An energy-based derivation of Lorentz transformation in one inertial frame

Abstract

The special theory of relativity is classically interpreted in terms of *space-time symmetry*, with the Lorentz transformation modifying the Galilean transformation as required to translate between 4-dimensional inertial frames as defined by Einstein and developed in relativistic texts.[1..6] We begin here an *energy-time asymmetrical interpretation* based on multiplying the Galilean transformation by an energy factor representing the difference in energy between a system at rest and a system moving with velocity $\vec{v}$, both systems existing in one universal inertial frame. Detailed consequences of this energy-time interpretation of Lorentz-based physics will be treated in following papers.

Introduction

Tests of the century-old *special theory of relativity* can be bifurcated into two categories: *space-time symmetry* and *energy-time asymmetry*. Space-time symmetry is characterized by 'gedanken' experiments, while energy-time asymmetry is present in all twentieth century particle physics experiments. The paradoxes of special relativity trace to space-time symmetry, while relativistic energy particle physics is largely paradox-free.

Not only is space-time symmetry linked to paradox in special relativity; Kauffmann argues that space-time symmetry is, at least indirectly, responsible for the nonphysical values predicted by Dirac's relativistic equation[8]. Also the covariance of the Schrödinger equation under the action of the Galilean group is of recent interest[9] and Galilean covariance in the context of open quantum systems is an area of active research; space-time symmetry in open quantum systems have been fully analyzed only for special Markovian dynamics. This is first of a series focused on the dichotomy between space-time symmetry and energy-time asymmetry in special relativity theory (SRT). We begin with the Galilean transformation and derive the Lorentz transformation via the addition of a quadratic velocity factor in support of energy $\sim mv^2$.

The untested space-time symmetry aspects of SRT are said to be implied by at least two things: first, the necessary Lorentz invariance of Maxwell's equations, and second, the necessity of two 'inertial frames' for derivation of the Lorentz transformation. Recent analysis [7] of Hertz's equations of electrodynamics has demonstrated Galilean invariance of Hertzian equations, while our derivation of the Lorentz transformation (LT) from a single inertial frame challenges the logic of the second argument.

Measuring distant objects in motion

Since Galileo's time, the physics of moving objects has been based on specifying the position x' of an object, initially at position x at time $t = 0$, moving with velocity $\vec{v}$ for time t as

$$x' = x \pm vt \,.$$

This is the Galilean transformation.

When a distant object moves with velocity $\vec{v}$ we can't touch or otherwise directly measure the size of the object. Einstein, addressing this problem, claimed to have spent considerable time imagining himself either *riding on* or *riding near* a beam of light, picturing himself as an observer in the 'photon's world'. In this way Einstein came to see moving objects as "other worlds" in which the laws of physics held; specifically Newton's laws of inertia. He framed his physics as '*inertial reference frames'* instead of as '*other physical worlds'*. Textbooks on special relativity often begin by defining *inertial reference frames*:

> *"An inertial frame is one in which spatial relations, as determined by rigid scales at rest in the frame, are Euclidian and in which there exists a universal time…* [such that Newton's laws of inertia hold.]" Rindler, (p.5)

The texts derive a mathematical transformation between these separate worlds in relative motion, essentially based on a "standard unit of velocity" chosen to be the speed of light, c. In order for there to exist such a standard it must be the same in all worlds, and this is Einstein's first postulate, principle, or axiom:

> *"The speed of light is constant in all inertial frames."*

For this to make sense, Einstein required a second principle:

> *"The laws of physics are the same in all inertial frames."*

We will analyze these principles elsewhere. Many textbooks start with these principles, apply them to two inertial frames, and derive the relevant Lorentz transformation between the two worlds. The following derivation of the Lorentz transformation is based on only *one real world*, in which space and time exist, the speed of light is constant, and moving objects obey Newton's laws of inertia, in terms of a 3D Euclidian space coordinate system and a 1D axis of 'universal' or 'absolute' time, simultaneous throughout all space.

In our framework the speed of light represents the maximum speed for signals to transfer from one position in space to another; information about objects moving with velocity $\vec{v}$ cannot reach us faster than light, and this introduces artifacts that distort the remote measurements mathematically.

The relevant transformations are:

$x' = f(x, v, t)$ Galilean transformation

$x' = g(x, v, v^2, c, t)$ Lorentz transformation

where factor $\gamma(v^2, c)$ connects the transformations: $g(x, v, v^2, c, t) = \gamma(v^2, c)\, f(x, v, t)$.

The Radar method of length measurement

We cannot lay our hands on distant objects in motion, so w*e can't make direct measurements on objects moving relative to each other.* Einstein's *inertial reference frame* is a frame in which Newton's Laws hold, and onto which he projected a 4D coordinate system centered on a moving object. This is essentially a model of the real world in which physics problems have classically been solved, but Einstein added a second inertial frame corresponding to *another* real world. Assume only one inertial frame based on a radar station; the remote object is a rocket moving toward or away from the radar with velocity *v*. Despite that the rocket is moving entirely in the world of the radar, with time *t* and speed of light *c*, relativists often ask about time in the 'real world' of the rocket, which is *assumed to carry its own time with it.* This complication changes the problem into Einstein's multiple world formulation, obscuring the simple 'one real world' energy basis of our derivation of the Lorentz transformation. In particular, a relativist, seeing time t in an equation, can become confused over exactly what t represents. In our derivation t represents the universal time of the inertial frame of the radar station, and parameter t stands for this universal time. For example, $x = ct$ represents the position of the wave front of light starting at $t = 0$ and traveling with speed c for time t. To specify a *specific* distance, say $x' = L$, we will use a specific timestamp or reading of the clock, such that $x' = ct' = L$. That is, *unlabeled* values of t represent the universal time parameter in the radar's inertial frame, while primed values t', t'', or subscripted values represent specific readings of the stopwatch that starts at $t = 0$ and runs until stopped at $t = t'$. So an equation with unlabeled t is a dynamic time relation, while primed values of t represent time measurement values.

We formulate the problem in one inertial frame, corresponding to the real world in which both our radar and the moving object we wish to measure are located. This has the advantage of utilizing only one clock, whereas Einstein's method requires at least two separate clocks. The radar emits electromagnetic pulses at a known time and place, and receives reflected pulses from objects moving with respect to our radar transmitter/receiver. The clock is used to time the duration of the signal from transmission to reception; a pulse is sent at time t_1, an echo arrives back at t_2; the instant of reflection is found by $\tau = (t_1 + t_2)/2$; the radial distance by $r = c(t_2 - t_1)/2$.

The radar method works as follows: First, assume that we are calibrating the radar clock by putting a rocket of known length $x_1 - x_0 = L$ on a test stand, with the nose of the rocket a known distance x_0 from the radar:

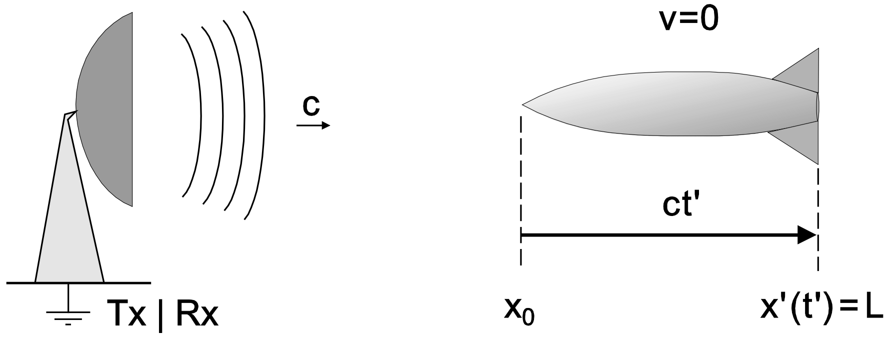

Fig 1. The emitted radar pulse strikes the nose of the rocket and reflects, yielding position x_0 at $t = 0$ after analysis. The pulse moves at speed c toward the tail fin at x_1 thus $x' = ct' = L$.

Since the nose-to-tail-fin length of the rocket is $x_1 - x_0 = L$ we measure from the nose ($x_0,\ t = 0$) as the rocket is stationary with respect to the radar, $v = 0$. As L is a measure of the units scale, and physics does not depend on our chosen scale, we could choose $L = 1$, the normalized rocket unit length so $|x_1 - x_0| = 1$. The *rest measurement*, $t' = L/c$ allows calibration of our radar clock based on *c*.

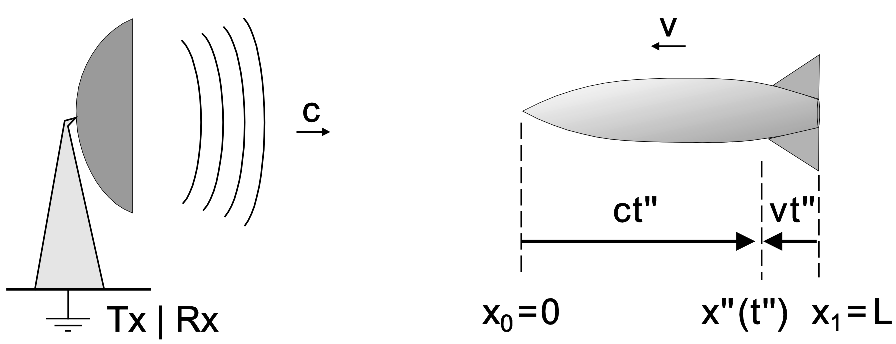

Fig 2. The emitted radar pulse strikes the nose of the rocket and reflects, yielding position x_0 at $t = 0$ after analysis. The rocket moves forward with velocity $\vec{v}$ while the pulse moves toward the tail fin at x_1. The radar pulse meets the tail at position x'' at time t'', such that $x'' < L$.

Having calibrated the radar's clock, measuring universal time in the radar's inertial frame, we now treat the rocket in motion with velocity v with respect to the radar (Fig 2). As the nose-to-tail-fin length of the rocket is $x_1 - x_0 = L$ we measure from the nose ($x_0,\ t = 0$) as the rocket moves toward the radar with velocity v. At time t'' radar sees the tail, which has moved forward distance vt'' while the pulse has traveled ct'', hence, as is clear from Fig 2,

$$(c + v)\, t'' = L \, .$$

If the rocket is stationary with respect to the radar, the pulse will travel length $x_1 - x_0 = L$ in time t', so $ct' = L$. If the rocket is moving toward the radar with velocity $+\vec{v}$, then the radar wave front will strike the nose at x_0, but while the wave front moves toward the tail fin at $x_1 (= L)$ the tail fin is moving toward the radar with velocity $+\vec{v}$. Thus the wave front will encounter the tail fin at $x'' = L - vt''$ at time $t'' < t'$. If the radar pulse has traveled for time t'' with speed c, then the total distance involved is $ct'' + vt'' = |x_1 - x_0| = L$, which is also the distance $ct' = L$ that would be measured when $v = 0$. Hence equation (1):

$$ct' = (c+v)t'' = L \quad \Rightarrow \quad t'' = \frac{L}{c+v} \qquad \text{rocket moving toward us} \qquad (1)$$

If the rocket is moving away from the radar the wave front will strike the rocket then continue toward the far end of the rocket. But the far end will be moving away with velocity $-\vec{v}$, thus, in order to reach the far end of the rocket, the radar pulse must travel the entire length of the rocket $x_1 - x_0 = L$ plus the extra distance moved: vt'''. Hence equation (2):

$$ct' = (c-v)t''' = L \quad \Rightarrow \quad t''' = \frac{L}{c-v} \qquad \text{rocket moving away from us} \qquad (2)$$

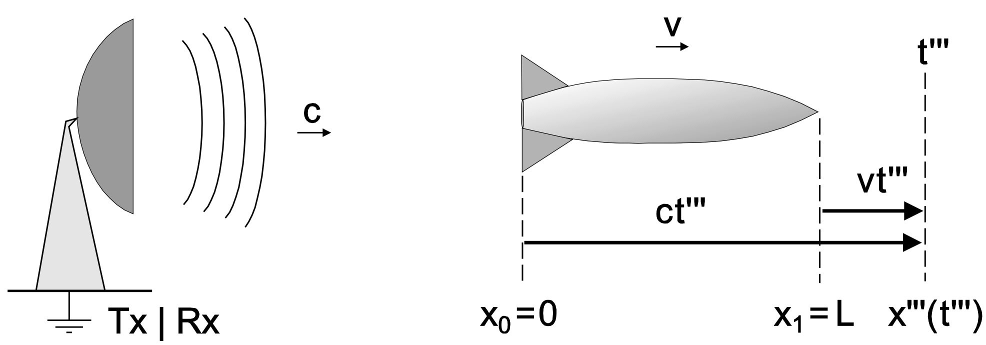

Fig 3. Radar pulses hit the tail of the rocket and reflect. While the wavefront continues to travel toward the nose, the nose moves away with velocity $-\vec{v}$ (relative to Fig 2.)

Equations (1) and (2) simply equate the lengths traveled when the pulse reaches the far end of the rocket to the length of the rocket when it is not moving $ct' = L$ and the times t', t'' and t''' represent the times the radar pulse reaches the far end (relative to $t = 0$, the time the radar pulse strikes the near end of the rocket) when the rocket is stationary or is traveling toward and away from the radar respectively. These relations are easily seen from figures 1, 2 and 3.

The assumptions underlying the derivation of equations (1) and (2) are based on Euclidian space and time in the 'inertial frame' of the radar. *There is no other inertial frame involved* (in the Einsteinian sense). The rocket is simply an object that obeys Newton's laws of inertia in the radar's inertial frame. Rocket speed and the speed of light are given relative to the radar system. The goal is to analyze the system in terms of measurements of the moving object when the speed of light is considered finite. From equations (1) and (2) we find:

$$t' = \frac{c+v}{c} t'' = \left(1+\frac{v}{c}\right) t'' \quad \text{and} \quad t' = \frac{c-v}{c} t''' = \left(1-\frac{v}{c}\right) t''' \qquad (3,4)$$

We see from figure 2 that the measured length of the rocket, x'', approaching the radar is apparently shorter than the actual rocket length and is a function of rocket speed v and speed of light c. We desire a function $\alpha(v,c)$ to represent this 'contraction' so we postulate that

$$x'' = \alpha(v,c)\, L \,. \qquad \text{rocket moving toward radar} \qquad (5)$$

From figure 3 we see that the apparent length x''' *increases* when the rocket is moving away from the radar, also a function of v and c. The Doppler-like radar effect 'stretches the length' when the rocket moves away with opposite velocity. We desire that the same function α be used when the rocket recedes from the radar, that is, when the velocity changes sign, and we write this as:

$$x''' = \alpha(-v,c)\, L \,. \qquad \text{rocket moving away} \qquad (6)$$

We attempt to solve for $\alpha(v,c)$. The three lengths of interest in this problem are:

$x' = L = ct'$	length of stationary rocket	$v \equiv 0, \quad x' \equiv L$
$x'' = L - vt''$	apparent length of approaching rocket	$x'' < L$
$x''' = L + vt'''$	apparent length of receding rocket	$x''' > L$

From the above equations and figures we replace t'' and t''' with expressions from equations (1) and (2):

$$x'' = L - vt'' = L - \frac{vL}{c+v} = \frac{L}{1+v/c} = \alpha(+v)L \qquad (7)$$

$$x''' = L + v\,t''' = L + \frac{vL}{c-v} = \frac{L}{1-v/c} = \alpha(-v)\,L \qquad (8)$$

From this we have obtained:

$$\alpha(+v) = \frac{1}{1+v/c} \qquad (9)$$

$$\alpha(-v) = \frac{1}{1-v/c} \qquad (10)$$

These functions describe apparent length contraction or expansion for a rocket approaching and receding from the radar. This does *not* imply actual physical contraction, only Galilean-based addition of rocket and light speeds. Lorentz length contraction has never been experimentally tested [11], while relativistic energy and momentum and relativistic "time dilation" have been demonstrated countless times in particle physics. Note that, when we replace L with x in equations (7) and (8), we have the Galilean transformation: $x = x'' + vt''$.

The Galilean transformation $x' = x - vt$ works for any $0 \le v \le c$ in one real world. Einstein *imagined a second real world moving with an object located in our real world* and created a model of the second real world by adding another *universal time* to a moving system in which Newton's laws of inertia are assumed to hold. He postulated that the speed of light is constant in the second world, independent of its velocity with respect to our real world. Since each world spans time and space, any point in the second world's coordinate space, (x', y', z', t') exists with coordinate (x, y, z, t) in our real world, and the coordinates can be transformed into each other via a Lorentz transformation, requiring only that we give up the idea of simultaneity, i.e., accept the imposition or projection of a second time dimension into our universe.

Modify Galilean transformation to incorporate energy dependence

The *relativity* of radar-based measurements shows apparent length contraction associated with the derivation of the Galilean transformation of spatial coordinates based on velocity $\vec{v}$ and universal time *t*. *Space-time relativity* is based on $\vec{v}$ and is independent of energy $\sim mv^2$. Key to space-time symmetry is Einstein's decision to transform the moving object into a 'rest frame' in the second inertial system. From the energy-time perspective, this $v = 0$ aspect defines the 'ground state' of the kinetic energy associated with this inertial reference frame: $mv^2 = 0$. The moving object seen by the observer in the rest frame has velocity v, so the energy is $\sim mv^2$. The difference in energy of the two systems is proportional to $\Delta(mv^2) = m\Delta(v^2) = m(v^2 - 0^2) = mv^2$. This relation is independent of the mass, so we summarize the relation as

$$\Delta v^2 = (v^2 - 0^2) = v^2 .$$

In terms of relative energies of particles in inertial frames, Einstein essentially set up the simplest possible formulation in terms of the differentials from the *ground state* or *rest frame* energy such that, since rest frame velocity is zero, the above relation always holds.

Consider the idea: modulo mc^2. *Modulo* mc^2 means that if a particle's energy exceeds mc^2 a new particle can be created, significantly changing the physics that we started with. So we limit our immediate concern to particle energies $\sim mv^2$ for $0 \le v < c$, hence $mv^2 / mc^2 < 1$.

The need for the Lorentz transformation arose in 20th century particle physics. *Measurement of length* using light speed signals yields apparent length contraction, but *remains Galilean in nature*, having no energy dependence. We wish to derive the Lorentz transformation required for particle physics while limiting our universe to *one inertial frame*, that is, one Euclidean 3-space [which can be rotated] and one universal time dimension [which cannot]. Adding an energy dependent factor $\beta(mv^2)$ modulo mc^2 to the Galilean translation $f(x,v,t)$ yields

$$g(x,v,v^2,c,t)=\gamma(mv^2/mc^2)\,f(x,v,t).$$

> *We view the Lorentz modification to the Galilean transformation not as a function of velocity, but of energy* $\sim v^2$ *and we imply that a Lorentz transformation describes an energy-phenomenon rather than a strictly space-translational phenomenon.*

We argue that particle physics of the twentieth century provides convincing proofs that Lorentz is needed in our real world: Galilean covariance (translation and boost) forces non-unitary dynamics to produce *an infinite growth of the system's energy on long timescales*, hence Galilean covariant maps yield a good approximation that can be used in experiments that run for sufficiently short times[9]. To derive the Lorentz transformation from our radar model we include a quadratic power of velocity to support mv^2-dependent energy, one that reduces to the identity function when $c\to\infty$.

$$\xi(v,c)=\beta(v^2,c)\alpha(+v)\,,\qquad \beta(v^2,\infty)=1 \tag{11}$$

We apply Rindler's[1] statement that *"The inverse of a Lorentz transformation is another Lorentz transformation, with parameter* $-\vec{v}$ *instead of* $\vec{v}$*."* We use this to define a transformation (11) and its inverse:

$$\xi^{-1}(v,c)=\beta(v^2,c)\alpha(-v)\,. \tag{12}$$

Although it is not always the case that an inverse is identical to a reciprocal, we intuit that this is the case here, since we aim for a known transformation. So dividing equation (11) by the inverse transformation (12) we obtain

$$\frac{\xi(v,c)}{\xi^{-1}(v,c)}=\frac{\beta(v^2,c)\alpha(+v)}{\beta(v^2,c)\alpha(-v)}=\frac{1-v/c}{1+v/c} \tag{13}$$

hence:

$$\xi^2=\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\;\equiv\;\frac{\left(1-\frac{v}{c}\right)^2}{\left(1-\frac{v^2}{c^2}\right)}\;\Rightarrow\;\xi(v,c)=\frac{1-v/c}{\sqrt{1-v^2/c^2}},\quad \xi^{-1}(v,c)=\frac{1+v/c}{\sqrt{1-v^2/c^2}} \tag{14}$$

Inserting this into the formula for translating an arbitrary length x to x':

$$x' = \xi x = \frac{x - \frac{vx}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{15}$$

We define $\gamma(v^2, c) = (1 - v^2/c^2)^{-1/2}$ and since parameter $x = ct$ then $t = x/c$ yields:

$x' = \gamma(x - vt)$ The Lorentz transformation: $x' = \gamma(x - vt)$ (16)

We check the other option: $x'' = ct'', v \to -v \quad \Rightarrow \quad x = \gamma x'' = \dfrac{x'' - (-v)x''/c}{\sqrt{1 - v^2/c^2}}$

$x = \gamma(x'' + vt'')$ The inverse Lorentz transformation: $x = \gamma(x' + vt')$ (17)

Deriving the Lorentz transformation from a second-order velocity modification of the Galilean transformation satisfies $g(x,v,v^2,c,t) = \gamma(v^2,c) f(x,v,t)$ if $f(x,v,t)$ is the Galilean transformation $x' = x - vt$ and $g(x,v,c,t)$ the Lorentz transformation $x' = \gamma(x - vt)$. Despite the *never-tested length contraction* implied by Lorentz, the energy-time interpretation is compatible with experimental physics of the 20th century yet avoids the paradoxes and logical nonsense of space-time symmetry. We use positive velocity to derive the function $\beta(v^2, c)$ from (9,11,14):

$$\xi(v,c) = \beta(v^2,c)\alpha(+v) \quad \Rightarrow \quad \frac{\beta}{1 + v/c} = \frac{1 - v/c}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad \beta = \sqrt{1 - mv^2/mc^2} \tag{18}$$

and change the sign of the velocity to derive the $\beta(v^2, c)$ from eqns (10,12,14):

$$\xi^{-1}(v,c) = \beta(v^2,c)\alpha(-v) \quad \Rightarrow \quad \frac{\beta}{1 - v/c} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad \beta = \sqrt{1 - mv^2/mc^2} \tag{19}$$

We have thus mapped the radar measurement model into the Lorentz transformation, providing the *necessary energy modification* $\beta(v^2, c)$ required for relativistic particle physics. A quick check:

$$\xi(v,c) = \frac{1 - v/c}{\sqrt{1 - v^2/c^2}} \times \left(\frac{1 + v/c}{1 + v/c}\right) \quad \Rightarrow \quad \frac{\sqrt{1 - v^2/c^2}}{1 + v/c} = \beta(v^2)\alpha(+v) \tag{20}$$

Only one coordinate system is used in our 'one world' derivation of the Lorentz transformation, and only energy arguments used to justify the derivation. A coordinate-free version of special relativity is Hestenes' '*Space-Time Algebra*'.[10]

We contrast this with the latest text on *Special Relativity* [12] in which Susskind derives the Lorentz transform and *assumes* the existence of multiple time axes (multiple 'real worlds'), and, given two or more 4D-coordinate systems, says:

> *"The obvious question is how do we go from one description to the other?"*

In other words, how do we go from the real world with space and time (x,y,z,t) to another world with space and time (x',y',z',t') he has attached to an object moving with velocity $\vec{v}$ in the (x,y,z,t) world. This object is considered 'at rest', $v=0$, in the second world, (x',y',z',t').

Since we assume that only one time dimension exists, we have $t' \equiv t$. An object moving with velocity $\vec{v}$ in this world can be considered to move with velocity $v=0$ in the moving space coordinates, but this does not change the universal nature of time. It does however change the kinetic energy $\sim mv^2$ when $\vec{v}$ is considered zero, so the energy-based Lorentz transformation transforms between (x,y,z,t,v^2) and $(x',y',z',t,v^2=0)$ energies. We then append energy factor $\sim v^2$ to the Galilean space-time transformation. As Susskind never mentions energy he must explain the v^2-factor as *space-time symmetry* of left and right:

> *"To summarize, writing $f(v^2)$ instead of $f(v)$ emphasizes the point that there is no preferred direction in space."*

Of course, since special relativity precludes gravity, there is no preferred local direction. Nevertheless, Susskind can't yet solve for $f(v^2)$, so he adds another factor, which he introduces as $g(v^2)$ in the t' transformation. He justifies this by "*inverting the roles of x and t*" [consistent with the "symmetry" of space and time.] He now has *two functions and two equations* and applies initial conditions, light ray paths, and Einstein's principle that speed of light is the same to conclude:

$$f(v^2) = g(v^2) . \tag{21}$$

But this still isn't enough to solve for $f(v^2)$ so he argues "*no preferred direction*" or "*space-time symmetry*", claiming to "*reason about the physical relationships between the two reference frames*".

Finally he argues his way (saying "*this may seem circular*") into a '*tedious but straightforward*' algebra to obtain

$$f(v^2) = \frac{1}{\sqrt{1 - v^2/c^2}} , \tag{22}$$

claiming "*this is the way Einstein did it.*"

Our energy-based approach

We can reject space-time symmetry as nonsense, and still know that *relativistic particle physics energy relations* have demonstrated the relevance of the Lorentz transformation to energy. So we reason that an energy factor should be applied to the Galilean space-time translation, implying a quadratic factor $\beta(v^2)$. As we wish to end up with the Lorentz transformation, we use the known fact [1] that

> *"The inverse of the Lorentz transformation is another Lorentz transformation with parameter $-v$ instead of $+v$."*

We therefore define the inverse function accordingly.

$$\xi = \beta(v^2)\,\alpha(+v) \qquad \text{and} \qquad \xi^{-1} = \beta(v^2)\,\alpha(-v) \tag{23}$$

The assumption of "*inverse as reciprocal*" works, but is not guaranteed, therefore we use an approach that is guaranteed: $\xi\xi^{-1} = 1$

$$\xi\xi^{-1} = 1 = \beta^2\alpha(+v)\,\alpha(-v) \tag{24}$$

$$\beta^2 = \left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right) \;\Rightarrow\; \beta = \sqrt{1-\frac{v^2}{c^2}} \tag{25}$$

$$\xi = \beta\,\alpha(+v) = \frac{\sqrt{1-v^2/c^2}}{1+v/c}\left(\frac{1-v/c}{1-v/c}\right) \;\Rightarrow\; \frac{1}{\sqrt{1-v^2/c^2}}(1-v/c) \tag{26}$$

Recognizing the Lorentz transformation as necessary for relativistic energies, we apply an energy-based factor to the Galilean space-time translation, and ask how to obtain the desired factor to yield the Lorentz transformation, making use of the inversion property of the transformation.

Susskind uses Einstein's unphysical principles to argue physically to obtain the Lorentz transformation with no mention of energy, but with an insistence on multiple time dimensions as physically real. He states:

> *"…Newton could not have known … that in going from one inertial frame to another, the space and time coordinates get mixed up with each other."*

As we have shown, this non-physical assumption is based on the erroneous idea of multiple time frames. The derivation of the Lorentz transformation in a *single* inertial frame, based on the difference in energy between a 'rest' object and an object moving with velocity $\vec{v}$, implies no such nonsense. And most of 20th century particle physics occurs in particle collisions at a *point* in time and space, where the time is obviously the same for each frame at the given point.

Next we derive *apparent time dilation* based on *radar*-type time measurement.

The Radar measurement of time

To measure time with a radar emitter/detector, we place the radar on the floor and a reflector on the ceiling a known distance $L/2$ above the floor. The light is emitted and travels to a mirror, $L/2$ distance above the floor, then travels back to the detector on the floor. The travel time is distance/speed. In the stationary (rest) frame the distance is twice $L/2$ and the travel time is $t = L/c$:

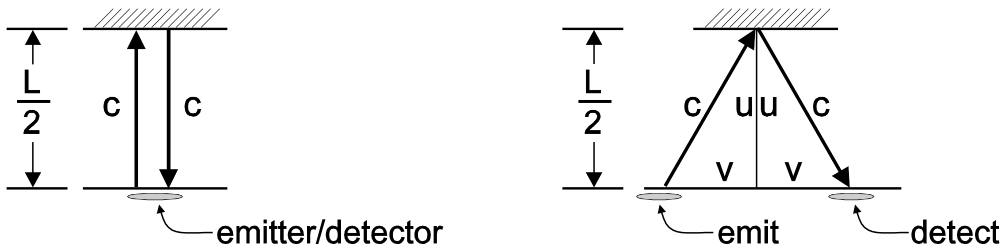

Fig 4. The same radar pulses, reflected from a mirror a known distance from the transmitter, establish a 'clock tick'. If the radar is in motion the time will appear to slow down with respect to measurement of time in the rest frame.

In the moving case the light still travels at the speed of light, but the mirror and detector are moving with velocity $\vec{v}$. Thus the effective speed u to travel distance $L/2$ is given by $c^2 = v^2 + u^2 \quad \Rightarrow \quad u = \sqrt{c^2 - v^2} \quad = \quad c\sqrt{1 - v^2/c^2}$.

In this moving case the 'equivalent' clock tick is

$$\tau = \frac{L}{u} \quad = \quad \frac{1}{\sqrt{1 - v^2/c^2}}\left(\frac{L}{c}\right) \quad = \quad \gamma t \qquad (27)$$

where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$. As $\tau = \gamma t$ the tick of the moving clock, τ, is greater than the tick of the rest clock, t, hence moving clocks "run slower" than rest clocks. Note that, like our radar-based length contraction, *time dilation is 'apparent'.*

$$d\tau = \gamma\, dt \quad \Rightarrow \quad \frac{d\tau}{dt} = \gamma \qquad \text{and} \qquad \frac{dt}{d\tau} = \sqrt{1 - \frac{v^2}{c^2}} \,. \qquad (28)$$

The proper time τ is Lorentz invariant: $(d\tau)^2 = (dt)^2 - \left(\frac{|d\vec{r}\,|^2}{c^2}\right)$ where $d\tau$ is the *infinitesimal Lorentz-invariant light-cone proper time interval.*

The standard Lorentz transformation equations

$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \qquad x' = \gamma(x - vt) \quad \text{with} \quad \gamma = \gamma(v,c) = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \qquad (29)$$

constitute a group, and the inverse transformation is

$$t = \gamma\left(t' - \frac{vx'}{c^2}\right), \qquad x = \gamma(x' - vt') \qquad (30)$$

The y and z equations remain $(y' = y, z' = z)$. Hence any relativistic transformation remains valid when the frames are 'switched' and v is replaced by $-v$.

Summary of energy-based Lorentz transformation

This treatment minimizes relativistic time dilation, treated elsewhere. We quote Rindler[1] about Einstein's relativistic postulate:

> *"Light propagates the same in all inertial frames... It is not for us to ask how!"*

If it made sense, we could ask how, so Rindler is admitting that it doesn't make sense. Rindler, as do most relativity texts, bases his derivation on Einstein's postulate of multiple "inertial frames":

> *"An inertial frame is one in which spatial relations, as determined by rigid scales at rest in the frame, are Euclidian and in which there exists a universal time...* [such that Newton's laws of inertia hold.]" (p.5)

Einstein's attachment of *a universal time* to a moving object essentially creates a *real physical world* with separate space and time in which objects exist and obey Newton's laws. As Rindler notes (p.8)

> *"Thus if a light signal recedes from me and I transfer myself to ever faster-moving frames in pursuit of it, I shall not alter the velocity of that light signal relative to me by one iota. This is totally irreconcilable with our classic concepts of space and time."*

Einstein's principle that is irreconcilable with classical physics leads to such issues as

> *...a man carrying a 20-foot pole into a 10-foot garage' by running at* $v = 0.886c$.

despite that Rindler admits

> *"No direct experimental verification of length contraction has yet been attempted."*

We derive the Lorentz transformation based on <u>one real world</u> ("inertial frame") using classical concepts of space, time, and energy. Rindler's derivation of the LT is based on Einstein's postulates, AP French[2] in *Special Relativity*, takes the same approach (pp.63-83) to deriving it. Similarly, Dixon [3], in *Special Relativity*, follows this path to Lorentz (pp.1-28). In this manner Jurgen Freund [4], (pp.37-43)

Special Relativity for Beginners derives the Lorentz transformation, beginning with the statement that

> *"The quantitative treatment of problems in special relativity necessitates two inertial reference frames…"*

Similarly, Einstein[5], in *Relativity: the Special and General Theory*, first presents his principles of relativity and only then derives (or presents) the Lorentz transformation, in terms of two inertial systems (pp.30-34). Still another text, Maudlin's *Philosophy of Physics: Space and Time* [6] does not derive the transformation, but discusses it in terms of *inertial frames*. As discussed above, Susskind [12] derives the transformation much as we do, but claims that the use of $f(v^2)$ rather than $f(v)$ is to "*emphasize the point that there is no preferred direction in space*", a space-time symmetry argument, and he makes no mention of energy.

> Every relativity text I've checked derives the Lorentz transformation *only* after presenting *Einstein's postulates of multiple inertial frames*.

Yet radar measurement of a moving object in *one inertial frame* leads to Galilean transformation in space and time. We only need the Lorentz transformation when energy is taken into account, such as is required for relativistic particle physics. Modifying the Galilean result with energy-dependent factor $\beta(v^2)$ avoids endless discussions of 'paradoxes' in special relativity based on length contraction and leads to an energy-based interpretation of *time dilation*.

Why would one assume two real worlds to derive the Lorentz transformation? We answer this question in our next paper.

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** An overview of the many issues involved in re-interpreting *space-time symmetry* in terms of *energy-time conjugation* is given in my response to the FQXi essay question '*What is Fundamental?*' at FQXI.org [13].

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