On the nature of the W boson

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December 9, 2017

Abstract

We study leptonic and semileptonic weak decays working in the framework of Hagen-Hurley equations. It is argued that the Hagen-Hurley equations describe decay of the intermediate gauge boson W. It follows that we get a universal picture with the W boson being a virtual, off-shell, particle with (partially undefined) spin in the $0 \oplus 1$ space.

1 Introduction

Decay of a spin 0 charged pion, $\pi^+ \longrightarrow \mu^+ + \bar{\nu}_\mu$, via formation of virtual intermediate gauge boson W, still raises question of spin conservation in the process. This is because the boson W has spin 1 while π^+ is spinless and the products μ^+, $\bar{\nu}_\mu$ are in spin 0 state. There are several explanations of this apparent spin nonconservation but it seems that none is absolutely convincing. More exactly, it is suggested that:

1. the $S_z = 0$ state of the W boson is more or less a spin-0 "particle" (a longitudinally polarized W boson) [1, 2],
2. the pion decays through the spinless Nambu-Goldstone boson χ, $\pi \longrightarrow q + \bar{q} \longrightarrow \chi \longrightarrow \mu + \nu$ [3],
3. the W boson is a virtual off-shell particle and thus conservation of angular momentum is debatable.

Recently, we studied properties of the W boson in the context of beta decay [4]. Working in the spinor formulation of Hagen-Hurley equations [5] we have demonstrated that these equations may describe decay of W boson in beta decay provided that its spin is not completely determined belonging to the $0 \oplus 1$ space [4]. This hypothesis agrees well with existence of two main mechanisms of beta decay, namely the Gamow-Teller (GT) and Fermi (F) transitions. The aim of the present work is to justify this hypothesis. Finally, we shall compare our hypothesis with the generally accepted explanation 1.

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2 The Hagen-Hurley equations

We base our theory on the Hagen-Hurley equations, see [6–9] and Subsection 6 ii) in [10]. These equations violate parity and thus can describe weakly interacting particles.

We wrote one of the Hagen-Hurley equations, in the interacting case, in spinor form [4, 5]:

$$\left.\begin{aligned} \pi^{A}{}_{\dot{B}}\ \zeta_{A\dot{D}} &= m\chi_{\dot{B}\dot{D}} \\ \pi_{A}{}^{\dot{D}}\ \chi_{\dot{B}\dot{D}} &= -m\zeta_{A\dot{B}} \end{aligned}\right\} \tag{1a}$$

$$\chi_{\dot{B}\dot{D}} = \chi_{\dot{D}\dot{B}} \tag{1b}$$

where Eq. (1b) is the spin 1 constraint [10]. To interpret Eqs. (1a) we put:

$$\chi_{\dot{B}\dot{D}}\left(x\right) = \eta_{\dot{B}}\left(x\right)\alpha_{\dot{D}}\left(x\right), \quad \zeta_{A\dot{B}}\left(x\right) = \xi_{A}\left(x\right)\alpha_{\dot{B}}\left(x\right), \tag{2}$$

where $\alpha_{\dot{A}}\left(x\right)$ is the Weyl spinor, describing massless neutrinos, while $\eta_{\dot{B}}\left(x\right)$, $\xi_{A}\left(x\right)$ are the Dirac spinors. Note that now $\chi_{\dot{1}\dot{2}} \neq \chi_{\dot{2}\dot{1}}$ and, accordingly, the spin is not determined – more exactly, the spin is in the $0 \oplus 1$ space. It means that we consider not real but virtual (off-shell) bosons [11]. Substituting (2) into Eqs. (1a) we obtained the Weyl equation and the Dirac equation, describing two spin $\frac{1}{2}$ particles, a Weyl neutrino and a massive fermion, whose spins can couple to $s = 0$ or $s = 1$, i.e. $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$ [4].

3 Weak decays

In this Section we consider several three-body leptonic weak decays of fermions involving a virtual W boson which decays yielding a lepton l and antineutrino $\bar{\nu}_l$:

$$A \longrightarrow B + W \longrightarrow B + l + \bar{\nu}_l \tag{3}$$

3.1 Beta decay

Results obtained in [4] cast new light on the Hagen-Hurley equations as well as on weak decays of spin 1 bosons. We have described transition from equation (1), describing a spin $s = 1$ particle to the Weyl and Dirac equations via substitution (2) [4]. This substitution means that the condition $s = 1$ is relaxed and now $s \in 0 \oplus 1$. We interpret this process as decay of the boson W into a Weyl antineutrino and a Dirac lepton [4].

The above description fits a mixed beta decay with formation of a virtual W^{-} boson, decaying into a lepton and antineutrino. For the sake of an example let us consider decay of the neutron:

$$n \longrightarrow p + W^{-} \longrightarrow p + e + \bar{\nu}_e \tag{4}$$

This is the mixed beta decay [12]:

$$n\,(\uparrow) \longrightarrow \begin{cases} p\,(\downarrow) + [e\,(\uparrow)\,\bar{\nu}_e\,(\uparrow)] & \text{GT transition} \\ p\,(\uparrow) + [e\,(\uparrow)\,\bar{\nu}_e\,(\downarrow)] & \text{F transition} \end{cases} \tag{5}$$

where products of the W^- boson decay (see [13]) are shown in square brackets and $(\uparrow)$ denotes spin $\frac{1}{2}$ [4]. Since spin of the products of decay of the virtual W^- boson belongs to the $0 \oplus 1$ space, their spin can be $s = 0$ or $s = 1$. Note that in the case of the Gamow-Teller transition there must be a spin-flip in the proton. Indeed, in the reaction (5) some neutrons (82%) decay according to the Gamow-Teller mechanism while some (18%) undergo the Fermi transition [12].

3.2 Muon and tau decays

Let us consider the three-body muon decay [13]:

$$\mu^- \longrightarrow \nu_\mu + W^- \longrightarrow \nu_\mu + e^- + \bar{\nu}_e \qquad (\approx 100\%) \tag{6}$$

There are two possible configurations of muon, electron and neutrinos spins corresponding to two extreme cases [14, 15]. In the case when the electron has the maximum allowed energy in the muon rest frame neutrino and antineutrino are going in the same direction and their total spin is zero, while the electron is ejected in opposite direction and carries the same spin as muon. In another case, when electron kinetic energy in the muon rest frame is very small neutrinos are emitted back-to-back and have total spin $s = 1$, therefore the electron and muon spins are antiparallel. These two situations can be depicted as:

$$\mu^-\,(\uparrow) \longrightarrow \begin{cases} \nu_\mu\,(\downarrow) + [e^-\,(\uparrow)\,\bar{\nu}_e\,(\uparrow)] & \mathcal{M}_- \\ \nu_\mu\,(\uparrow) + [e^-\,(\downarrow)\,\bar{\nu}_e\,(\uparrow)] & \mathcal{M}_+ \end{cases} \tag{7}$$

where $\mathcal{M}_-$ dominates when the decay electron has the energy near the maximum allowed energy in the muon rest frame, while $\mathcal{M}_+$ prevails when the electron kinetic energy is close to zero in the muon rest frame. We interpret the mechanism $\mathcal{M}_-$ as implying spin 1 of the W boson, while $\mathcal{M}_+$ corresponds to its spin 0. There is also another possibility of $\mathcal{M}_-$ reaction, namely $\mu^-\,(\uparrow) \longrightarrow \nu_\mu\,(\uparrow) + [e^-\,(\uparrow)\,\bar{\nu}_e\,(\downarrow)]$, but this would mean that the intermediate boson carries spin 0 only. We note that the mechanisms $\mathcal{M}_-$, $\mathcal{M}_+$ correspond to Gamow-Teller and Fermi transitions, respectively. In the case of tau lepton decay there are two leptonic decays:

$$\tau^- \longrightarrow n + W^- \longrightarrow \nu_\tau + e^- + \bar{\nu}_e \qquad (17.82\%) \tag{8a}$$

$$\tau^- \longrightarrow n + W^- \longrightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu \qquad (17.39\%) \tag{8b}$$

In these two cases arguments described after Eq. (6) apply again and lead to the same two mechanisms of decay:

$$\tau^-\,(\uparrow) \longrightarrow \begin{cases} \nu_\tau\,(\downarrow) + [e^-\,(\uparrow)\,\bar{\nu}_e\,(\uparrow)] & \mathcal{M}_- \\ \nu_\tau\,(\uparrow) + [e^-\,(\downarrow)\,\bar{\nu}_e\,(\uparrow)] & \mathcal{M}_+ \end{cases} \tag{9a}$$

$$\tau^-\,(\uparrow) \longrightarrow \begin{cases} \nu_\tau\,(\downarrow) + [\mu^-\,(\uparrow)\,\bar{\nu}_\mu\,(\uparrow)] & \mathcal{M}_- \\ \nu_\tau\,(\uparrow) + [\mu^-\,(\downarrow)\,\bar{\nu}_\mu\,(\uparrow)] & \mathcal{M}_+ \end{cases} \tag{9b}$$

4 Conclusions and discussion

We have demonstrated that in muon and tau lepton decays, as well as in the mixed beta decay [4], spin of the W boson is not completely determined belonging to the $0 \oplus 1$ space. There are also many examples of three-body semileptonic decays of fermions of type (3), see [13] and Table 1 for a sample:

Table 1	
$\Sigma^- \longrightarrow n + W^- \longrightarrow n + e^- + \bar{\nu}_e$	1.017×10^{-3}
$\Sigma^- \longrightarrow n + W^- \longrightarrow n + \mu^- + \bar{\nu}_\mu$	4.5×10^{-4}
$\Xi^- \longrightarrow \Lambda + W^- \longrightarrow \Lambda + e^- + \bar{\nu}_e$	5.63×10^{-4}
$\Xi^- \longrightarrow \Lambda + W^- \longrightarrow \Lambda + \mu^- + \bar{\nu}_\mu$	3.5×10^{-4}
$\Lambda_C^- \longrightarrow \Lambda + W^- \longrightarrow \Lambda + e^- + \bar{\nu}_e$	3.6%

In all cases it is assumed that decay involves formation of the W boson. Although these are not the main channels of decay, yet reactions of this kind consistently appear in weak decays. We thus expect that in decays (3) there are always two competing mechanisms, $\mathcal{M}_-$ and $\mathcal{M}_+$:

$$A\,(\uparrow) \longrightarrow \left\{ \begin{array}{ll} B\,(\downarrow) + [l\,(\uparrow)\,\bar{\nu}_l\,(\uparrow)] & \quad \mathcal{M}_- \\ B\,(\uparrow) + [l\,(\downarrow)\,\bar{\nu}_l\,(\uparrow)] & \quad \mathcal{M}_+ \end{array} \right. \tag{10}$$

Finally, we compare our hypothesis with the accepted views on the nature of the W boson. The prevailing interpretation of weak decays involving W boson is possibility 1 listed in the Introduction, i.e. decay mediated by virtual, off-shell, longitudinally polarized W boson. Accordingly, since W boson is a virtual particle its spin is physically unobservable in the process of decay. This is in harmony with our hypothesis that spin of the W boson is (partially) undefined. Additionally, we propose two mechanisms of decay, $\mathcal{M}_-$ and $\mathcal{M}_+$, analogous to Gamow-Teller and Fermi transitions, see Eq. (10), based on the idea that spin of the W boson belongs to the $0 \oplus 1$ space.

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