Physical interpretation of de Broglie's wave particle duality relation in a phonon

Arijit Bag[*1]

[1] *Indian Institute of Science Education and Research Kolkata, Mohanpur, Nadia, West Bengal, India, 741246*

Abstract

Using Planck's postulates for black body radiation and wave particle duality relation of de Broglie we can get a relation between mass of a phonon and emission frequency of a body at different temperatures. Using boundary condition lower limit of mass of a phonon is calculated which is $7.36 \times 10^{-51} kg$.

Keywords : ***Planck's postulates; Black body radiation; wave particle duality; phonon***

PACS :44.40.+a, 84.40.Az, 63.20.dk

1 Introduction

German physicist Max Planck initiated to explain the black body radiation [1, 2, 3] with the help of few assumptions which leads to the formation of quantum theory. Planck assume that every radiator has numbers of resonator which vibrate with a finite frequency. Vibrational frequency of a resonator increases with increase of temperature and hence energy maximum shifted to words higher energy region on heating of a black body. According to Planck, emission frequency of a black body is same as the frequency of its resonator. Hence, we can say that a Gaussian type distribution of energy density of a black body with respect to wave length at a fixed temperature implies that vibrational frequency of every resonator is not unique. It follows some distribution law. But, here the question arises why resonators of a homogeneous body vibrate with different frequencies? There are two different possibilities. First possibility is that mass of the resonators are different from one another. The second possibility is that every resonator has same mass but their energy levels are quantized and hence at a fixed temperature number of resonators in different quantized states are different and follow the Boltzmann distribution law.

At present day science resonators are termed as phonon [4, 5, 6]. According to Planck, phonon has finite mass. Now, if we consider condition one is true which means different phonon has different mass, it could be possible to calculate mass of a phonon knowing the value of temperature. From the spectra we should get the value of frequency, ν, which would enable to calculate the mass of a phonon. From the intensity measurement of the spectra one may get the number of phonon present in a system with same mass. On the other hand, if second condition is true, *i.e.* all resonators have same mass, then also we should be able to calculate the mass of a phonon using Boltzmann distribution [7].

In 1925, French physicist Louis de Broglie proposed that every moving particle has a wave nature [8]. Wave length(λ) of a moving particle is related to its momentum (p) as

$$p = \frac{h}{\lambda} \tag{1.1}$$

Thus, wave length (λ) and hence the frequency (ν) of the resonator of a black body should be calculated using de Broglie's relation. Comparing theoretical and experimental frequency we can calculate the mass of a phonon.

*Corresponding author: E-mail: bagarijit@gmail.com

In this article, I have applied wave particle duality relation in Planck's black body equation and get a relation between mass of a resonator and frequency of its corresponding radiation. Using this relation we can get exact frequency of a radiation of a particular resonator at different temperature which is not possible using Planck's equation for black body radiation. Using boundary conditions, lower limit of mass of a phonon is calculated and presented here. Lower limit of emission frequency of a body at room temperature $(300K)$ is also calculated using the lower limit of mass of a phonon. It is also observed that if the wave nature of a resonator due to its vibration is not its trajectory then Planck's equation for black body would not valid. Or, in the other way we can say that de Broglie's wave of a particle is due to the wave nature of its trajectory.

2 Theory

In Planck's theory of electromagnetic radiation [9] frequency of radiation depends on the monochromatic vibrations of the resonators of the body which is absorbing or emitting electromagnetic radiation. Let p is the momentum of a resonator and m is the rest mass of that resonator. Thus, kinetic energy of that resonator is $\frac{p^2}{2m}$. From equipartition principle we know that at any temperature $T^\circ K$ kinetic energy of one mode of vibration is $\frac{1}{2}kT$ where k is Boltzmann constant. Thus, comparing kinetic energy from equipartition principle and from Newton's laws of motion we get

$$\frac{1}{2}kT = \frac{p^2}{2m}$$

or,

$$kT = \frac{p^2}{m} \tag{2.1}$$

As every vibrational degrees of freedom has two modes, at $T^\circ K$ temperature total vibrational kinetic energy (U) of a resonator is kT. From equation 2.1 we get

$$\begin{aligned} U &= kT \\ &= \frac{p^2}{m} \end{aligned} \tag{2.2}$$

Now, from de Broglie's wave particle duality relation we get

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{h\nu}{c} \end{aligned} \tag{2.3}$$

where h is Planck's constant, λ is wave length of that resonator due to vibration, ν is vibrational frequency corresponds to λ and c is the velocity of light in vacuum. Putting the value of p from equation 2.3 in equation 2.1 we get

$$kT = \frac{h^2\nu^2}{mc^2} \tag{2.4}$$

Equation 2.4 would valid if and only when de Broglie's wave length of the resonator is identical with the wave length of vibration in real sense. Else, every resonator would have two different vibrational frequency - one for the thermal excitation and another for its momentum *i.e.* frequency corresponds to its de Broglie wave length. But, in real case we get only one radiation from one resonator. Thus, we can say equation 2.4 is a valid relation and hence, **de Broglie wave nature of a particle is due to the wave nature of its trajectory.** Similar conclusion is made by Bag in a very recent work [10] where wave nature of a moving particle is derived from Newtonian mechanics similar to de Broglie's equation except the proportionality constant.

In equation 2.4, m is the rest mass of the resonator. Hence, mc^2 is the energy equivalence of the resonator following Einstein's $E = mc^2$ equation [11]. Again, we know, $E = h\nu$. Thus, we get

$$\begin{aligned} mc^2 &= E \\ &= h\nu_0 \end{aligned}$$

or,

$$\frac{mc^2}{h} = \nu_0 \tag{2.5}$$

In equation 2.5, ν_0 is different from ν. ν is the vibrational frequency of the resonator while, ν_0 appears for mass energy conversion of the resonator. Replacing $\frac{mc^2}{h}$ by ν_0 in equation 2.4, we get

$$kT = \frac{h\nu^2}{\nu_0} \tag{2.6}$$

If we consider that for an emitter mass of all resonators are same *i.e.* ν_0 is constant then we get a direct relation between temperature (T) and frequency (ν) of emission as

$$\nu = \sqrt{T} \tag{2.7}$$

In Planck's equation ν and T are related through energy density and no direct relation. Thus, equation 2.7 has some advantage; we can calculate frequency of emission at any temperature of a body exactly if we know the frequency of emission at any one temperature or mass of the resonator. In other words, if we know ν for any temperature then ν_0 and hence mass of the resonator would be calculated. In this context we can say that where we get a distribution plot rather a monochromatic radiation, resonators of different masses are present there.

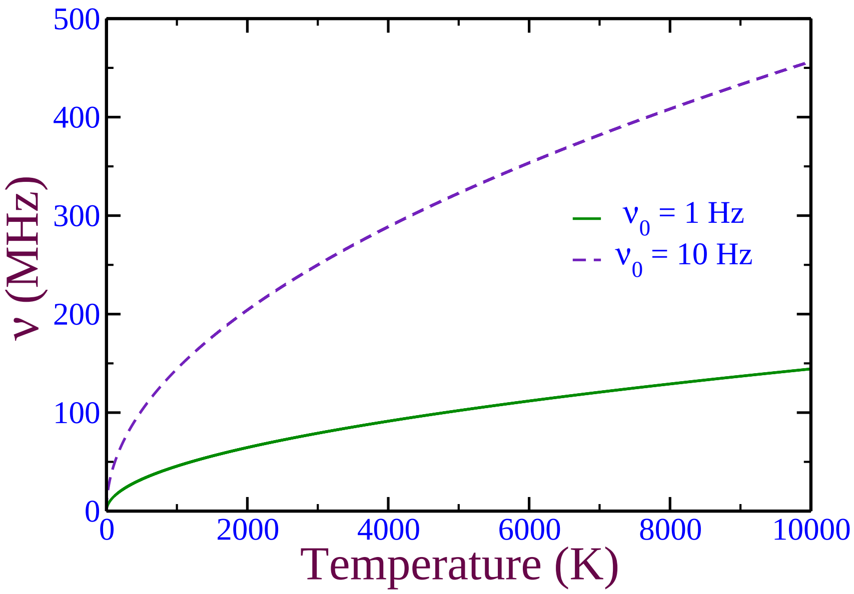

Figure 1: Variation of emission frequency (ν) with heating temperature (T)

3 Results and discussions

3.1 Calculation of lowest mass of phonon

If we put $T = \frac{h}{k}$ in equation 2.6 we get

$$\nu_0 = \nu^2 \tag{3.1}$$

Now, putting the value of $h = 6.6207 \times 10^{-34} JS$ and $k = 1.3806 \times 10^{-23} JK^{-1}$ we get $T = 4.8 \times 10^{-11} K$. This temperature is very near to $0K$. Hence, at this temperature all resonators would be in ground state. Frequency at this temperature would be the lowest energy radiation. Now, let us consider the lowest possible frequency of an electromagnetic radiation is xHz. Hence, for the smallest resonator *i.e.* resonator with lowest mass, ν_0 would be xHz. Again, lowest value of ν is also xHz. So, for smallest resonator at its lowest energy state we can get a solution from equation 3.1 as

$$\begin{gathered} x = x^2 \\ or, \\ x = 1 \end{gathered} \tag{3.2}$$

From equation 3.2 we get lowest value of ν_0 is $1Hz$. Putting the value of $\nu_0 = 1$ in Equation 2.5 we get the mass of the smallest phonon as $7.36 \times 10^{-51} kg$.

3.2 Temperature dependence of emission frequency

In theory section it is discussed that in the present approach it is possible to calculate the emitted frequency of a monochromatic hot body from its temperature if we know the mass of the resonator. In previous subsection it is proved that lowest possible mass of a resonator is $7.36 \times 10^{-51} kg$ and lowest value of ν and ν_0 is $1Hz$. At very low temperature when $T \rightarrow 0K$ for the lightest resonator $\nu = 1Hz$. From equation 2.7 we know that with increase of temperature ν will increase if ν_0 remains constant. In figure 1 change of ν is plotted against temperature (T) for $\nu_0 = 1Hz$ and $\nu_0 = 10Hz$. It is observed that for higher value of ν_0, rate of change of ν with temperature would be high which is obvious from equation 2.6 subject to the condition that ν_0 remains constant with change of temperature. If mass of the resonator changes with temperature we can't get such simple variation of ν with temperature.

3.3 Lowest value of emission frequency at room temperature

From equation 2.4 we get the expression for mass of a resonator as

$$m = \frac{h^2\nu^2}{kTc^2} \tag{3.3}$$

Using the standard values of h, k, and c and lowest value of mass of phonon *i.e.* $m = 7.36 \times 10^{-51} kg$, we get the lowest value of emission frequency(ν) at $300K$ temperature as $\nu = 2.5 \times 10^6 Hz$ or $2.5MHz$.

4 Conclusions

In this article a relationship is derived from Planck's equation for black body radiation and de Broglie's wave of a particle to calculate the mass of a resonator (phonon). It is also shown that de Broglie's weave of a particle with finite mass implies the wave nature of its trajectory. Thus, a tiny particle shows wave nature due to its wave like motion not for its mass to energy conversion. Lower limit of mass of a phonon and lowest value of emission frequency of a phonon at $300K$ is calculated and reported here. If mass of a phonon changes with change of temperature we should be able to predict its mass at different temperature using this approach.

5 Acknowledgments

Author thanks IISER Kolkata for financial support and Dr. Pradip Ghorai, author's post doctorate research guide to give an opportunity to publish this work independently.

References

[1] M Planck *Sitzungsberichte Ber. Akad.* 440 (1899)

[2] M Planck *Ann. d. Phys.* **1** 99 (1900)

[3] M Planck *Berichte der Deutschen Phys. Ges.* **2** 19 (1900)

[4] M Pawel and L Jacak *Physical Review B* **69(19)** 193302 (2004) doi:https://doi.org/10.1103/PhysRevB.69.193302

[5] A Lanzara, P V Bogdanov, X J Zhou, S A Kellar, D L Feng, E D Lu, T Yoshida, H Eisaki, A Fujimori, K Kishio and J I Shimoyama *Nature* **412** 510 (2001) doi:10.1038/35087518

[6] P K Schelling, S. R. Phillpot and P Keblinski *Appl. Phys. Lett.* **80(14)** 2484 (2002) doi: http://dx.doi.org/10.1063/1.1465106

[7] T Constantino *J. Statistical Phys.* **52(1)** 479 (1988) doi: 10.1007/BF01016429

[8] L de Broglie *Ann. d. Phys.* **3** 22 (1925)

[9] M Planck *Annalen der Physik* **4** 553 (1901)

[10] A Bag *Int. J. Res. Sc. Technol.* **5**(IV) 68 (2015) doi: 10.21275/v4i11.NOV151702

[11] A Einstein *Sitzungsber Preuss Akademie Wiss* 142 (1917)