Reflections on the reality of the wavefunction

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Abstract: This paper further explores the structural similarities between the elementary quantum-mechanical wavefunction ($a \cdot e^{-i \cdot \theta} = a \cdot cos\theta - i \cdot a \cdot sin\theta$) and circularly polarized electromagnetic waves to further tune a possible *physical* interpretation of the wavefunction. The interpretation that is offered analyzes the real and the imaginary part of the wavefunction as oscillations which each carry half of the total energy of the particle. These oscillations are perpendicular to each other, and the interplay between both may describe how energy propagates through space over time. The model is based on three fundamental premises:

1. The dimension of the matter-wave field vector is force per unit *mass* (N/kg), as opposed to the force per unit *charge* (N/C) dimension of the electric field vector. This dimension is an acceleration (m/s^2), which is the dimension of the gravitational field.

2. This gravitational disturbance may cause a charged *mass* to move about some center, combining linear and circular motion. This interpretation may reconcile the wave-particle duality to some extent: fields interfere but if, at the same time, they do drive a pointlike particle, this may explain why, as Feynman puts it, "when you do find the electron some place, the entire charge is there." This hybrid hypothesis is supported by an elegant yet simple derivation of the Compton radius of an electron.

3. In light of the direction of the magnetic moment of an electron in an inhomogeneous magnetic field, the plane which circumscribes the circulatory motion of the electron should also *comprise* the direction of its linear motion. Hence, unlike an electromagnetic wave, the *plane* of the two-dimensional oscillation can*not* be perpendicular to the direction of motion of our electron.

Finally, this paper addresses an issue which has hampered other physical interpretations of the wavefunction: amplitude transformations when going from one representation (or reference frame) to another. Indeed, the author re-visits the original mathematical arguments here and shows that the proposed model, and the related physical interpretation of the wavefunction, are not incompatible with the key results and mainstream logic in this regard.

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Introduction

Mainstream physics textbooks usually stop short of offering geometric or *physical* interpretations of the wavefunction, and warn the student against viewing quantum states as being, somehow, *real*. However, recent discoveries —most notably CERN's 2011-2012 ATLAS and CMS experiments, which observed a new particle in the mass region around 126 GeV which is consistent with the theoretical Higgs boson, and LIGO's 2015-2017 detections of gravitational waves—have prompted many physicists to re-examine the nature of some core mathematical concepts in quantum theory[3].

Truth be told, despite their dislike of philosophers, physicists such as Richard Feynman and Stephen Hawking leave plenty of space—and have made major contributions to—a better understanding of how the wavefunction—and quantum states—may bridge reality and our perception of it. One of the best illustrations in this regard may well be Feynman's "long and abstract side tour"—as he puts it in his *Lecture* on spin-1/2 particles[4]—in which he derives the transformation matrices to go from one representation to another. In fact, the thought experiments in this chapter come tantalizing close to a

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[2] *The Quantum-Mechanical Wavefunction as a Gravitational Wave*, 26 September 2017 (http://vixra.org/abs/1709.0390, accessed on 6 December 2017)

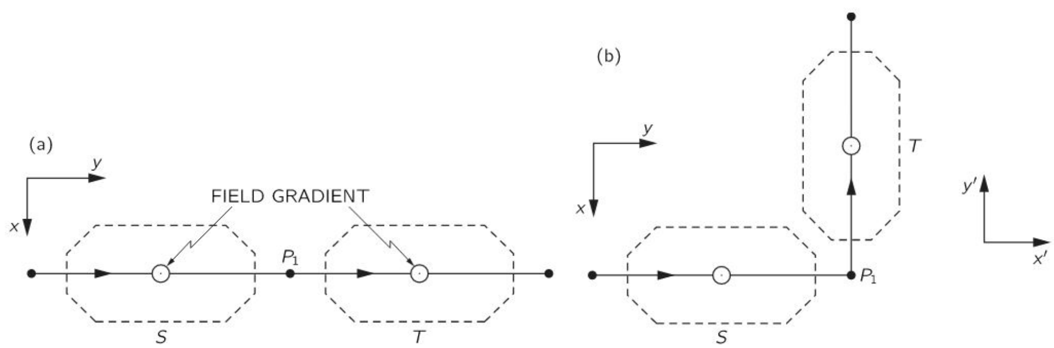
[3] See, for example, Pusey, Barrett and Rudolph, *On the reality of the quantum state*, in: *Nature Physics* 8, 475–478, 2012 (https://www.nature.com/articles/nphys2309, accessed on 6 December 2017).

[4] *Feynman Lectures on Physics* (http://www.feynmanlectures.caltech.edu/III_06.html, accessed on 6 December 2017). Feynman's Lectures will be used as a standard reference to mainstream physics throughout this paper and will, therefore be referenced elsewhere too. The references indicate the volume, chapter and section. For example, Feynman, III, 6-3 refers to Volume III, Chapter 6, Section 3).

physical interpretation not only of quantum states but also of the wavefunction of spin-1/2 particles. Hence, it is probably worthwhile to give an example of Feynman's line of reasoning in this introduction.[5]

In the illustration below (Feynman, III, 6-3), Feynman compares the physics of two beam splitters[6] with a different *relative* orientation: in (a), the angle is 0°, while in (b) we have a (right-handed) rotation of 90° about the *z*-axis. He then proves—using geometry and logic only—that the probabilities and, therefore, **the *magnitudes* of the amplitudes** (denoted as C_+ and C_- and C'_+ and C'_- in the *S* and *T* representation respectively) **must be the same, but the amplitudes *must* have different phases**, noting—in his typical style, mixing academic and colloquial language—that "there must be some way for a particle to tell that it has turned a corner in (b)."

Figure 1: A rotation of 90° about the z-axis



The various interpretations of what actually *happens* here may shed some light on the heated discussions on the *reality* of the wavefunction—and of quantum states. We *know*, from theory *and* experiment, that the amplitudes *are* different. For example, for the given difference in the *relative* orientation of the two apparatuses (90°), we *know* that the amplitudes are given by $C'_+ = e^{i\cdot\phi/2}\cdot C_+ = e^{i\cdot\pi/4}\cdot C_+$ and $C'_- = e^{-i\cdot\phi/2}\cdot C_+ = e^{-i\cdot\pi/4}\cdot C_-$ respectively.[7] The more subtle question here is the following: is the *reality* of the particle in the two setups the same?

Feynman notes that, while "the two apparatuses in (a) and (b) are different", "the probabilities are the same". He refrains from making any statement on the particle itself: is or is it *not* the same? The common sense answer is obvious: of course, it is! The particle is the same, right? In (b), it just took a turn—so it is just going in some other direction. That's all. However, common sense is seldom a good guide when thinking about quantum-mechanical realities. Also, from a more philosophical point of view, one may argue that the reality of the particle is *not* the same: something might—or *must*[8]—have *happened* to the electron because, when everything is said and done, the particle *did* take a turn in (b). It did *not* in (a).

[5] We will re-visit his argument and, hence, advise the reader *not* to skip this example.
[6] Modified or 'improved' Stern-Gerlach apparatuses, as he terms it.
[7] The amplitude to go from the down to the up state, or vice versa, is zero.
[8] The difference between 'might' and 'must' here is, obviously, the difference between a deterministic and a non-deterministic world view.

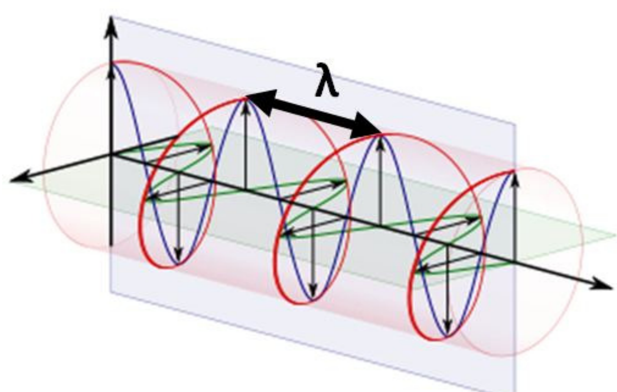
One may shrug this off as a moot point, but it is not. In fact, Feynman himself, despite his dislike of philosophers, does *not* shrug it off as a moot point. For example, he notes that, if we rotate the *T* apparatus by 360°, the system will, in effect, be indistinguishable from the zero-degree situation but that the amplitudes will also be different: $C'_+ = e^{i\cdot\phi/2}\cdot C_+ = e^{i\cdot\pi}\cdot C_+ = -C_+$ and $C'_- = e^{-i\cdot\phi/2}\cdot C_+ = e^{-i\cdot\pi}\cdot C_- = -C_-$. Both amplitudes are multiplied by −1. Feynman says the following in this regard (Feynman, III, 6-3): "It is very curious to say that, if you turn the apparatus 360°, you get new amplitudes. They aren't really new, though, because the common change of sign doesn't give any different physics." However, in a footnote, he acknowledges the *reality* of the situation might not be the same. Referring to a continuity assumption he had used earlier, he notes the following: "If something has been rotated by a sequency of small rotations whose net result is to return it to the original orientation, *it is possible to define the idea that it has been rotated by 360°−as distinct from zero net rotation*−if you have kept track of the whole history."

These are weird philosophical questions. Is an apparatus that has been turned 360° a different apparatus? Is an electron that takes a turn a different electron? Even if one's answer to these questions is negative, one should not dismiss—or not out of hand, at least—the suggestion that the wavefunction must, somehow, represent something real. When everything is said and done, Einstein's intuition that, if there is interference and diffraction, *something* must be interfering or diffracting, makes sense. This paper offers a tentative *model* to think of the wavefunction in this way.

1. The flywheel model of an electron

We explored the geometry of the wavefunction in a previous paper.[9] We noted the *mathematical* similarities between the elementary quantum-mechanical wavefunction ($a\cdot e^{-i\cdot\theta} = a\cdot cos\theta - i\cdot a\cdot sin\theta$) and a circularly polarized electromagnetic wave. Both consist of two plane *component* waves—a sine and a cosine function—as illustrated below.

Figure 2: A circularly polarized wave



[9] The Quantum-Mechanical Wavefunction as a Gravitational Wave (http://vixra.org/abs/1709.0390, accessed on 6 December 2017).

The analogy has some obvious, immediate and interesting implications. For example, assuming the dimension of the matter-wave field vector is force per unit *mass* (N/kg)—as opposed to the force per unit *charge* (N/C) dimension of the electric field vector—we were able to interpret Schrödinger's wave equation as an energy diffusion equation, and we were also able apply the concept of the Poynting vector to the matter-wave. Most importantly, we were able to show that the probabilities must reflect energy densities. Last but not least, this physical interpretation of the wavefunction also explains relativistic length contraction.[10]

We were encouraged to explore the geometry of the wavefunction because of another *structural* similarity, which we interpreted as an *equivalence*: the $E = m \cdot a^2 \cdot \omega^2$ and the $E = m \cdot c^2$ relations. The first captures the energy of an oscillation in *two* dimensions. [11] The second is Einstein's mass-energy equivalence relation, whose mathematical shape encourages us to think of energy as a two-dimensional oscillation of mass.[12]

A short recap may be useful here. We developed the metaphor of a twin-engined *perpetuum mobile*. Think of a 90° V-twin engine without petrol (Figure 3).[13] With permanently closed valves, the air inside each cylinder compresses and decompresses as the piston moves up and down. It provides, therefore, a restoring force. As such, it will store potential energy—just like a spring—and the motion of the pistons will also reflect that of a mass on a spring: it is described by a sinusoidal function, with the zero point at the center of each cylinder. We can, therefore, think of the moving pistons as harmonic oscillators, just like mechanical springs.[14]

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[10] For more detail, see the referenced paper. It is tempting to rehash the derived results here, but we will not do so.

[11] The total energy (potential and kinetic) of *one* oscillator is given by $E = m \cdot a^2 \cdot \omega^2/2$. Hence, the energy of *two* oscillators adds up to $E = E = m \cdot a^2 \cdot \omega^2$.

[12] The interpretation treats mass as a simple scalar field, which is entirely consistent with the Standard Model. To put it simply, mass is that what gets accelerated by a force. In that regard, our previous paper also includes some thoughts on the *physical* significance of the absolute nature of the speed of light. These thoughts can be summarized as follows. Einstein's $E = mc^2$ equation implies the ratio between the energy and the mass of *any* particle is always the same:

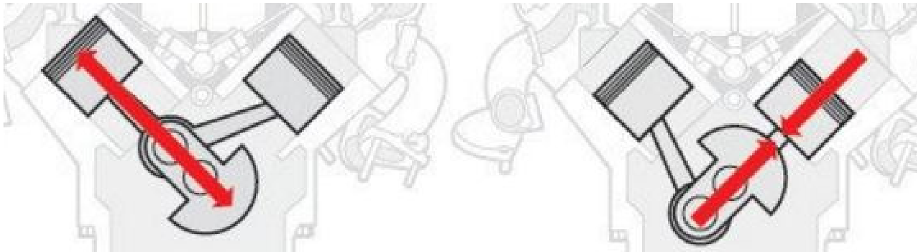
$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photon}}{m_{photon}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

This reminds us of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ of harmonic oscillators once again. The key difference is that the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom. In contrast, $c^2 = E/m$ for *any* particle, *always*. That is exactly the point: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges here as *the* defining property of spacetime – the resonant frequency, so to speak. We have no further degrees of freedom here.

[13] We will resist a comparison between the efficiency and power of a Ducati Monster and a Harley-Davidson. ☺

[14] Instead of two cylinders with pistons, one may also think of connecting two springs with a crankshaft, but that would require springs that would *not* be able to move sideways.

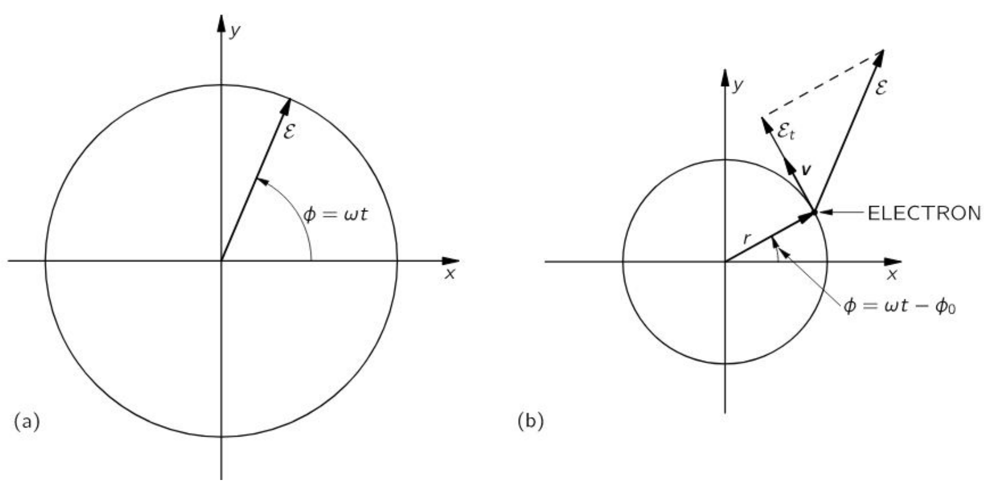
Figure 3: The V-2 metaphor



This inspired us to think of a what we refer to as a *flywheel* model of an electron (or of a charged spin-1/2 particle in general).[15] Such flywheel model is also used in Feynman's remarkable geometric interpretation (see Feynman, III, 17-4) of how a rotating electron might absorb the energy of a light beam. Other authors have also continued to explore the so-called *Zitterbewegung* interpretation of quantum mechanics.[16] The *Zitterbewegung*—a term which was coined by Erwin Schrödinger himself—is, effectively, a local circulatory motion of the electron, which is presumed to be the basis of the electron's spin and magnetic moment.

Feynman's illustrations—Figure 4 (a) and (b)—speak for themselves. Photons—and the electromagnetic wave itself—carry angular momentum. As such, we have a *rotating* electric field vector, which is denoted as a large ε (epsilon) here so as to not cause any confusion with the E that is used to denote energy (Figure 4-*a*). Hence, the *tangential* component of this field may drive the electron and, thereby, transfer energy to the electron (Figure 4-*b*).

Figure 4: Feynman's flywheel model of an electron



[15] Jean-Louis Van Belle, *The flywheel model of an electron*, 19 November 2017 (https://readingfeynman.org/2017/11/19/the-flywheel-model-of-an-electron/, accessed on 6 December 2017).
[16] See, for example, David Hestenes, *The Zitterbewegung Interpretation of Quantum Mechanics*, in: Found. Physics., Vol. 20, No. 10, (1990) 1213–1232 (https://pdfs.semanticscholar.org/ba8f/fcbacbe33a4819ec065e160a9f014ad9f634.pdf, accessed on 6 December 2017)

We should note that, in Feynman's model, the electron is assumed to be in some orbit around a nucleus. However, the author of this paper is of the opinion that the model may also be used to describe the motion of as *stand-alone* electron as *a harmonic oscillator which can be driven by an external electric field*.[17]

It is probably worth to briefly recap Feynman's mathematical argument here. Photons are spin-1 particles, so the angular momentum will be equal to ± ħ. The *total* angular momentum of a polarized beam consisting of *N* photons will be equal to $J_z = N \cdot \hbar$.[18] Now, the Planck-Einstein equation tells us that the energy of each photon is equal to $E = \hbar \cdot \omega = h \cdot f$. Hence, the total energy of the beam is equal to $W = N \cdot E = N \cdot \hbar \cdot \omega$. Combining the $W = N \cdot \hbar \cdot \omega$ and $J_z = N \cdot \hbar$ equations, we get: $J_z = N \cdot \hbar = W/\omega$. Now, a charge (read: the electron) will experience a force which is equal to $\mathbf{F} = q \cdot \boldsymbol{\varepsilon}$, but only the tangential component needs to be taken into account. Hence, we write: $F = q \cdot \varepsilon_t$.

As our light beam—the photons—are being absorbed by our electron (or, in Feynman's original argument, by the atom as a whole), it absorbs *angular momentum*, because there is a *torque* about the central axis. The relevant formulas for the angular momentum and the torque here are the usual ones: $\mathbf{L} = \boldsymbol{r} \times \mathbf{p}$ and $\boldsymbol{\tau} = \boldsymbol{r} \times \mathbf{F}$.[19] Now, the time rate of change of the angular momentum of an object is the vector sum of all torques acting on it. Hence, if the torque is equal to $\tau = F_t \cdot r = q \cdot \varepsilon_t \cdot r$, then $dJ_z/dt = q \cdot \varepsilon_t \cdot v$.

Now, energy is force over a distance distance and, therefore, it is easy to see that the following formula will capture the time rate of change of the energy of the electron[20]: $dW/dt = q \cdot \varepsilon_t \cdot v$. Taking the *ratio* of dW/dt and dJ_z/dt, we get the following interesting equation:

$$\frac{dJ_z/d\mathrm{t}}{d\mathrm{W}/d\mathrm{t}} = \frac{dJ_z}{d\mathrm{W}} = \frac{\mathrm{q} \cdot \varepsilon_\mathrm{t} \cdot r}{\mathrm{q} \cdot \varepsilon_\mathrm{t} \cdot v} = \frac{r}{v} = \frac{r}{r \cdot \omega} = \frac{1}{\omega}$$

Feynman then relates this to the $J_z = N \cdot \hbar = W/\omega$ formula, but our analysis has to parts way with his here, as Feynman seems to forget the (angular) frequency of the photons should *not* be equated to the (angular) frequency of the electron. Having said that, the analysis remains interesting. Feynman suggests to integrate dJ_z and dW over some time interval—which makes sense because W is, obviously, the energy that is carried by the beam *in a certain time*. Hence, if we integrate dW over this time interval, we get W. Likewise, if we integrate dJ_z over the same time interval, we should get the total angular momentum that our electron is absorbing from the light beam.

[17] We should quote Feynman himself in this regard: "We have often described the motion of the electron in the atom as a harmonic oscillator which can be driven into oscillation by an external electric field. We'll suppose that the atom is isotropic, so that it can oscillate equally well in either direction. Then in the circularly polarized light, the *x* and the *y* displacements are the same, but one is 90° behind the other. The net result is that the electron moves in a circle." (Feynman, III, 17-4)

[18] A polarized beam presumably consists of photons that are all polarized *in the same direction*.

[19] These products are *vector* cross-products, and the $\boldsymbol{\tau} = \boldsymbol{r} \times \mathbf{F}$ formula explains why we need to find the *tangential* component of the force ($\mathbf{F}_t$), whose magnitude is equal to $F_t = q \cdot \varepsilon_t$.

[20] It may help to remind yourself of the fact that *v* is equal to $ds/dt = \Delta s/\Delta t$ for $\Delta t \to 0$, and to re-write the equation above as $dW = q \cdot \varepsilon_t \cdot v \cdot dt = q \cdot \varepsilon_t \cdot ds = F_t \cdot ds$.

Hence, because $dJ_z = dW/\omega$, we do concur with Feynman's conclusion: the total angular momentum which is being absorbed by the electron is proportional to the total energy of the beam, and the constant of proportionality is equal to 1/ω. However, and here we effectively part ways with Feynman's analysis, we should remind ourselves that the ω in this constant is the angular frequency of the electron, *not* the angular frequency of our light beam.

Let me put it differently: Feynman's model seems to assume an electron at rest, so to speak, and then the beam drives it so it goes around in a circle with a velocity that is, effectively, given by the angular frequency of the beam itself. In contrast, our flywheel model pushes the analysis a bit further along, because we think of our electron of being a flywheel *always—even at rest*. Having said that, both models raises interesting questions. How and where is the absorbed energy being stored? What is the mechanism here?

In Feynman's analysis, the answer is quite simple: the electron did not have any motion before but does spin around after the beam hit it. So it has more energy now: it was *not* a tiny flywheel before, but it is now! In contrast, in our interpretation of the matter-wave, the electron was spinning around already, so where does the extra energy go now? The intuitive answer is simple: its velocity (*v*), and the radius *r*, should increase as the electron acquires more angular momentum. However, the analysis is, perhaps, not so simple, as we will show below.

2. The radius of an electron

Our flywheel model of an electron suggests that the real and the imaginary part of its wavefunction may be interpreted as two oscillations which each carry half of the total energy of the particle, and that the interplay between these two oscillations describe how energy propagates through space over time. To simplify the analysis, we should probably first consider a particle (think of an electron) at rest. Hence, **p** = **0** and the *elementary* wavefunction reduces to $\psi = a \cdot e^{-i\cdot\theta} = a \cdot e^{-i\cdot E\cdot t/\hbar}$. The E and t in the argument are, of course, the *rest* energy and the *proper* time of the electron and, hence, we should write them as E_0 and t', but let us not complicate the notation here.

> **Note**: The E and **p** in the argument of the wavefunction ($\theta = \omega\cdot t - \mathbf{k}\cdot\mathbf{x} = (E/\hbar)\cdot t - (\mathbf{p}/\hbar)\cdot\mathbf{x} = (E\cdot t - \mathbf{p}\cdot\mathbf{x})/\hbar$) are, of course, the energy and momentum as measured in the reference frame of the observer. Hence, we will want to write these quantities as $E = E_v$ and $p = p_v = p_v\cdot v$. If we then use *natural* time and distance units (hence, the *numerical* value of *c* is equal to 1 and, hence, the (relative) velocity is then measured as a fraction of *c*, with a value between 0 and 1), we can relate the energy and momentum of a moving object to its energy and momentum when at rest using the relativistic transformation formulas: $E_v = \gamma\cdot E_0$ and $p_v = \gamma\cdot m_0\cdot v = \gamma\cdot E_0\cdot v/c^2$. The argument of the wavefunction can then be re-written as: $\theta = [\gamma\cdot E_0/\hbar]\cdot t - [(\gamma\cdot E_0\cdot v/c^2)/\hbar]\cdot x = (E_0/\hbar)\cdot(t - v\cdot x/c^2)\cdot\gamma = (E_0/\hbar)\cdot t'$. The γ in these formulas is, of course, the Lorentz factor, and t' is the *proper* time: $t' = (t - v\cdot x/c^2)/\sqrt{1-v^2/c^2}$. Two essential points should be noted here:
>
> 1. The argument of the wavefunction is invariant. There is a primed time (t') but there is no primed θ (θ'): $\theta = (E_v/\hbar)\cdot t - (p_v/\hbar)\cdot x = (E_0/\hbar)\cdot t'$.
> 2. The $E_0/\hbar$ coefficient pops up as an angular frequency: $E_0/\hbar = \omega_0$. We may refer to it as *the* frequency of the elementary wavefunction.

Hence, the angular velocity of both oscillations, at some point **x**, is given by ω = –E/ħ. Now, the energy of our particle includes all of the energy – kinetic, potential and rest energy – and must, therefore, be equal to E = mc^2. We should now relate this to the m·a^2·ω^2 energy formula for the energy of our V-2 *perpetuum mobile*.

Let us first consider the amplitude of the oscillation (*a*). We equate this factor to the amplitude of the wavefunction, which is why we use the same notation (*a*). Indeed, if the oscillation of the real and imaginary parts of our wavefunction store the energy of our particle, then their amplitude should surely matter. In fact, the energy of an oscillation is, in general, proportional to the *square* of the amplitude: E ∝ a^2, which is consistent with the a^2 factor in the E = m·a^2·ω^2 formula.

Of course, we do have an added complication here: the Uncertainty Principle tells us that an *actual* particle should *not* be represented by the elementary wavefunction. We must build a wave *packet* for that: a sum of wavefunctions, each with their own amplitude a_i, and their own angular frequency ω_i = –E_i/ħ. Each of these wavefunctions will *contribute* some energy to the total energy of the wave packet. To calculate the contribution of each wave to the total, both a_i as well as E_i will matter. However, we do not perceive this as a major constraint at this point of the analysis. Consider the following logic. What is E_i? E_i will vary around some *average* E, which we can associate with some *average* mass m = E/c^2. Hence, the analysis becomes more complicated, but a formula such as the one below might make sense:

$$\mathrm{E} = \sum \mathrm{m}_i \cdot a_i^2 \cdot \omega_i^2 = \sum \frac{\mathrm{E}_i}{c^2} \cdot a_i^2 \cdot \frac{\mathrm{E}_i{}^2}{\hbar^2}$$

We can re-write this as:

$$c^2\hbar^2 = \frac{\sum a_i^2 \cdot \mathrm{E}_i^3}{\mathrm{E}} \Leftrightarrow c^2\hbar^2\mathrm{E} = \sum a_i^2 \cdot \mathrm{E}_i^3$$

We may look at this equation as some sort of *physical* normalization condition when building up the *Fourier sum*.[21] Of course, we should, preferably relate this to the *mathematical* normalization condition for the wavefunction. Our intuition tells us that the probabilities must be related to the energy *densities*, but how exactly? We have dealt with this matter in a previous paper[22] and, hence, will refer the interested reader there, as we want to proceed with the main line of the argument here.

To ground the analysis, we may consider the following. The frequency and *angular* frequency are, obviously, related through the *f* = ω/2π = (E/ħ)/2π = E/h formulas. Alternatively, and perhaps more elucidating, we get the following formula for the *period* of the oscillation: T = 1/*f* = h/E. This is interesting, because **we can look at the period as a *natural* unit of time for our particle**. This period

[21] The value of $c^2\hbar^2$ is about 1×10^{-51} $N^2 \cdot m^4$. Let us also do a dimensional analysis: the physical dimensions of the E = m·a^2·ω^2 equation only make sense if we express m in kg, *a* in m, and ω in *rad*/s. We then get: [E] = kg·m^2/s^2 = (N·s^2/m)·m^2/s^2 = N·m = J. The dimensions of the left- and right-hand side of the physical normalization condition are equal to $N^3 \cdot m^5$.

[22] The Quantum-Mechanical Wavefunction as a Gravitational Wave (http://vixra.org/abs/1709.0390, accessed on 6 December 2017).

is *inversely* proportional to the (rest) energy of the particle, and the constant of proportionality is h. Substituting E_0 for $m_0 \cdot c^2$, we may also say it's *inversely* proportional to the (rest) mass of the particle, with the constant of proportionality equal to h/c^2. The period of an electron, for example, would be equal to about 8×10^{-21} s. That is *very* small, and it only gets smaller for larger objects ! But what does all of this actually *mean*?

At first sight, the analogy between our flywheel model of an electron and the V-twin engine seems to be complete: the 90 degree angle of our V-2 engine makes it possible to perfectly balance the pistons and we may, therefore, think of the flywheel as a (symmetric) rotating mass, whose angular momentum is given by the product of the angular frequency and the moment of inertia: $L = \omega \cdot I$. Of course, the moment of inertia (aka the angular mass) will also depend on the *shape* of the flywheel and the mass distribution. We have two obvious *candidate* formulas here:

1. $I = m \cdot a^2$ for a rotating *point* mass m or, what amounts to the same, for a circular *hoop* of mass m and radius $r = a$.
2. For a rotating (uniformly solid) *disk*, we must add a 1/2 factor: $I = m \cdot a^2/2$.

How can we relate those formulas to the $E = m \cdot a^2 \cdot \omega^2$ formula? The *kinetic* energy that is being stored in a flywheel is equal to $E_{kinetic} = I \cdot \omega^2/2$, so that is only *half* of the $E = m \cdot a^2 \cdot \omega^2$ product if we substitute I for $I = m \cdot a^2$. For a disk, we get a 1/4 factor, so that's even worse.[23] The issue may be cleared by noting that our flywheel model incorporates potential energy too. In fact, we should remind ourselves that the $E = m \cdot a^2 \cdot \omega^2$ formula adds the (kinetic and potential) energy of *the oscillators*. The essence of the metaphor is *not* the flywheel: the flywheel just *transfers* energy from one oscillator to the other. Hence, we should not include it in our energy calculations.

The essence of our model is the two-dimensional oscillation which *drives* the electron, and which is reflected in Einstein's $E = m \cdot c^2$ formula. That two-dimensional oscillation—the $m \cdot a^2 \cdot \omega^2 = m \cdot c^2 \Leftrightarrow a^2 \cdot \omega^2 = c^2$ equation—tells us that **the *resonant* (or *natural*) *frequency* of the fabric of spacetime is given by the speed of light measured in units of *a***, as evidenced by the fact we can re-write the the $a^2 \cdot \omega^2 = c^2$ equation as $\omega = c/a$: the radius of our electron appears as a *natural* distance unit here.

[23] Textbook models usually do not worry about a 1/2 factor in didactic models. For example, when deriving the size of an atom, or the *Rydberg* energy, even Feynman casually writes that "we need not trust our answer [to questions like this] within factors like 2, π, etcetera." We *do* worry about them: factors like 2, 1/2, π or 2π are pretty fundamental numbers, and so they need an explanation. As for Feynman's model of an atom, we suggest the ½ factor may be there because, when thermal motion does *not* come into play, electrons want to pair up: we should think of the Cooper pairs when explaining superconductivity. Likewise, the 1/2 factor in Schrödinger's equation also has weird consequences (when substituting ψ for the elementary wavefunction, and doing the derivatives and deriving the conditions for the left- and right-hand side of the equation to be equal, one gets a weird energy concept: $E = m \cdot v^2$). This problem may also be solved when assuming we are actually calculating orbitals for a *pair* of electrons, rather than orbitals for just one electron only.

Now, if our electron is effectively spinning around, then we can calculate its tangential velocity as being equal to $v = a \cdot \omega = c$.[24] Now, if that is the case, we get a remarkably simple and elegant formula for the (reduced) Compton radius of an electron:

$$c = a \cdot \omega = a \cdot E/\hbar = a \cdot m \cdot c^2/\hbar \Leftrightarrow a = \hbar/(m \cdot c) \approx 3.8616 \times 10^{-13} \text{ m}$$

Now, I promised to answer the following question in the previous section: if the electron is spinning around already in this flywheel model—*even when at rest* —and it absorbs the energy of an incoming beam of light, where does the extra energy go? I already gave the intuitive answer: its velocity (v), and the radius r, should increase as the electron acquires more angular momentum. However, as its energy increases, $\omega = E/\hbar$ must increase. At the same time, the velocity $v = r \cdot \omega$ must still be equal to $v = r \cdot \omega = [\hbar \cdot /(m \cdot c)] \cdot (E/\hbar) = c$. So… If ω increases, but $r \cdot \omega$ must equal the speed of light, then r must actually *decrease*. This is a weird but inevitable conclusion, it seems.[25]

3. An interpretation of the matter-wave

The model, the metaphor and its interesting implications do not answer the fundamental question: what *is* that rotating arrow? We cannot *prove* anything in this regard. We can only advance hypotheses, which may or may not sound reasonable to the reader. Our *hypothesis* is that it is, in effect, a *rotating field vector*. As such, it does resemble the electric field vector of a (circularly polarized) electromagnetic wave. However, our hypothesis also includes major differences. The following assumptions may be highlighted in particular:

1. The (physical) dimension of the field vector of the matter-wave is different. We would like to associate the real and imaginary component of the wavefunction with a force *per unit mass* (as opposed to the force per unit charge dimension of the electric field vector). Of course, the newton/kg dimension reduces to the dimension of acceleration (m/s^2), which is the dimension of a gravitational field.

2. We suggest this gravitational disturbance, so to speak, does cause an electron to move about some center, and we suggest it may do so at the speed of light. In contrast, electromagnetic waves do *not* involve any mass: they're just an oscillating *field*. Nothing more. Nothing less. This interpretation may reconcile the wave-particle duality to some extent. The field vectors interfere but, at the same time, they do drive a pointlike particle, which explains, as Feynman puts it, that "when you do find the electron some place, the entire charge is there." (Feynman, III-21-4)

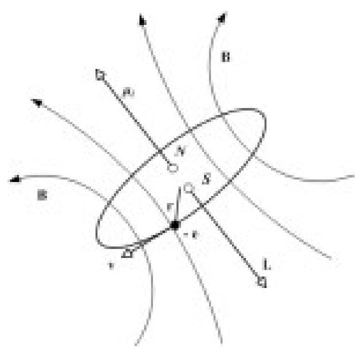
[24] As recent research suggests black holes may be spinning around at speeds approaching the speed of light (see, for example, https://phys.org/news/2014-02-fast-black-holes.html, accessed on 6 December 2017), this is a weird but interesting consequence of the model.

[25] The author is grateful to Dr. Inés Urdaneta for the literature suggestions which may help solve this obvious issue in the interpretation.

3. The third difference is that the *plane* of the oscillation can*not* be perpendicular to the direction of motion of our electron, because then we would not be to explain the direction of its magnetic moment, which is either up or down when traveling through a Stern-Gerlach apparatus (or, in a standardized reference frame, along the *z*-axis).

The latter element of our model deserves some additional remarks. The basic point is the following: the direction of the angular momentum (and the magnetic moment) of an electron—or, to be precise, its component as measured in the direction of the (inhomogeneous) magnetic field through which our electron is *traveling*[26]—can*not* be parallel to the direction of motion. On the contrary, it must be *perpendicular* to the direction of motion. In other words, if we imagine our electron as spinning around some center (see Figure 1), then the disk it circumscribes will *comprise* the direction of motion.

Figure 5: Angular momentum and magnetic moment: the classical view



We need to add another detail here, of course. As the readers will know, we do not really have a precise direction of angular momentum in quantum physics. While there is no fully satisfactory explanation of this, the classical explanation—combined with the quantization hypothesis—goes a long way in explaining this: an object with an angular momentum ***J*** and a magnetic moment ***μ*** that is *not exactly* parallel to some magnetic field **B**[27], will *not* line up: it will *precess*—and, as mentioned, the quantization of angular momentum may well explain the rest.[28]

We already mentioned the major implications of our model in this and our previous paper, and we intend to further explore them in future publications. Hence, to conclude this paper, we will want to address one major theoretical objection to our *physical* interpretation of the wavefunction. It is the following one and, we admit, it is a *major* objection—which is why we want to tackle it head-on[29] : ***at***

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[26] We refer to the beam splitters, or modified Stern-Gerlach apparatuses, that are being used in the thought experiments which are used to derive the transformation coefficients for amplitudes. For more detail, see the introduction to this paper.

[27] As elsewhere, the bold-face notation is used to denote *vector* quantities.

[28] We have detailed our attempts in this regard in various posts on our blog (https://readingfeynman.org/, accessed on 6 December 2017). While these attempts are, admittedly, not *fully satisfactory*, they effectively do go a long way in relating angles to spin numbers.

[29] While the author had acknowledged this undefended outpost in his first paper, he was surprised—positively—by the reactions to it. In fact, the encouragements of one reader prompted him to re-visit the, admittedly, "long and

first sight, our model would not seem to be compatible with the transformation formulas for amplitudes when switching reference frame, or *representations* as they are referred to in quantum mechanics. We wrote this paper to argue it is, and the next, last and final section of this paper will show why.

Before we proceed, however, we would like to mention one more implication of our model. The elementary wavefunction is, apparently, left-handed. Indeed, we write: $\psi = a \cdot e^{-i \cdot \theta} = a \cdot (\cos\theta - i \cdot \sin\theta)$. Now, surely, *Nature* cannot be bothered about our convention of measuring phase angles clockwise or counterclockwise. Also, the angular momentum can be positive or negative: $J = +\hbar/2$ or $-\hbar/2$. Hence, we would probably like to think that an actual spin-1/2 particle (think of an electron once more) may be represented by a wave packet consisting of left-handed as well as right-handed elementary waves. To be precise, we may think they *either* consist of (elementary) left-handed waves or, *else*, of (elementary) right-handed waves. An elementary right-handed wave would be written as:

$$\psi = a \cdot e^{i \cdot \theta} = a \cdot (\cos\theta + i \cdot \sin\theta)$$

Of course, the reader will immediately have the following question: how does that work out with the E·t argument of our wavefunction? Position is position, and direction is direction, but time? Time has only one direction. Right?

Well… Yes and no. It has *one* direction, obviously, but *Nature* surely does not care how we *count* time: counting it like 1, 2, 3, etcetera or like −1, −2, −3, etcetera is just the same. If we count like 1, 2, 3, etcetera, then we write our wavefunction like:

$$\psi = a \cdot \cos(E \cdot t/\hbar) - i \cdot a \cdot \sin(E \cdot t/\hbar)$$ [30]

If we count time like −1, −2, −3, etcetera then we can write it as:

$$\psi = a \cdot \cos(-E \cdot t/\hbar) - i \cdot a \cdot \sin(-E \cdot t/\hbar) = a \cdot \cos(E \cdot t/\hbar) + i \cdot a \cdot \sin(E_0 \cdot t/\hbar)$$

We will leave it to the reader to further think about this. The point is: if we can have left- or right-handed circular polarization of electromagnetic waves, we can have both for the matter-wave too. In fact, this should explain why we can have *either* positive *or* negative quantum-mechanical spin (+ħ/2 or −ħ/2). It is the usual thing: we have two *mathematical* possibilities here, and so we *must* have two *physical* situations that correspond to it.

In what follows, we will write the elementary function as $\psi = a \cdot e^{i \cdot \theta}$.

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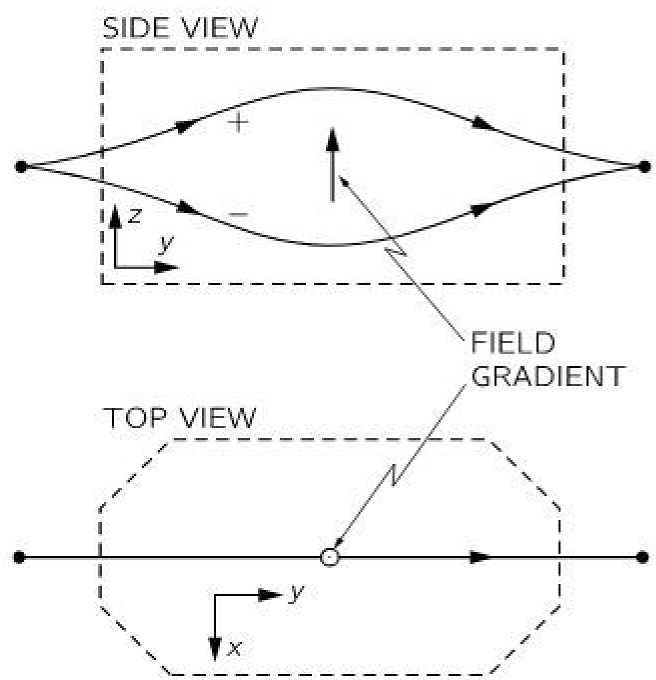
abstract side tour" on transformations in Feynman's *Lecture*, which resulted in this draft (or pre-publication) paper.

[30] E is, once again, the *rest* energy of our particle. Hence this would be the (elementary) wavefunction in the reference frame of the particle itself.

4. Reference frames and transformations

If the *z*-direction is the direction along which we measure the angular momentum (or the magnetic moment) of our electron, then the *up*-direction will be the *positive* z-direction. We will also assume that the *y*-direction is the direction of travel of our particle. Hence, we are, effectively, in the reference frame which Feynman (*Lectures*, III-6) uses to derive the transformation matrices for spin-1/2 particles (or for two-state systems in general). His 'improved' Stern-Gerlach apparatus—which may be referred to as a beam splitter—illustrates this geometry (see Figure 6).

Figure 6: The reference frame of the measurement apparatus



Our flywheel model assumes the magnetic moment—or the angular momentum, really—comes from an oscillatory motion in the *x*– and *y*-directions. To be precise, we imagine an oscillations in two (linear) dimensions simultaneously: one is given by the *real* component of the wavefunction, while the other is given by the imaginary component. When visualizing this, one may think of a polarized wave but one, somehow, needs to imagine that the circular motion is *not* in the *xz*-plane, but in the *yz*-plane.

Now what happens if we change the reference frame, or the *representation* as it is referred to in quantum mechanics? We request the reader to abandon standard definitions for a while and think through the following logic. What do we *mean* by changing the reference frame? What *is* our reference frame? The reference frame is given by the measurement apparatus above, or by our *perspective* of it. Indeed, the apparatus gives us two directions: (1) the *up* direction, so that's the positive direction of the *z*-axis, and (2) the direction of travel of our particle, which coincides with the positive direction of the *y*-axis. We want the reader to think about the *relativity* of these: our *observation* of what *physically* happens here does not give these two directions any *absolute* character, but the reader will have to admit they are more than just some mathematical construct: when everything is said and done, we will have to admit that these two directions are *real*.[31]

What is their *reality*? We request the reader to think through the following. Suppose that we are looking in the positive *y*-direction—so that's the direction in which our particle is moving—then we might imagine how it would look like when *we* would make a 180° turn and look at the situation from the other side, so to speak. We do not change the reference frame (i.e. the *orientation*) of the apparatus here: we just change our *perspective* on it. Now, if we *would* want to change the reference frame because of our changed *perspective* (we are looking at the same thing from the back side, so to speak), we will probably want to keep the *z*-axis as it is—pointing upwards[32]—, and we will also want to re-define the *x*– and *y*-axis using the familiar right-hand rule for defining a coordinate frame, so as to ensure we do not get in trouble when discussing the physics of the situation with a colleague. ☺ Hence, our new *x*-axis and our new *y*-axis will be the same as the old *x*- and *y*-axes but with the sign reversed. In short, we'll have the following mini-transformation[33]: (1) $z' = z$, (2) $x' = -x$, and (3) $y' = -y$.

Hence, if we are effectively looking at something *real* that was moving along the *y*-axis, in the *positive* direction, then it will now still be moving along the *y'*-axis, but in the *negative* direction. Hence, our elementary wavefunction $e^{i\theta} = cos\theta + i \cdot sin\theta$[34] will *transform* into $-cos\theta - i \cdot sin\theta = cos\theta - i \cdot sin\theta$. It describes the same reality. It has to. We just changed our reference frame: we didn't change reality.

Of course, the mainstream physicist will shrug this off as nonsense, because the transformation matrix for an amplitude—and, hence, presumably, for a wavefunction[35]—for a rotation about the *z*-axis is the following one:

Figure 7: Transformation matrix for a rotation about the *z*-axis

$R_z(\phi)$

$\langle jT \mid iS \rangle$	$+S$	$-S$
$+T$	$e^{i\phi/2}$	0
$-T$	0	$e^{-i\phi/2}$

[31] The terms 'relative' and 'absolute' are ambiguous and, hence, we should probably avoid using them. One may object that the term 'real' and its opposite (unreal?) are ambiguous too but… Well… The reader is free to suggest better language.

[32] We might refer to Kant here, but that would be too much of a philosophical digression. The point is: the mathematical idea of a three-dimensional reference frame is grounded in our intuitive notions of up and down, and left and right. In this regard, we advise the skeptical reader to think about the necessity of the various right-hand rules and conventions that we cannot do without in math, and in physics. We elaborated on that in other posts on our physics blog. See, for example https://readingfeynman.org/2017/03/14/symmetries/, accessed on 6 December 2017,

[33] This transformation is *not* a regular one. In fact, its irregularity will explain the point we want to make here.

[34] As mentioned above, we believe the $\psi = a \cdot e^{i \cdot \theta}$, as opposed to the $\psi = a \cdot e^{-i \cdot \theta}$, is an equally valid elementary wavefunction. They represent *two* distinct physical possibilities: spin *up* versus spin *down*, in this case. Hence, the reader may want to double-check the calculations for the $\psi = a \cdot e^{-i \cdot \theta}$ function.

[35] The two concepts are not the same, obviously. We will come back to this in a moment.

Hence, if ϕ is equal to 180° (remember: we walked around the apparatus and, hence, the relevant angle is, effectively, 180°), then these $e^{i\phi/2}$ and $e^{-i\phi/2}/\sqrt{2}$ factors are equal to $e^{i\pi/2} = +i$ and $e^{-i\pi/2} = -i$ respectively. Hence, our $e^{i\theta} = cos\theta + i\cdot sin\theta$ wavefunction now becomes…

Here we need to think about the difference between an amplitude and a wavefunction. Are they two different things? They are and they are not. The $e^{i\theta} = cos\theta + i\cdot sin\theta$ is an *elementary* wavefunction which, we presume, describes some real-life particle—we talked about an electron with its spin in the *up*-direction—while these transformation matrices are to be applied to amplitudes describing… Well… An *up*– or a *down*-state respectively. Hence, for all practical purposes, they are the same thing in this situation. But… Well… If the $e^{i\theta} = cos\theta + i\cdot sin\theta$ wavefunction would describe an *up*-electron, then we still have to apply that $e^{i\phi/2} = e^{i\pi/2} = +i$ factor, right? Hence, we get a new wavefunction that will be equal to $e^{i\phi/2}\cdot e^{i\theta} = e^{i\pi/2}\cdot e^{i\theta} = +i\cdot e^{i\theta} = i\cdot cos\theta + i^2\cdot sin\theta = sin\theta - i\cdot cos\theta$, right?

So how can we reconcile that with the $cos\theta - i\cdot sin\theta$ function we *observe* when walking around the apparatus? The $cos\theta - i\cdot sin\theta$ and the $sin\theta - i\cdot cos\theta$ functions are not *very* different but… Well… They *are* different. *Same-same but different* is *not good enough* here. How can we reconcile them? The answer is: we can*not*. Hence, either *my* theory is wrong or… Well… Feynman—and the mainstream physicsts who shrugged our model off as nonsense—cannot be wrong, can they?

They cannot. The answer is: Feynman, and the mainstream physicist, are not talking about the same situation. *Our electron in the thought experiment does, effectively, make a turn of 180°, so it is going in the other direction now !* That *is* a different situation. It is different than what we did, and that is to just go around the apparatus and look at the situation from the other side.[36]

Hence, the proposed model, and the related physical interpretation of the wavefunction, are not incompatible with the mainstream logic and accepted quantum math which—let us not forget—have been validated through countless experiments.

Let us, to conclude this paper, think about the difference between the $sin\theta - i\cdot cos\theta$ and $cos\theta - i\cdot sin\theta$ functions. First, note that they will give us the same probabilities: the square of the absolute value of both complex numbers is the same. [It's equal to 1 because we didn't bother to put a coefficient in front.] Secondly, we should note that the sine and cosine functions are essentially the same. They just differ by a phase factor: $cos\theta = sin(\theta + \pi/2)$ and $-sin\theta = cos(\theta + \pi/2)$. Hence, we can write the following:

$$sin\theta - i\cdot cos\theta = -cos(\theta + \pi/2) - i\cdot sin(\theta + \pi/2) = -[cos(\theta + \pi/2) + i\cdot sin(\theta + \pi/2)] = -e^{i\cdot(\theta + \pi/2)}$$

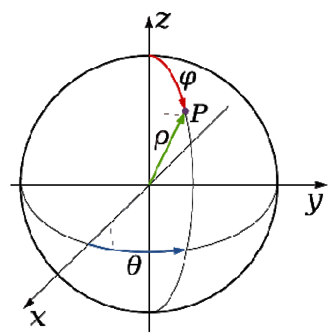
Well… This is nice result. The $e^{-i\cdot\theta} = cos\theta - i\cdot sin\theta$ and $e^{i\cdot(\theta + \pi/2)} = sin\theta - i\cdot cos\theta$ functions differ by (1) a phase shift and (2) a minus sign in front of the argument. Hence, that is, apparently, what it takes to reverse the direction of an electron.

[36] It may take the reader a few moments to understand the argument. The mentioned articles on transformation and symmetries (see, for example, https://readingfeynman.org/2017/03/17/some-more-on-cp-and-cpt-symmetry/, accessed on 6 December 2017) may help.

Conclusions

There are, of course, other ways to look at the matter—literally. For example, we can imagine two-dimensional oscillations as *circular* rather than linear oscillations. Think of a tiny ball, for example, whose center of mass stays where it is (see Figure 8). Any rotation – around any axis – will be some combination of a rotation around the two other axes. Hence, we may want to think of a two-dimensional oscillation as an oscillation of a polar and azimuthal angle.

Figure 8: Two-dimensional *circular* movement



The point of this paper is not to make any definite statements. That would be foolish. Its objective is just to challenge the simplistic mainstream viewpoint on the *reality* of the wavefunction. Stating that it is a mathematical construct only without *physical significance* amounts to saying it has no meaning at all. That is, clearly, a non-sustainable proposition.

The interpretation that is offered here combines the particle and wave character of the matter wave. Its implications are interesting and, so far, no contradictions with mainstream theory have been found. The author would like to thank the readers of his previous article for their constructive comments, and looks forward to receiving more of them.

6 December 2017

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References

This paper explores interpretations and analyzes some key results of mainstream physics only. Hence, most references would be references to standard physics textbooks. For ease of reading, any reference to such material has been limited to a popular textbook that can be consulted online: Feynman's Lectures on Physics (http://www.feynmanlectures.caltech.edu). References are per volume, per chapter and per section. For example, Feynman, III, 19-3 refers to Volume III, Chapter 19, Section 3.

More specific references have been mentioned in the footnotes of this paper. Such references include, but are not limited to:

— Matthew F. Pusey, Jonathan Barrett, Terry Rudolph, *On the Reality of the Quantum State*, in: Nature Physics 8, 475–478, 2012 (https://www.nature.com/articles/nphys2309, accessed on 6 December 2017).

— David Hestenes, *The Zitterbewegung Interpretation of Quantum Mechanics*, in: Found. Physics., Vol. 20, No. 10, (1990) 1213–1232 (https://pdfs.semanticscholar.org/ba8f/fcbacbe33a4819ec065e160a9f014ad9f634.pdf, accessed on 6 December 2017)

In addition, the author has started exploring some of the references suggested by *Dr.* Inés Urdaneta, but these will be probably be mentioned in a later paper, when the author has come to terms (or not) with the conclusions reached in those papers.

All of the illustrations in this paper are open source or have been created by the author.