# Axiom that explains Duality and non-local Action at a Distance Sarma N. Gullapalli

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#### Abstract

A quantum physical Axiom is presented which (a) explains duality, (b) eliminates "which way" issue (c) explains Wheeler's duality thought experiment [1] and experimental implementations of it such as reported in [2], (d) narrows scope of "retro-causality" to entanglement [3, 4] (e) explains non-locality in E.P.R. quantum entanglement [5] and (d) emphasizes Albert Einstein's unanswered question of incompleteness of quantum mechanics.

#### I. INTRODUCTION

The wave-particle duality question: "is the photon (electron) a particle or a wave?" has been discussed for almost a century from the earliest days of quantum mechanics to the present. The generally accepted view has been that it *physically* behaves *either* like a particle *or* like a wave depending on the context of its interaction. The implied *physical change* from particle nature to wave nature or vice versa has motivated many investigations. Young's double slit experiment (Figure 1) has been a valuable test bed. Because it has been established by experiments that the interference pattern persists even in single photon situation, in the particle-picture the "which way" question (Figure 2) arises. Wheeler [1] proposed a *delayed choice* thought experiment (Figure 3) which is a Mach–Zehnder interferometer wherein the exit beam splitter is switched in or out at the last moment, forcing interference (wave like behavior) or no interference (particle like behavior) condition on the photon *after* the photon has already "committed" itself to the opposite nature. The Wheeler experiment has been realized in practice by several researchers with some modifications. Roch et al [2] (Figure 4) claim closest realization of Wheeler experiment. They report that the photon *changes its behavior retrospectively*. Using proposed axiom we shall explain the experimental results without any such retrospective behavior on the part of the photon.

Delayed choice has also been investigated using two entangled photon pairs [3, 4] in which the interference or no-interference choice is made for one pair (say "idler" photons) *later* than for the other pair (say "signal" photons) and is shown to influence the behavior of signal photons *retrospectively*. The proposed axiom eliminates the "which way" observation as the reason, but does not quite explain retro-causality. By thus narrowing the scope of retro-causality to entanglement, this axiom suggests focusing attention on entanglement for future research.

In his 1935 paper [5] Albert Einstein expressed skepticism about completeness of quantum mechanical theory by describing a thought experiment which implied instantaneous action at a distance, a non-local causality that violates the luminal speed limit set by theory of relativity ("E.P.R. Paradox"). Erwin Schrodinger immediately responded in 1935 [6] coining the term "entanglement" to describe the phenomenon and pointed out that it is an inevitable consequence of quantum mechanics. To resolve this issue, the notion of a *hidden variable* at the source that predetermines the outcome was proposed, but it was felt that by definition a hidden variable could not be verified. But the 1964 landmark quantitative inequality test developed by J.S. Bell [7] enabled experimental verification of hidden variable. Improved upon in 1969 by J.F. Clauser, M.A. Horne, A. Shimony and R.A. Holt [8], considerable experimental research has been done on this topic, steadily refined to remove loopholes, as for example reported by NIST in 2015 [9]. As a result, it has now been firmly established that there are no hidden variables, and thus non-local "action at a distance" has been confirmed. We shall explain this non-local action at a distance using our axiom, and also present a larger more complete perspective that includes propagation of wave function from the source.

# II. AXIOM

# Wave function is not a physical entity, it is a purely mathematical probability construct whose probability basis must necessarily include all possible paths from the time it is generated until it is terminated

This axiom codifies what we already generally understand when we use wave functions, but with the important difference that this axiom states that *the wave function is non-physical* and so the question of both particle and its wave nature being physical does not arise. In contrast, current understanding of duality is that a particle can *physically* behave *either* as a particle *or* as a wave depending on interaction such as *observation* of *which way* it goes in a multipath setup. Wave function terminates when the photon or particle (or entangled particle pair or entangled multi-particle group) transfers all its energy to another form in irreversible process.

The non-physical nature of wave function is evident from the following reasoning. A wave function  $\psi$  is a complex probability amplitude function such that  $|\psi|^2$  is a probability density function. A probability density function by itself does not represent anything physical. For example, a Gaussian probability density function does not represent any bell shaped physical entity. There is no one-to-one mapping between points on a mathematical probability function and points on anything physical. In fact, if we were to venture to think of any mapping (which in itself is pointless), each point on a probability density function involves an ensemble of physical entities, often an infinity of them, that too only in the limit as a dimensionless ratio between zero and one. On the other hand, in the classical picture a mathematical wavelet represents a physical entity such as the pressure of an acoustic wave or a field component of an electromagnetic wave, and there is one-to-one mapping between the mathematical wavelet and the physical wavelet. Additionally, by definition of probability the wave function must *necessarily* propagate (explore) along all possible paths (all possible alternatives) from the time it is originated until it is terminated when outcome is decided such as by detection. By noting that the wave function is *non-physical*, this axiom avoids the troubling aspect of current perspective of duality which is that the *physical particle* can also be a *physical wave*.

However, the wave function obeys Schrodinger's wave equation which connects it to physical quantities, and through Schrodinger's equation the purely mathematical wave function interacts with physical environment! We know "how" Schrodinger's equation explains physical reality, but not "why". This unanswered question is essentially Albert Einstein's question regarding completeness of quantum mechanics, not just non-local action at a distance or duality.

### If we accept this axiom it readily follows that

(a) In any system with multiple paths (interferometers such as double slit, Mach-Zehnder, Michelson, etc, or just a point source radiating in space), from the time a physical photon and its non-physical wave function are generated by a source, the non-physical wave function evolves along all probable paths available until the wave function is terminated such as by detection of the photon, and at any terminal point in space the superposition of wave function along all paths defines the probability density for the photon for that point. Moreover, because all probable paths to terminal point are always covered by the wave function, there always exists a single path for the single photon to reach the terminal point, and so the single photon can follow only one path to the terminal point as dictated by the probability determined by the wave function. Note that the evolution of the wave function may change dynamically depending on any dynamic path changes in the system. Thus, if we accept this axiom, it would be wrong to expect a single photon to follow multiple paths simultaneously. This becomes clear in the examples given below.

(b) The purely mathematical non-physical wave function obeys Schrodinger's wave equation which connects it to physical quantities and thereby determines its velocity of propagation which does not exceed speed of light in free space, and determines its interactions with physical components. But because it is non-physical it can change *instantly* from one state (which may also be entangled) to another, both obeying Schrodinger equation, *without violating any laws of physics, including theory of relativity.* This explains non-local action at a distance.

## III AXIOM APPLIED TO SINGLE PHOTON YOUNG'S DOUBLE SLIT EXPERIMENT

Figure 1 shows a basic schematic of Young's double slit experiment with single photons. Fringes are observed only when the coherence length (=  $c \cdot T_c$  where  $T_c$  is coherence time of the source and c is velocity of light for the medium of the paths) is longer than the optical path difference between the two paths, and the angle between the two paths at a detector in the array is sufficiently small to ensure well aligned superposition. In quantum mechanical picture coherence and alignment is that of the wave function associated with the photon (particle).

But a single photon generates just one point on the interference pattern. Successive single photons overlay successive points on successive interference patterns. For this overlay not to be smeared, the wave functions of successive single photons must have *mutual coherence* (with time delay adjusted), for which the coherence time of source must be longer than the frame time over which interference is recorded. This is true for any interferometer, including the ones used in Wheeler experiments. This condition is usually readily met with laser sources.

Figure 2 shows a conceptual set up to investigate the "which way" question that arises in the particle-picture. Detectors  $D_A$ ,  $D_B$  and EMCCD (Electron Multiplying CCD) array are all single photon detectors and the source pulse is attenuated such that statistically only one photon per pulse reaches the detectors. BS<sub>A</sub> and BS<sub>B</sub> are beam splitters. Using similar set ups it has been experimentally confirmed that typically either  $D_A$  or  $D_B$  or *one* of EMCCD detectors

goes off per pulse. EMCCD data collected over a number of pulses (for pulses when neither  $D_A$  nor  $D_B$  goes off) shows interference pattern. The "which way" question is: When interference fringes form (by superposition of both paths) which path did the single photon take?







This "which way" question does not arise if we accept our axiom. The non-physical wave function goes through both slits, the physical photon goes through only one slit, its path *always* leading to the detector that goes off. The interference pattern is probability density due to superposition of non-physical wave function from both paths, not of physical photon paths. At beam splitter BS<sub>A</sub> the wave function through slit A splits into two, one going towards detector  $D_A$  and the other going toward EMCCD array. At beam splitter BS<sub>B</sub> the wave function through slit B splits into two, one going towards detector  $D_B$  and the other going toward EMCCD array. The superposed amplitudes of wave function components at EMCCD array result in interference pattern due to coherence and spatial alignment. Thus there are probability amplitudes at  $D_A$ ,  $D_B$  and at various detectors of EMCCD array. *Note that by above axiom, until detection the wave function follows all possible paths, all probabilistic alternatives*. That is, when physical photon is on one path, its wave function is on both paths until detection terminates it. Thus the single photon ends up at *either*  $D_A$  or  $D_B$  or one EMCCD detector with corresponding respective probabilities, *following a single path that exists for it, through either slit A or slit B, not both*. After slit A, the single photon goes either to  $D_A$  or EMCCD, not both; after slit B the single photon goes to either  $D_B$  or EMCCD, not both. Because the single photon goes through

only one path (either to  $D_A$  or to  $D_B$  or to one EMCCD detector), and probability at EMCCD is according to interference fringes due to superposition of wave function amplitudes, "which way" question simply does not arise.

Moreover, when the photon goes to  $D_A$  or  $D_B$  and not to EMCCD, there is no contribution to the interference pattern and so there is no need to conjecture that mysteriously "observation" by  $D_A$  or  $D_B$  "destroys" the interference pattern. The truth is, *different photons* go to  $D_A$ ,  $D_B$  and EMCCD respectively according to probabilities determined by the non-physical wave function, the three possibilities are mutually exclusive, a single photon never goes to all three, not even to two, and so there is no "destruction" of interference pattern by "observing"  $D_A$  or  $D_B$ .

# IV AXIOM APPLIED TO WHEELER'S DELAYED CHOICE THOUGHT EXPERIMENT

An interesting question arises as to what happens if the setup is *changed dynamically after the photon is already on one path but has not reached the detector array*. Wheeler ([1] p 183) proposed a *delayed choice* thought experiment (Figure 3) using Mach-Zehnder interferometer which is changed dynamically when the photon is *in mid-flight inside the interferometer*.

Referring to Figure 3 (assuming ideal components), conventional view has been that when a single photon from source S enters the interferometer, (a) when BS<sub>2</sub> is in place, for the observed constructive interference in D<sub>1</sub> (which therefore registers count) and destructive interference in D<sub>2</sub> (which therefore registers no count) *the photon has to travel both paths* (Path 1 and Path 2) *in a wave-like behavior*, (b) when BS<sub>2</sub> is not in place there is no interference and so either D<sub>1</sub> or D<sub>2</sub> is observed to register a count (not both) with 50% probability each, *in a particle-like behavior*. Now the question is: what happens if BS<sub>2</sub> is present (so there must be wave-like behavior for interference) but is removed *after the photon has passed BS<sub>1</sub> committed to both paths*, which path will be dropped to avoid interference? Similarly what happens if BS<sub>2</sub> is absent (so there must be particle-like behavior with no interference) but is inserted *after the photon has passed BS<sub>1</sub> and therefore committed to one of two paths*, will there be interference?



Figure 3 Wheeler's delayed choice thought experiment

We can readily answer these questions based on our axiom. *The key is that it is the non-physical wave function that travels all possible paths, not the physical photon.* The wave function propagates along all possible paths in the set up (which could change dynamically) until it is terminated such as by irreversible measurement by a detector. Probability density function which is defined by the wave function is according to all possible (dynamic) paths *until detection.* Because (due to our axiom as pointed out earlier) wave function is not a physical entity and is a purely mathematical probability amplitude, *it can change instantly.* The physical photon ends up where the probability density function says it should be, *regardless of which path it takes.* When BS<sub>2</sub> is in place when photon arrives, even if it is inserted in the last moment, photon ends up at D<sub>1</sub> with probability 1 as dictated by wave function, and path to D<sub>1</sub> is available to photon regardless of whether the photon took Path 1 or Path 2. When BS<sub>2</sub> is not in place when the photon arrives, even if it is removed in the last minute, the photon ends up at either D<sub>1</sub> or D<sub>2</sub> with 50% probability each depending on which path it was on, again dictated by the wave function. Thus we see that our axiom fully explains the expected results.

#### V AXIOM APPLIED TO EXPERIMENTAL IMPLEMENTATION BY ROCH ET AL [2]

Among several experimental realizations, Roch et al [2] claim to be closest to Wheeler's thought experiment. Referring to simplified schematic in Figure 4 (see [2] for details) source S is a single N-V (Nitrogen-

Vacancy) color center in a diamond nanocrystal, which when excited by a laser pulse emits a single linearly polarized photon within 45 ns of the narrow 800 ps excitation pulse, enabling precision timing of the photon emission. The photon goes through a polarizing beam splitter PBS in BS<sub>1</sub>, whose H and V orthogonal polarization outputs (single photon goes to either H or V channel) are separated into two 48 meter long paths, Path1 for H and Path2 for V. After 48 meters these paths enter BS<sub>2</sub> consisting of a half wave plate followed by polarization beam splitter PBS which combines the V and H paths, followed by an electro-optic-modulator (EOM) followed by a Wollaston Prism WP which separates H and V polarizations which then terminate in single photon detectors D<sub>1</sub> and D<sub>2</sub> respectively. The transit time of 160 ns to traverse 48m allows practical implementation of dynamic change while photon is in midflight.



Figure 4 Simplified schematic (see [2] for details) of Roch et al implementation of Wheeler's delayed choice thought experiment

When EOM is off, H and V polarizations are unaffected and because the two polarizations are orthogonal there is no interference, and either  $D_1$  or  $D_2$  detects the photon depending on whether photon was on H or V polarization path. When EOM is on it introduces a rotation of polarizations by 22.5 degrees which results in alignment of projections of H and V polarizations in D<sub>2</sub> channel (causing constructive interference) and anti-alignment in D<sub>1</sub> channel (causing destructive interference), and so only  $D_2$  detects the photon. Thus the EOM off condition corresponds to removing  $BS_2$  in Wheeler's thought experiment (Figure 3) and the EOM on condition corresponds to inserting  $BS_2$ in Wheeler's thought experiment. EOM which is very fast (40 ns to switch on or off) is controlled by a Quantum Random Number Generator (QRNG) which is controlled by the same laser pulse that triggers source S. QRNG is located at  $BS_2$  to ensure there can be no "knowledge" of its random choice at  $BS_1$  when the photon passes through  $BS_1$ . The timings are such that when the random EOM choice is made for the nth photon (by n-1 clock) it is in midflight, about 12 to 25m from BS<sub>1</sub> inside the interferometer so its path is already "committed". The experiment confirms that interference results when EOM is on, and there is no interference when EOM is off. Based on this (surmising conventionally that single photon needs to travel both paths for interference, and either path for no interference) Roch et all conclude by quoting Wheeler "we have a strange inversion of the normal order of time" with the photon changing its behavior in midflight inside the interferometer from one path to both paths when EOM is switched from off to on, and from both paths to single path when EOM is switched from on to off.

We can readily explain the results using our axiom. The non-physical probability amplitude wave function travels along both H and V channels till it terminates upon detection either by  $D_1$  or  $D_2$ . Let the photon be on one channel, say H channel, inside the interferometer (about 12 to 25m from BS<sub>1</sub>) when EOM is switched, say from off to on. When wave function (and photon) reach EOM, the probability amplitude is accordingly 1 for  $D_2$  and 0 for  $D_1$ , and so the photon goes to  $D_2$ . Note that when EOM is on there is path for the single photon to go from the H channel to  $D_1$  or  $D_2$  because of the projection in PBS in BS<sub>2</sub>, but probability for  $D_1$  is zero, and 1 for  $D_2$  and so photon goes to  $D_2$ . If, on the other hand EOM were switched from on to off, when the wave function (and photon) reach EOM, the wave function accordingly sets probability of 0.5 for  $D_1$  and 0.5 for  $D_2$ , and the photon goes to  $D_1$  (if it were on V channel it would go to  $D_2$ ). Thus physical photon does not change its behavior in midflight from one polarization to both polarizations (H to H and V) or vice versa, it simply follows the probability density determined by the non-physical wave function which travels on both channels. Photon follows only one path. Note that wave function can respond instantaneously to EOM, and so choice can be delayed till the wave function (and photon) just reach EOM.

## VI AXIOM APPLIED TO DELAYED CHOICE EXPERIMENT WITH ENTANGLED PHOTON PAIRS

Figure 5 which reproduces Figure 2 of Kim et al [3] shows an interesting delayed choice experiment performed by them involving two entangled photon pairs prepared by sending pump laser pulse (train) through a double slit to be incident at two adjacent points A and B on non-linear optical crystal (BBO) resulting in generation at A of "signal" photon in one direction and its entangled "idler" photon in a different direction, and corresponding entangled pair generated at B, the two signal photons from A and B focused by lens on detector  $D_0$ , and the two "idler" photons from A and B sent along separate paths through beam splitters BSA and BSB respectively. One output of BSA goes to detector D<sub>3</sub> while the other is reflected off mirror M<sub>A</sub> and sent through beam splitter BS with one path to detector  $D_1$  and the other to detector  $D_2$ . Similarly one output of BSB goes to detector  $D_4$  and the other is reflected off mirror  $M_B$  and through BS to  $D_1$  and  $D_2$ , spatially aligned with the respective paths from BSA. The path lengths are such that path to  $D_0$  is much shorter than paths to  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$ , all four of which are made equal. The time delay between detection at  $D_0$  and detections at  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  is 7 ns, sufficiently longer than the 2 ns response time of detectors to ensure that detection at  $D_0$  is in the *past* of detections at  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$ . The outputs of  $D_0$  and each of  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  are correlated in the coincident circuit. Detector  $D_0$  is translated laterally, and for each position a number of counts are recorded. The resulting correlation between  $D_0$  and  $D_1$  shows interference and likewise between  $D_0$  and  $D_2$  with a phase difference whereas correlation between  $D_0$  and  $D_3$  and that between  $D_0$  and  $D_4$  show no interference. This means that when idler photons reach either  $D_1$  or  $D_2$  the interference at  $D_0$  is retrospectively erased!



Figure 5 Kim et al [3] delayed choice experiment with entangled photon pairs

Let us now apply our axiom to this experimental set up. Signal and idler wave functions of entangled pair A follow all possible respective paths till detection. Similarly for entangled pair B. Because total wave function (signal and idler) of A is not separable due to entanglement, and similarly for B, for interference we require coherence and spatial alignment of signal wave functions as well as coherence and spatial alignment of idler wave functions. For the moment let us *assume* (we will come back to this shortly) that the signal-idler composite wave function of either A or B terminates only when both signal and idler photons are detected. Because there is no spatial alignment at D<sub>3</sub> and D<sub>4</sub> or when one idler goes to D<sub>1</sub> and the other idler to D<sub>2</sub>, it follows that unless *both* idler photons go either to D<sub>1</sub> or to D<sub>2</sub> there can be no interference at D<sub>0</sub>. *Note that there is no which way observation involved*. Now let us go back to the assumption we made, according to which detection at D<sub>0</sub> does not terminate the entangled wave function of both signal and idler (of A or B). If we remove this assumption clearly there is retro-causality, with detections at D<sub>1</sub> and D<sub>2</sub> determining what D<sub>0</sub> detected in the past. As there is no rationale for our assumption because our knowledge of detection process at D<sub>0</sub>, our axiom falls short of explaining retro-causality. But still our assumption does not require "which way" "observation" to change photon physically from wave to particle or vice versa.

In this experiment, separation of paths was used to detect "which way". In another experiment [4] Herzog et al used change of coherence was used to detect "which way" while maintaining spatial alignment. Their experiment also reports retro-causality. Application of our axiom to their experiment will also fall short of explaining retro-causality, but does not require "which way" information because it is the non-physical wave function that travels all paths, not the physical photon which follows only one path.

Thus our axiom suggests focusing future retro-causality research on entanglement, not on "which way". There may be more to wave function and detection of entangled particles than meets the eye.

# VII LOCALITY AND CAUSALITY IN EPR EXPERIMENT AND QUANTUM COMMUNICATION

Referring to Figure 6, a pair of polarization-entangled photons a and b generated by source S at time  $t_0$  travel in two different spatial directions, and the state of polarization **a** of a and **b** of b are measured by respective instruments, at A at time  $t_A > t_0$  corresponding to distance  $L_{SA} = c_A \cdot t_A$  where  $c_A$  is velocity of light in channel SA and at B at time  $t_B > t_A$  corresponding to distance  $L_{SB} = c_B \cdot t_B$ . Because there are no hidden variables that could define polarization of a and b before measurement, polarization of a and b remain *undefined* due to the mixed state of entanglement till the first measurement at  $t_A$  at which time b *instantly* becomes polarized parallel to **a**, at point B<sub>1</sub> at distance  $L_{SB1}$  from S,  $L_{SB1} = c_B \cdot t_A < L_{SB}$ . Treating the measurement **a** at A as the cause and **b** (parallel to **a**) as its *instantaneous* effect at B<sub>1</sub>, and noting that the distance from A to B<sub>1</sub> is greater than zero, it is seen that the effect is non-local with respect to A because it reaches B<sub>1</sub> faster than speed of light in free space which is the upper limit set by Einstein's theory of relativity (hence the EPR paradox). However, while this is true, it does not represent the full picture of cause and effect because *the effect will be non-existent if* a *and* b *were not entangled by* S *at time*  $t_0$  *to begin with*.

That is, the full cause-effect relationship is a two-input { $\underline{a}$  AND entanglement} single output { $\underline{b}$ } relationship, and because entanglement occurred at S at time  $t_0$ , and the wave function propagates from S at speed less than or equal to speed of light in free space, locality is not violated with respect to S in the full picture, while it is violated in the limited picture with respect to A. This does not diminish in any way the enormous significance of the instantaneous response at B<sub>1</sub>, but merely reminds us that the propagation of wave function must *also* be kept in the picture of locality and causality. Unlike in non-entangled quantum systems and classical systems, in entangled quantum systems the management of distance from source to measurements are critical for the causal order. In the example in Figure 6 causal order is A to B because **a** determines **b**, that is, A  $\alpha$  B, symbol  $\alpha$  denoting A is in the causal past of B. If for some reason (such as for maintenance rerouting or due to mobile conditions)  $t_A$  becomes longer than  $t_B$ , then **b** set by measurement B determines  $\mathbf{a}$  and the causal order is reversed, B is in causal past of A (B  $\alpha$  A), and in such a case communication from A to B becomes impossible! Thus, timing which directly depends on propagation of wave function from source S is critical to ensuring the desired causal orders in entangled quantum communication systems. In contrast, in non-entangled quantum systems and classical systems timing and distances are not critical and there are no such constraints. The benefit of entangled quantum communication is the promise of security. Because  $t_B > t_A$ , causality is not violated in the entangled case illustrated in Figure 1. Einstein's concern was with regard to locality. But, using our axiom, we see that the non-physical wave function can change its state instantly, explaining the nonlocal behavior in entanglement.



Figure 6: Communication with polarization-entangled photons

#### CONCLUSION

By removing the troublesome "observation" of "which way" from the duality picture, the proposed axiom simplifies analysis of quantum systems. One simply needs to follow all possible paths of the wave function from the time it and the particle are generated till it is terminated regardless of which path the particle is on, because there always exists a single path for the single particle to the termination point. The termination of entangled wave functions needs more research for better understanding of retro-causality.

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