Proof of Beal's conjecture by deduction

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Abstract

Beal's conjecture solution is identified as an identity. Each term on the LHS of the solution is converted to an improper fraction with the base kept as the denominator. In fractional form, addition of fraction requires like quantities and therefore common denominator. Since the "discreteness" of the terms as numbers in exponential form is preserved upon conversion and we keep the bases as the denominators, the equation will be proven by congruence upon transformation of the terms from exponential form to fractional form.

1. Introduction and conclusion

Beal's conjecture states that if $a^x + b^y = c^z$, where a, b, c, x, y and z are positive integers with x, y, $z > 2$, then a, b, and c have a common factor. The conjecture was made by math enthusiast Daniel Andrew Beal in 1997 [1]. So far it has been a challenge to the public as well as to mathematicians to prove the conjecture and no counterexample has been successfully presented to disprove it.

Identifying Beal's equation $a^x + b^y \equiv c^z$ as an identity allows treating the expression on the LHS as an independent entity. Upon conversion of the terms on the LHS from exponential form to fractional form we deductively prove Beal's conjecture by asserting that the bases of the numbers in exponential form must have common factor in congruence with the bases treated as denominators in fractional form.

By comparing numbers in exponential form to those in fractional form we can see that a fractional basis-unit to a fractional number is as the base to an exponential number, i.e. the fractional unit of $\frac{1}{3}$ to the improper fractional number of $\frac{81}{3}$ is as the exponential unit of base 3 to the exponential number of 3^3 in terms of modularity of the two numbers since they are equal numbers. To add two numbers in fractional form together we must add by proportion. For that we need to express them in terms of a common fractional multiplier by multiplying the two numbers by multiplicative identity made from the LCD, so each fraction must be written as multiple of a common fraction

, i.e. $\frac{5}{3} + \frac{7}{2} = \left(\frac{5}{3} + \frac{7}{2}\right)\frac{6}{6} = 10\left(\frac{1}{6}\right) + 21\left(\frac{1}{6}\right) = 31\left(\frac{1}{6}\right)$. There is then 31 of common fractional units of $\frac{1}{6}$ in the sum, complying with the fractional identity equation of $10\left(\frac{1}{6}\right) + 21\left(\frac{1}{6}\right) = 31\left(\frac{1}{6}\right)$. Here the "fractional multiplicative identity" is $\frac{6}{6}$ to make the terms in the form of $\propto P$ where $\propto$

is an integer and P is the fractional basis-unit. Likewise, we can express a number in exponential form by $\propto P$ where $\propto$ is an integer and P is an exponential basis-unit; the base, i.e. 3^3 has $\propto = 9$ and P is the base 3 and so its counterpart fractional number of $\frac{81}{3} = 9\left(\frac{9}{3}\right)$. In other words, numbers in both forms exponential and fractional are made of an integer number of proportional basis-unit, base and fractional unit respectively. Therefore, adding of exponential numbers can be by proportion as well.

To add by proportion, addition of two fractions needs a multiplicative identity of common number of divisions that make both the numerator and denominator of the identity and addition of numbers in exponential form needs an "exponential multiplicative identity" of common number of divisions in the numerator and denominator as well but expressed in exponential form to yield a basis-unit in exponential form. In the example above, the sum of the two fractions of $\frac{5}{3} + \frac{7}{2}$ needs a fractional multiplicative identity of common number of LCD of 6 divisions on the numerator and on the denominator (a whole number made of 6 divisions, i.e. pizza made of 6 slices). Likewise, to produce a common multiplier in exponential form, addition of $3^3 + 6^3$ to yield 3^5 needs exponential multiplicative identity made of common number of 3 divisions, but 9 of them on the numerator and on the denominator (a whole number made of 9 of the 3 divisions, where 3 is the base i.e. pizza made of 9 of 3 slices with total of 27 slices). The specific number of divisions on the numerator and denominator of the identity is made in exponential form since we are adding exponential numbers. Multiplying the sum $3^3 + 6^3$ by the exponential multiplicative identity of $\frac{3^3}{3^3}$ converts the expression to $1 \cdot 3^3 + 2^3 \cdot 3^3$ with the numerator and denominator of the multiplicative identity is made of the GCD in exponential form. There is then $(1 + 2^3)$ of the common basis-unit of 3^3. The expression becomes $3^2 \cdot 3^3$ and yields the RHS 3^5. This is in line with the fact that Beal's conjecture equation is identified as an identity equation and that the RHS of the equation is a term in exponential form and must be represented as $\propto P$ and that both $\propto$ and P must combine by the power rules. It follows that the two terms on the LHS must have a common exponential factor (CF) of P. The CF is identified by the proper application of multiplicative identity. The CF is explicit in the case of like terms of fractional or exponential form but it is implicit in the case of unlike fractional terms.

The role of the multiplicative identity is to identify a common proportion to add proportionally. The existence of a common factor is the consequence of the crucial process of identifying a common proportion. Discrete mathematics obligates addition by proportion for numbers in exponential form in Beal's conjecture since their structure of positive integer base raised to positive integer power is discrete in nature and therefore can be represented as multiples of common exponential number of $\propto P$. For powers of 2, Fermat's Last Theorem and Fermat-Catalan conjecture are at play.

We can see now how we can identify a common basis-unit of 3^3 when evaluating $(3^3 + 6^3)\frac{3^3}{3^3} = 1 \cdot 3^3 + 2^3 \cdot 3^3$ by multiplying by the proper identity that allows us to add by proportion just as we do with fractions as in the example $\left(\frac{8}{6} + \frac{7}{6}\right)\frac{6}{6} = 8\left(\frac{1}{6}\right) + 7\left(\frac{1}{6}\right)$ that allows us to identify a common fractional basis-unit of $\frac{1}{6}$. In like-basis numbers, such as adding the two exponential numbers $(3^3 + 6^3)\frac{3^3}{3^3}$ or in like-fractional numbers of adding the two fractional numbers $\left(\frac{8}{6} + \frac{7}{6}\right)\frac{6}{6}$, the process of applying a multiplicative identity reveals an explicit common factor. In both cases, adding like or unlike numbers, we see that we need to apply a multiplicative identity to identify the proper proportion when adding by proportion. We conclude that we need the presence of a common exponential number identified by the application of exponential multiplicative identity to add numbers proportionally in exponential form.

We will prove Beal's conjecture deductively by converting exponential numbers to their respective fractional numbers of the conjecture's solution and asserting that in fractional form the LHS of the conjecture solution needs a common fractional factor and by congruence property, the solution in exponential form must have a common factor as well but in exponential form. We seek an exponential common factor that identifies the proper number of divisions of the exponential multiplicative identity for the sum on the LHS of Beal's conjecture solution. In the example above, the number of divisions of the number on the numerator as well as on the denominator of the exponential identity is 9 of the base 3 that makes the GCD 3^3. Since we have a common factor in exponential form, the base of the factor is common to both terms on the LHS of Beal's conjecture equation as well as with the sum on the RHS since we identified the equation as identity equation.

2. Modularity and the discreteness value of numbers

Discreteness value of numbers is defined here as the unit-value, fractional or exponential, that repeats and defines the number. In the case of exponential numbers and fractional numbers that are converted from exponential numbers, the numbers must evaluate to 0 mod (discreteness value).

2.1. Numbers in exponential form and its unique discreteness

Any number in exponential form is made of multiples of its base, i.e., the number 2^{374} can be made of multiples of only the prime number 2 with no remainder, i.e. cannot be made of multiples of 3 or 5 with no remainder; the number 3^{473} can be made of multiples of the prime number 3 and the number 6^{743} can be made of multiples of the base 6 or multiples of either the prime number 3 or 2, and therefore any number in exponential form can be represented by αP, where α is a positive integer and P is a unique prime number as described by the fundamental theorem of Arithmetic which asserts that the prime divisors of a given integer are determined uniquely; meaning, $P_1^{k_1}\ldots.P_n^{k_n}$ is divisible by $P_1\ldots.\ P_n$, where P is a prime number that represents the discreteness variable that repeats to construct the whole number without remainder; $\alpha P \equiv 0\ mod\ P$. Therefore, adding two numbers in exponential form and single power like that in Beal's conjecture equation, the equation maybe represented as $\alpha P_1 + \gamma P_2 = \delta P_3$, where P_1, P_2 and P_3 are prime numbers and each term must be in exponential form. Since adding by proportion is a valid process for addition of fractional numbers and since numbers in exponential form can be converted to fractional numbers on the basis of their modularity, adding by proportion is a valid process for adding numbers in exponential form as well.

2.2. Numbers in fractional form

Any number in fractional form is made of multiples of its "basis-unit", e.g. $3\left(\frac{1}{4}\right)$ is made of 3 multiples of the basis-unit of $\frac{1}{4}$. Therefore any fractional number can be represented by αP, where α is a positive integer and P is a common multiplier; a fractional basis-unit of the number.

2.3. Equal numbers

Numbers are equal if they have the same value and therefore the same discreteness regardless of the form they are expressed in. e.g. the number in exponential form 2^4 is made of 8 multiples of the prime number 2 while its counterpart in fractional form $8\left(\frac{4}{2}\right)$ has the same discreteness of 2

as well; that is 8 multiples of 2. The two numbers 2^4 and $\frac{32}{2}$ evaluate to 0 mod 2 and have the same size (value) of 16. In other words they have congruence properties upon transformation from one form to another since both have the same discreteness of the basis-unit of 2 that repeats to construct the number without remainder.

2.4. Conversion of numbers

Since any number in exponential form and single power is made of multiples of its base, it can be converted to improper fraction keeping the base as the denominator. If we divide and multiply the number in exponential form by its base we produce the following conversion formula,

(2.4.1) $$x^l = \frac{x^{l+1}}{x} = x^{l-1}\left(\frac{x^2}{x}\right)$$

The base x of the exponential number x^l is kept as the multiplicative inverse of the discreteness variable in fractional form upon transformation. Therefore the denominator of the fractional number here is representative of the discreteness of the number. An example is $2^4 = \frac{2^5}{2} = 2^3\left(\frac{4}{2}\right)$. The term $8\left(\frac{4}{2}\right)$ is in the form of αP. The two numbers in exponential and fractional form are equivalent and with discreteness value of 2.

2.5. Unique discreteness value of numbers in fractional form

The conversion process from exponential form to improper fractional form expressed as multiples of the same discreteness value is the key to ensure that the common factor of their discreteness value is unique. In the example of $2^4 = 32\left(\frac{1}{2}\right) = 8\left(\frac{4}{2}\right)$, the coefficient 8 is multiples of the discreteness value of the prime number 2, since 8 is 2^3 of base 2, complying with the conversion formula of $x^{l-1}\left(\frac{x^2}{x}\right)$.

2.6. Discreteness implies congruence

If two numbers have the same congruence with respect to a common mod (integer), the integer is the common factor.

Lemma 2.6.1 *Let $a, b, c \in \mathbb{Z}^+$. Let a, b be in exponential form and fractional form respectively. If $a \equiv b \equiv 0 \; mod(c)$, then a, b have a common factor of c.*

Proof c is a common factor of a and b by modularity property of mod c.

3. Addition of numbers and the proof of Beal's conjecture

The purpose of conversion of numbers from exponential form to improper fractional form keeping the base as the denominator of the fractional number is to reflect the constraints projected by the denominators when adding numbers in fractional form onto their corresponding bases when adding numbers in exponential form.

3.1. Addition of numbers in fractional form: adding by proportion

To add proper or improper fractions containing unlike quantities it is required to add by proportion and convert all amounts to like quantities (same proportion) to proceed with the addition process.

Example 3.1.1 To evaluate the expression of the sum of whole numbers of $2 + 3$ expressed as unlike improper fractions of $\frac{8}{4} + \frac{9}{3}$, we follow rules of addition of fractions. We need to multiply the whole expression by identity made from the LCD $\left(\frac{12}{12}\right)$ since the two fractions $\frac{8}{4}$ and $\frac{9}{3}$ are made of different fractional-basis of $\frac{1}{4}$ and $\frac{1}{3}$ respectively, i.e., $8\left(\frac{1}{4}\right)$ is made of 8 of fractional-basis of $\frac{1}{4}$ and $9\left(\frac{1}{3}\right)$ is made of 9 of fractional-basis of $\frac{1}{3}$. The expression becomes $\left(\frac{8}{4} + \frac{9}{3}\right)\frac{12}{12}$ and simplifies to $24\left(\frac{1}{12}\right) + 36\left(\frac{1}{12}\right)$, respective to the terms in the expression. Proportionally, the first fraction in the expression has 24 of the common fractional-basis proportion of $\frac{1}{12}$ while the other one has 36. Notice that the identity of $\frac{12}{12}$ does not change the value of the fractional numbers in the expression but it only changes the fractions' forms.

To add numbers in fractional form then we need to make them in the form αx where the coefficient α is positive integer that corresponds to the common fraction proportion x that repeats to form the sum. Adding proportions then, the fractional expression above ads to 60 multiples of the proportion of $\frac{1}{12}$ necessitating that both fractions in the expression must be made of multiples of the

proportional quantity $\frac{1}{12}$ for a successful addition. The fractional solution $24\left(\frac{1}{12}\right) + 36\left(\frac{1}{12}\right) = 60\left(\frac{1}{12}\right)$ then requires a common factor of the proportion $\frac{1}{12}$ in fractional form aided by multiplication rules of fractions. Any such equation forms an identity by simply adding fractions proportionally. Such an identity may constitute an identity whose terms are in exponential form if all of the three terms can be converted to exponential form if they are whole numbers, since a number in exponential form is a whole number, keeping the discreteness variable which ensures congruence between the terms in the two identities in two distinct forms, fractional and exponential. Therefore, whole numbers expressed as improper fractional numbers may be added together by the method of common proportions. Since numbers in exponential form are whole numbers, the method of addition by common proportion may be employed.

Example 3.1.2 The number 81 cannot be added to the number 16 by the method of discrete common proportion since both have different discreteness value; $81 \equiv 0\ mod(3)$ while $16 \equiv 0\ mod(2)$, but the two numbers of 81 and 243 can be added by addition by proportion since we can take the common discreteness value of the prime number of 3 as a common factor; $81 + 243 = 27(3) + 81(3) = 108(3)$. This represents an identity solution but unfortunately does not comply with Beal's conjecture since the sum on the RHS cannot be made in exponential form.

3.2. Addition of numbers in exponential form

If the whole numbers of 2 and 3 that makes the expression $2 + 3$ in example 1 were numbers made in exponential form, the addition by proportion applies, i.e. $27 + 216 = 3^3 + 6^3$.

The construct of numbers in exponential form requires a repetition of a base to form a whole number since it must evaluate to 0 modulus the base and it can be represented as $\propto (P)$. Since the conversion process of numbers in exponential form to numbers in fractional form retains the base P of the number in exponential form as the denominator in fractional form, which requires a common factor upon addition, deductively we conclude that a common factor is required for addition of numbers in exponential form as well. When adding whole numbers in exponential form of mod (base) that evaluate to 0, the route through conversion of the numbers to fractional form with the same modularity criteria first is unavoidable since adding fractions by proportion is an intermediate step that preserves the numbers' modularity. The intermediate step is important since

when whole numbers in exponential form are added together we must ensure that the sum is also in exponential form and must follow rules of exponentiation and must evaluate to 0 mod (base) as well. In example 2, only the first term of the equation $27(3) + 9(3) = 36(3)$ can be expressed in exponential form. For any Beal's equation solutions, all of the three terms must be made in exponential form of single power.

4. Proof of Beal's conjecture

Beal's conjecture equation can naturally be proved by recognizing it as identity and adding by proportion by applying an exponential multiplicative identity over the LHS terms and simplifying to get the RHS. As explained in the introduction, we multiply the expression on the LHS by the proper exponential multiplicative identity to find its modularity factor that defines the common discreteness value of its terms in congruence with addition of the terms in fractional form. To prove the conjecture we will convert the expression on LHS of the equation from exponential form to fractional form. Since the expression is equivalent in both forms we will prove the conjecture by proving that if the sum of the expression must have a common factor in fractional, so it must be in the exponential form.

Theorem 4.1. *Let x,y,z,l,k,n* $\in \mathbb{Z}^+; l, k, n > 2$ *;*$x^l + y^k = z^n$. *Then x,y,z have a common factor.*

Proof. Each term of the expression on the LHS of Beal's equation $x^l + y^k = z^n$ can be converted to fractional form as,

$$\frac{x^{l+1}}{x} + \frac{y^{k+1}}{y}$$

By Lemma 2.6.1, the first term in fractional form must share a common factor x with the first term in exponential form $\left(\frac{x^{l+1}}{x}, x^l\right)$ and the second term must share y with the corresponding second term $\left(\frac{y^{k+1}}{y}, y^k\right)$ since they both are the same number expressed in different forms. Since the two terms $\left(\frac{x^{l+1}}{x}, \frac{y^{k+1}}{y}\right)$ must share a common factor by the rules of addition of fractions, so must the two terms (x^l, y^k) in exponential form by the equivalence properties of congruence. ■

Furthermore, Beal's equation can be expressed as,

(4.1) $$\frac{x^{l+1}}{x} + \frac{y^{k+1}}{y} = z^n$$

Recognizing the equation as identity, the denominators *x, y* must have a common factor by Theorem 4.1 to yield the RHS by the rules of addition of fractions. Let $x = \alpha n$ and $y = \alpha s$, the expression on the LHS becomes,

$$\frac{(\alpha n)^{l+1}}{\alpha n} + \frac{(\alpha s)^{k+1}}{\alpha s}$$

Where $n, s \in \mathbb{Z}^+$. The common factor α crosses to the bases in the numerators. Simplifying, the expression above becomes

$$(\alpha n)^l + (\alpha s)^k,$$

Let $k = f + l$, the expression becomes,

(4.2) $$(n^l + \alpha^f s^{f+l})\alpha^l$$

The coefficient $(n^l + \alpha^f s^{f+l})$ must combine with the exponential multiplicative factor α^l by power rules and must yield the number in exponential form on the RHS of equation (4.1). Therefore addition of fractions as well as addition of numbers in exponential form must be made only by relative gain against a common proportion (discreteness value) to form an identity.

Equation (4.2) ensures that Beal's conjecture is true for powers > 2 when α^l has powers $l > 2$. This concludes the proof that all three terms of equation (4.1) must have a common factor of α^l to constitute an identity, hereby proving Beal's conjecture by congruence upon transformation from exponential form to fractional form.

Equation (4.2) is the representative equation to solve for the RHS of any solution of Beal's conjecture equation, which explicitly says that to add two whole numbers in exponential form to yield a number in exponential form we must add in proportion like we do with fractions. The common proportion of α^l is the GCD obtained from the exponential multiplicative identity of $\frac{\alpha^l}{\alpha^l}$ in equation (4.2). To solve Beal's equation then, the theme is to employ equation (4.2) to solve for the RHS on the basis of the identity of the equation. We need to first factor α^l from the equation;

the GCD of the two terms on the LHS. By the power rules we then need to combine it with the resulting coefficient $(n^l + \alpha^f s^{f+l})$ to obtain the RHS.

5. Examples of identity addition of Beal's conjecture solutions

Example 5.1 To add $3^3 + 6^3$ to yield 3^5 as an identity, let's convert the two terms on the LHS into fractions keeping the bases as the denominators in the fractional form. The expression becomes $3^4\left(\frac{1}{3}\right) + 6^4\left(\frac{1}{6}\right)$. We convert the second term to like proportion of $\frac{1}{3}$. The expression becomes $81\left(\frac{1}{3}\right) + \frac{6^4}{2}\left(\frac{1}{3}\right) = 81\left(\frac{1}{3}\right) + 648\left(\frac{1}{3}\right) = 729\left(\frac{1}{3}\right) = 3^5$. Notice that we have used the discreteness value of 3 as the denominator as well as the base as described in the introduction. Alternatively, following equation (4.2) and factoring the GCD of α^l as 3^3, the expression on the LHS becomes $(1 + 2^3)\, 3^3$. Simplifying yields $3^2 \cdot 3^3$, which yields the RHS. The equation as an identity simply says that, if 1 proportion of the common proportion α^l of 3^3 obtained from applying the exponential multiplicative identity $\frac{3^3}{3^3}$ is added to 2^3 proportions of the common proportion of 3^3, the sum yields 3^2 proportions of the common proportion 3^3 according to the identity equation $1 \cdot 3^3 + 2^3 \cdot 3^3 = 3^2 \cdot 3^3 = 3^5$.

Remark The fact that we can convert exponential form to fractional form and vice versa of the two numbers on the LHS of Beal's conjecture solution keeping the base as the denominator, it follows that the addition rules of the two numbers in fractional form must be maintained and a common factor of the terms' denominators necessarily obligates the same common factor of the terms' bases in exponential form since we maintained the bases to remain as the denominators upon conversion of the two numbers.

Example 5.2 The equation $70^3 + 105^3 = 35^4$ complies with Beal's conjecture. Multiplying the expression $70^3 + 105^3$ by the identity $\frac{35^3}{35^3}$ we obtain $2^3 \cdot 35^3 + 3^3 \cdot 35^3$ and by factoring the GCD and simplifying we obtain the RHS 35^4. This is only possible because with the expression $2^3 \cdot 35^3 + 3^3 \cdot 35^3$ we can add by proportion according to equation (4.2) while for the coefficients' expression $2^3 + 3^3$ we cannot add by proportion, complying with Theorem 4.1 that adding by proportion of exponential numbers is a must.

Example 5.3 For the equation $7^6 + 7^7 = 98^3$, employing equation (4.2) one possible α^l is 7^3. The sum of the coefficient terms yields 2744 which can be shaped to 14^3 by taking the third root, which produces the RHS of the equation upon combining the terms by the power rule of the product of two numbers having the same power. If we factor out the GCD of 7^6 from the LHS of the equation, the expression becomes $(1+7)7^6$ and can further be expressed as $2^3 \cdot 7^6$. To combine the resulting two terms we divide the power of the second term by 2, to make use of one of the methods to combine two numbers of same power, and square its base to get $2^3 \cdot 49^3$. This expression yields the same term on the RHS of the equation. The three terms then share a GCD of 7^6 and therefore a common prime of 7 as Beal's conjecture requires. The GCD of 7^6 defines the multiplicative identity of $\frac{7^6}{7^6}$.

Example 5.4 For the equation $34^5 + 51^4 = 85^4$, an obvious prime common factor of the bases of the two numbers on the LHS of the equation is 17. Multiplying by the identity $\frac{17^4}{17^4}$ we obtain the sum as $544 \cdot 17^4 + 3^4 \cdot 17^4$. The two numbers are then reduced to $(625)17^4$ by factoring the GCD 17^4. The expression can be reduced to $5^4 \cdot 17^4$ and yields the RHS. We can see that the coefficients' sum was $17 \cdot 2^5 + 3^4$ with the first term expressed in non-exponential form. The CF transformed the first term into exponential form. We conclude that the exponential number 17^4 is the GCD of the terms on LHS of the equation and 17 is a common factor of their bases as Beal's conjecture implies. There are then 625 of α^l of 17^4 that makes up the number 85^4. The prime base-unit of all three terms is then the prime number 17 and all terms must evaluate to 0 modulus 17.

Example 5.5 Adding by proportion, we employ equation (4.2) on the LHS of the equation $760^3 + 456^3 = 152^4$. The equation can be broken down to the base's primes and becomes $5^3 \cdot 2^9.19^3 + 3^3 \cdot 2^9.19^3$. The two terms now can be combined to yield $(3^3 + 5^3)\, 2^9.19^3$, and by shaping 2^9 to 8^3 the expression becomes $(3^3 + 5^3)\, 8^3.19^3$ with a GCD of 152^3 to yield $152 \cdot 152^3$, which yields the RHS of the equation. The identity $\frac{152^3}{152^3}$ has been used to identify the common multiplier of the GCD so we can add by proportion. There are then 152 of α^l of 152^3 that makes up the term 152^4 with α^l necessarily makes up all three terms of the solution; where there is 125 of α^l that

makes up the 760^3 term and there is 27 of them that make up the term 456^3. The common base-factor of all three terms is 152 which have two distinct prime base-units made of the 2 and 19.

Example 5.6 Let's consider the Beal's conjecture solution of $27^4 + 162^3 = 9^7$. By factoring α^l of 27^4 we get $(1+8)27^4$, which becomes $3^2 \cdot 3^{12}$ and produces a final result of 3^{14}, which can be shaped to produce 9^7, the RHS of the equation. It is important to make sure that the sum-term on the RHS of the equation has not been shaped differently before we judge whether the resulting equation is identical to the given one.

Example 5.7 Another example to beware of the end result as deemed different is the equation $33^5 + 66^5 = 1089^3$. By following equation (4.2) the sum of the two terms on the LHS of the equation is 33^6, which can easily be shaped to 1089^3. The same goes with the equation $8^3 + 8^3 = 4^5$. By following equation (4.2) we get the sum as 2^{10} or 32^2 which can be shaped to 4^5. The last example simply can be simplified by shaping the terms as $2^9 + 2^9 = 2^{10}$, which simplifies as $2 \cdot 2^9 = 2^{10}$. It is obvious here that both terms on the LHS have the same prime base-unit of 2.

Example 5.8 By factoring α^l of 19^3 from the LHS of the equation $19^4 + 38^3 = 57^3$ we obtain $(19+8)\ 19^3$. Simplifying we get $27 \cdot 19^3$ which by shaping 27 becomes $3^3 \cdot 19^3$ and yields the RHS of the equation. It is clear that the primitive solution of the equation of $19 + 8 = 27$ can be shaped to $19 + 2^3 = 3^3$ with α^l of 19^3 used to upgrade the primitive equation to higher powers to comply with Beal's conjecture.

Example 5.9 By factoring out α^l of 80^{12} from the LHS of the equation $80^{12} + 80^{13} = 1536000^4$ we obtain $(1+80)\ 80^{12}$. Obviously the LHS of the primitive equation involved here of $1 + 80 = 3^4$ cannot be shaped to higher powers to comply with the conditions of Beal's conjecture. Only when its terms are changed by multiplying by the proper CF of α^l. Simplifying we get $81 \cdot 80^{12}$ which becomes$3^4 \cdot 80^{12}$, and by shaping 80^{12} as 512000^4 we get the RHS.

Example 5.10 By factoring out the GCD of 28^3 from the LHS of the equation $84^3 + 28^3 = 28^4$ we obtain $(27+1)\ 28^3$. Simplifying, we get $28 \cdot 28^3$ which becomes the RHS.

Example 5.11 By factoring out the GCD of $1838^3$from the LHS of the equation $1838^3 + 97414^3 = 5514^4$ we obtain $(1+148877)\ 1838^3$. By borrowing 1838 factor from the coefficient

term and simplifying we get $81 \cdot 1838^4$. The 81 can be shaped to 3^4 and the product yields the RHS.

Remark we have successfully evaluated every solution of Beal's equation we have stumbled upon using the logic of adding by proportion and employing equation (4.2), considering all equations as identity ones.