Force between two parallel current wires and Newton's third law

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Abstract: In this paper, the essence of the interaction between two parallel current wires is analyzed. During the analysis we found that if the interaction is originated from the resultant Lorentz force of the charge carriers in the currents, the Newton's third law will be violated in this system. However, if the essence of the interaction of the two current wires is considered as the attraction or repulsion of the two circular magnetic fields created by each current, the trouble of violating Newton's third law can be avoided.

Introduction

When a current-carrying wire is placed in an external magnetic field B, the wire will experience a force exerted by the magnetic field. It is widely believed that the physical origin of this force is from the resultant Lorentz force of the charge carriers in the current. The interaction of attraction or repulsion between two current-carrying wires is thought coming from this force. Each wire generates a magnetic field, and the other wire experiences a magnetic force as a consequence. Based on this, it is derived that the force between two parallel current-carrying wires can be expressed as,

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} L \qquad (1)$$

Where, $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ are the current intensities, r the distance between the two wires, L the length of the wire, and μ_0 the permeability of vacuum [1, 2, 3, 4]. It is called Ampère's force law.

When deriving formula (1), it is also thought by default: a) the two wires are identical in geometry and material; b) the two wires are identical in environment, such as they are both in vacuum or air, etc. So, the magnitude of the force derived for each wire is equal. However, if the two wires are made from different materials or they are in different environments, like, one of them is covered with magnetic shielding material, will the derived force for the two wires be equal or not?

In the following three cases, we have the two parallel wires made from different materials with different permeability or they are in different environments. The analysis shows that if we think the essence of the interaction between the two current wires is the resultant Lorentz force of charge carriers in the current, the forces for the two wires are not same in magnitude. Obviously, it conflicts with the Newton's third law. However, if the essence of the interaction is considered as the force between the two magnetic fields produced by the two currents, the trouble against the Newton's third law can be avoided.

Cases and analysis

Case1. Two wires are made from different materials with different permeability

When we put a material into an applied uniform magnetic field, the total field inside the object would then be varies with its permeability. For the two parallel wires with the same electric current, according to Ampère's circuital law the same concentric circles' magnetic field is produced around each wire. Because of the different permeability of the two wires, the total magnetic field strength in each wire induced by the opposite magnetic field is different. If the essence of the force between the two current wires is the resultant Lorentz force of charge carriers, the two wires will feel different force in the field produce by the other current wire. This is obviously against the Newton's third law.

In figure1, there are two parallel wires having different permeability, μ_1, μ_2. The electric currents of the two wires are $\mathbf{I}_1$and $\mathbf{I}_2$. According to Ampère's force law,

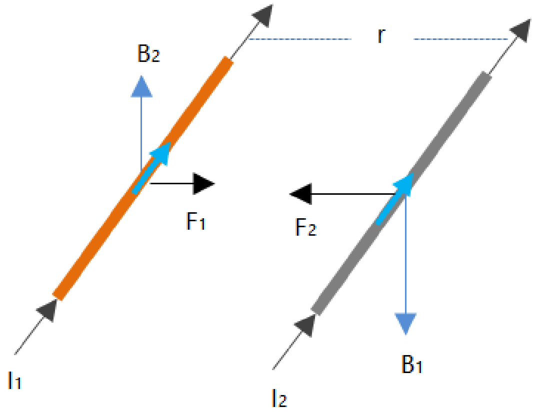

Fig.1. Two parallel wires, with different permeability, μ_1, μ_2, have current $\mathbf{I}_1$and $\mathbf{I}_2$.

The force exerted on wire1 is $F_1 = I_1 L B_2$, where L is the length of the wire1.

$$B_2 = \frac{\mu_0 \mu_1 I_2}{2\pi r} \qquad (2)$$

$$F_1 = \frac{\mu_0 \mu_1 I_1 I_2}{2\pi r} L \qquad (3)$$

The force exerted on wire2 is $F_2 = I_2 L B_1$,

$$B_1 = \frac{\mu_0 \mu_2 I_1}{2\pi r} \qquad (4)$$

$$F_2 = \frac{\mu_0 \mu_2 I_1 I_2}{2\pi r} L \qquad (5)$$

Because $\mu_1 \neq \mu_2$, we get $F_1 \neq F_2$.

Case2. One of the two parallel wires is wrapped up with a magnetic shielding tube

As shown in figure2, there are two parallel wires with identical geometry and made from the same material. But, one of them is wrapped up with a tube made of magnetic shielding material and the other not.

If the two currents are in same direction and without this tube, from the Ampere Force Law we get that the two wires pull together and the force on each wire is same in magnitude.

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} L \qquad (6)$$

Now the wire2 is wrapped up in a magnetic shielding tube which keeps the magnetic field produced by current1 free from the space inside the tube, so the wire2 does not feel the magnetic field produced by current1. However, the wire1 feels the magnetic field produced by current2. As a result, wire1 feels a force from the magnetic field produced by current2, but wire2 feels no force from the magnetic field produced by current1. This does not accord with the Newton's third law.

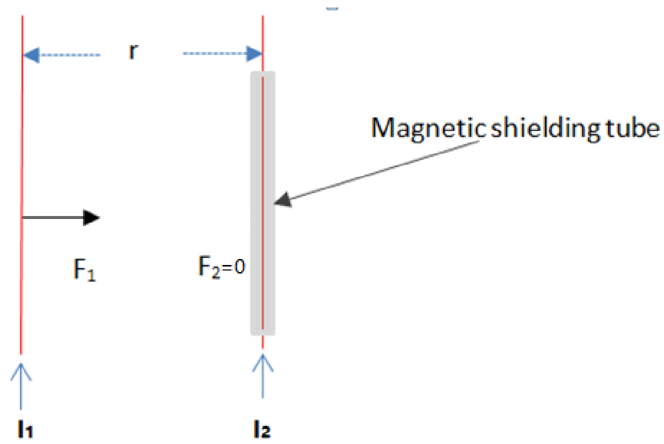

Fig.2. Two parallel wires with identical geometry and made from the same material. The wire2 is wrapped up with a tube made of magnetic shielding material. Electric currents $\mathbf{I}_1$and $\mathbf{I}_2$ flow through the two wires.

Case3. One of the two parallel wires is wrapped up with a superconducting tube

Here, the situation is quite similar to that in case2. Because of perfect diamagnetism of the superconducting tube, the magnetic field produced by current1 cannot reach to wire2. However, the magnetic field produced by current2 can reach to wire1. So, according to Ampere's force law, wire1 feels a force from the magnetic field produced by current2, but wire2 feels no force from the magnetic field produced by current1. The result conflicts with the Newton's third law, again.

Discussion

Up to now, a question may be asked that if any force on the tube is neglected during the derivation above and just this result in the trouble of conflicting with Newton's third law？

At first, let's see the case2. Suppose there is a force between the current and the magnetic shielding tube. From the aspect of symmetry, there should not be net interaction between the tube and the wire2. If any interaction, it may only exist between the tube and current wire1. The force may be attractive or repulsive. However, physically, the force being attracting or repelling will have nothing to do with the direction of the current1, so the problem against the Newton's third law is still not be resolved. From the same logical analysis, the trouble of conflicting with Newton's third law in case3 is not resolved, either.

By the analytical argument for the three cases above, we see that the conflict with the Newton's third law is unavoidable if the Lorentz force is considered as the essence of the interaction between two parallel current wires.

However, if the force between the two current wires is regarded as the interaction of the two magnetic fields produced by each current rather than the Lorentz force, the trouble violating Newton's third law can be avoided.

A current wire could be regarded as a kind of bar magnet. Unlike the normal one, this bar magnet has no poles, but a circular field around the wire. The interaction of the two current wires is very similar to the interaction of two parallel line charges. The force between two parallel line charges is expressed as,

$$F = \frac{2\lambda_1\lambda_2}{r} L \qquad (7)$$

Where λ_1 and λ_2 are the linear charge density, r the distance between the two linear charges, L the length of the linear charge [5, 6].

If we regard I_1、I_2 as the measure of strength of the magnetic field source of the current wire bar magnet, formula (1) will almost be the same as formula (7). In formula (1), if we think that it is not visual and intuitionistic enough to use I to indicate a magnet's strength, we can change formula (1) into another form, in which the I is replaced with the strength of the circular magnetic field of the current wire.

For two parallel wires with current I_1 and I_2, and radius R_1, R_2, according to Ampère's circuital law, the circular magnetic field created by each current wire is,

$$B_1(r) = \frac{\mu_0 I_1}{2\pi r} \qquad (8)$$

$$B_2(r) = \frac{\mu_0 I_2}{2\pi r} \qquad (9)$$

Where, r is the perpendicular distance to the wire.

For each wire, the circular magnetic field on the surface of the wire produced by the current itself is,

$$B_1(R_1) = \frac{\mu_0 I_1}{2\pi R_1} \qquad (10)$$

$$B_2(R_2) = \frac{\mu_0 I_2}{2\pi R_2} \qquad (11)$$

In formula (8) and (9), we time $\frac{R_1}{R_1}$ and $\frac{R_2}{R_2}$ then we get,

$$B_1(r) = \frac{\mu_0 I_1}{2\pi r} = B_1(R_1)\,\frac{R_1}{r} \qquad (12)$$

$$I_1 = \frac{2\pi R_1 B_1(R_1)}{\mu_0} \qquad (13)$$

$$B_2(r) = \frac{\mu_0 I_2}{2\pi r} = B_2(R_2)\,\frac{R_2}{r} \qquad (14)$$

$$I_2 = \frac{2\pi R_2 B_2(R_2)}{\mu_0} \qquad (15)$$

The force exerted on each wire is,

$$F_1 = F_2 = I_1 B_2 L = I_2 B_1 L = \frac{2\pi R_1 R_2 B_1(R_1) B_2(R_2)}{\mu_0 r} L \qquad (16)$$

Equation (16) is identical with equation (1). $B_1(R_1)$, $B_2(R_2)$ are the magnetic field strength on the surface of the wires. $R_1 B_1(R_1), R_2 B_2(R_2)$ can be regarded as the source strength of the current wire bar magnets. So, the force between the two current wire bar magnets is proportional to the product of the two bar magnets' strength and inversely proportional with the distance between them. It is so similar to the interaction of two linear charges.

Conclusions

1. For the interaction between two parallel current wires, if it is originated from the resultant Lorentz force of the charge carriers in the current, the Newton's third law will be violated in this system.
2. If the force between the two current wires is originated from the interaction of the two circular magnetic fields created by each current, the trouble of violating Newton's third law can be avoided.

References:

[1] Feynman, R. et al., "Feynman lectures on physics volume 2, Mainly Electromagnetism and Matter, Chapter 13", Addison-Wesley, 2011.

[2] Raymond A Serway and John W. Jewett, Jr., "Physics for Scientists and Engineers with Modern Physics", Chapter 29, Ninth Edition, Technology Update, CENGAGE Learning, 2015.

[3] Raymond A Serway and John W. Jewett, Jr., "Physics for Scientists and Engineers with Modern Physics", Chapter 30, Ninth Edition, Technology Update, CENGAGE Learning, 2015.

[4] Purcell, Edward M. and Morin, David J. "Electricity and Magnetism", chapter 6, (Berkeley Physics Course, Vol. 2), Cambridge University Press; 3 edition (January 21, 2013)

[5] Purcell, Edward M. and Morin, David J. "Electricity and Magnetism", chapter 1, (Berkeley Physics Course, Vol. 2), Cambridge University Press; 3 edition (January 21, 2013)

[6] Feynman, R. et al., "Feynman lectures on physics volume 2, Mainly Electromagnetism and Matter, Chapter 5", Addison-Wesley, 2011.