutron $(1.67 \times 10^{-27} kg)$. This also supports the conclusion made from this work as we know that $\gamma - ray$ radiation from an atom is related to neutron and proton.

We have found that if lower limit of ν_0 is $1Hz$ then lower limit of mass of a resonator(phonon) is $7.367 \times 10^{-51} kg$. In fact, according to $E = mc^2$ relation we can conclude that this is the lower limit of mass of an object which can exist as a particle if frequency of electromagnetic radiation is an integer. Though, frequency of electromagnetic radiation is not an integer, still we may fix $\nu_0 = 1Hz$ as cut off frequency as we found that near this value we observe change of properties. Cut off frequency may be found in a different way. Let us now consider a particle which has a mass equal to the lower limit of mass. If we consider it as spherical and having mass density same as the highest mass density of a black hole [6] which is $1.5 \times 10^{20} kgm^{-3}$ we get the radius of that particle as $2.272 \times 10^{-24} m$. This radius is well above the Planck length [7] which is $1.6162 \times 10^{-35} m$. Planck length is an important parameter in string theory [8–10]. The characteristic length scale of strings is assumed to be on the order of the Planck length, the scale at which the effects of quantum gravity are believed to become significant [11]. If we consider present scale of Planck length is true then using same mass density we get mass of the smallest resonator as $2.654 \times 10^{-84} kg$ which is equivalent to $\nu_0 = 3.6 \times 10^{-34} Hz$. Obviously, this value of ν_0 is very very less than our previous assumption *i. e.* $\nu_0 = 1Hz$. On the other hand, if we consider both present scale of Planck length and the lower limit of mass is true, then we get mass density of strings as presented in Table II. From the calculated values we found that for three dimensional string mass density is of the order of $10^{53} kgm^{-3}$ ($4.166 \times 10^{53} kgm^{-3}$ for sphere) which is 10^{33} times higher than the highest mass density of a black hole [6]. This implies that three dimensional string is not possible or mass density in the center of a black hole is 10^{33} times higher than the present value.

TABLE II: Mass density of different strings

Dimension of the string	Shape of the string	Mass density of the string
1-Dimensional	Linear	$4.558 \times 10^{-16} kgm^{-1}$
2-Dimensional	Circular	$8.978 \times 10^{18} kgm^{-2}$
3-Dimensional	Sphere	$4.166 \times 10^{53} kgm^{-3}$

There is another way we can solve the anomaly we have arrived by considering that a particle may have mass less than $7.367 \times 10^{-51} kg$. Then the frequency of its energy equivalent (using $E = mc^2$ relation) would be less than $1Hz$, which may be possible if it violets $E = mc^2$ relation or it violets quantum boundary conditions. Violation of $E = mc^2$ relation implies that matter having this property would may not be converted to energy at any cost or if in some process it is converted to energy, it would create different amount of energy from the expected value as predicted by $E = mc^2$ relation. Violation of quantum boundary condition by such a tiny mass implies that it could not be kept in any bound state and hence it would not interact with any kind of electromagnetic radiation because interaction of a matter with an electromagnetic radiation occurs only when at least one of its states, arises due to any kind of boundary condition, changes. As it is not in any kind of boundary condition it will expand spontaneously and hence its pressure would be negative. These properties are similar to Dark matter [12–15]. Thus, we can consider the lower limit of mass as termed earlier which is $7.367 \times 10^{-51} kg$, is in fact the **Critical mass** of a fundamental particle above which we get normal matter and bellow the dark matter.

From above discussions we can conclude that normal matter and dark matter are related and partitioned by critical mass barrier. Till date, it is not undoubtedly proved that matter and dark matter are related and convertible. The other possibility i.e. matter and dark matter are two different things, is also not proved. Present hypothesis supports in favor of matter-dark matter inter relation. Recent research works prove that Einstein's familiar formula ($E = mc^2$) should be scaled [16–19], though different researchers proposed different values of the scaling factor. Thus, critical mass calculated here should be scaled according to the scaling factor for $E = mc^2$ relation. It is also possible that we need not any more scaling factor as total mass is divided in two different parts according to the present hypothesis.

In space energy theory [20] it is considered that frequency of any electromagnetic radiation would decrease with time even it travels through vacuum. Change of wave length (λ) of an electromagnetic radiation with time (t) is given as -

$$\lambda = \lambda_0 (1 + \theta t^{\frac{1}{2}}) \tag{17}$$

where, θ is a constant. According to equation 17, every electromagnetic radiation after a certain time should reach to a frequency less than $1Hz$ and would behave as dark matter. In this process energy in our universe is spontaneously converted to dark matter which creates space and hence we observe an accelerating universe at present [21–24]. Thus, present hypothesis and space energy theory [20] are complement to each other.

Following space energy theory formation of dark matter from electromagnetic radiation is explained in the previous section. But, how matter to dark matter conversion takes place? This is not clear from present assumptions. We know if mass of an individual resonator be less than the critical mass then it would behave as a dark matter. Still we do not know whether mass of a resonator is a constant quantity or not. If mass of a resonator is a fixed quantity then matter to dark matter conversion would not take place. According to present assumption matter to dark matter conversion would take place if mass of the resonator changes with temperature. A relation between rest mass of the resonator and temperature of the object is presented in equation 7. In this equation vibrational frequency of the resonator is also a variable of temperature. Thus, we can't draw any conclusion unless we have any experimental proof. But, here is only two possibilities: mass of the resonator may increase or decrease with temperature. Increase of mass with increase of temperature means association of resonator takes place due to increase of temperature. This implies that at very very high temperature, for example, at the stage of the first few seconds of our universe as considered in big bang theory, all resonators were associated as one resonator in a very compact form with extremely high mass density. On cooling, from one resonator of high mass, numbers of resonator with low mass generates and the process continues until mass of a resonator cross the critical mass and form dark matter. This supports big bang theory. On the other hand, if with increase of temperature mass of a resonator decreases, we may conclude that matter is created from dark matter i.e. at the initial stage of our universe there was only dark matter; on cooling, dark matters condensed to form matter. But, in that condition total mass of dark matter of the universe would decrease with time. Thus, a measurement of the change of mass of dark matter of a confined space would prove which process is the actual process for matter dark matter conversion. If no change is observed, we can say there is no interchange occurs between matter and dark matter.

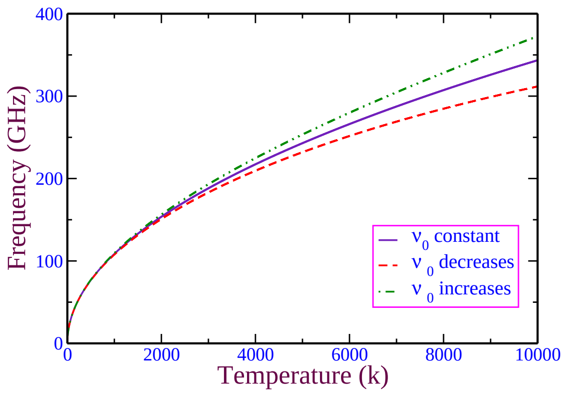

FIG. 3: Variation of ν with Temperature for static and variable ν_0

At present, there is no experimental data to prove the assumptions made here. But, two different experiments may be performed which would justify whether these assumptions are true or false. If we do the experiment to measure the wave length of a monochromatic radiation after a finite time interval and found that its wave length increases due to time travel then we can conclude that both the space energy theory and assumptions made here are true. In other experiment, we can measure the frequency of emitted radiation of a monochromatic source at different temperature and plot ν vs T. If the experimental values of ν at different temperature are less (red line in figure 3) than the corresponding theoretical values considering ν_0 as constant (violet line in figure 3), we can conclude that mass of a resonator decreases with increase of temperature. If experimental values are higher than the corresponding theoretical values (green line in figure 3), then we should say mass of resonator increases with increase of temperature. In either case, i.e. any deviation from the ideal plot, we can conclude that assumptions proposed in this research article is legal. But from experiment only we can conclude whether matter is created from dark matter or the reverse is true.

IV. CONCLUSIONS

In the present work by incorporating de Broglie's equation in Planck's equation of black body radiation we reached equation 16 which opens up few questions and possibilities. Equation 16 may be considered as the universal equation of state which correlate electromagnetic radiation with string theory, dark energy and dark matter, space energy theory and quantum gravity. Using this equation we can calculate mass of a resonator i.e. phonon. From the value of critical mass we can calculate the limit of length of a string using maximum mass density and vice verse. From density constrained we can say both maximum mass density and string length may be justified only if strings are dark matters. Then different types of strings would be the fundamental particles we are searching for.

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