A proof of the falsity of the axiom of choice.

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Abstract

We observe two things in this paper : namely that the Banach Tarski paradox is false and that the correct part of the proof leads to a violation of the axiomof choice.

1 Proof.

The standard argument behind the Banach Tarski paradox goes as follows; one constructs two rotations a, b around an angle $r2\pi$ with r irrational around the x and z axis respectively. One considers the free group F_2 constructed by a, b which is split into five disjoint parts $S(a), S(a^{-1}), S(b), S(b^{-1}), e$ where $S(a)$ contains all irreducible words starting with the letter a. Clearly, $S(a) \sim S(b)$ geometrically and equally so when inverses are taken. The *axiom of choice* allows one to substract a set M containing one representant of each F_2 orbit on the two sphere. Consider the sets

$$A = S(a)M, B = S(a^{-1})M, C = S(b)M, D = S(b^{-1})M, M$$

and consider the Σ algebra generated by the sets xM where $x \in F_2$. Notice further that $b^n D \subset b^{n+m} D$ for $n, m > 0$ and that $\lim_{n \to \infty} b^n D = S^2$ so that actually $D = S^2$ up to a set of measure zero. However, if D were to miss points then the above formula could not be true and therefore we reach the stronger conclusion that $D = S^2$. This cannot be given that generically for any value of r, there exists a countable number of orbits such that $S(a), S(a^{-1}), S(b), S(b^{-1})$ determine disjoint suborbits. Hence, M does not exist which proves the falsity of the axiom of choice.

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