Proof of Beal conjecture with illustrative examples

Kamal Barghout

PMU

kbarghout@pmu.edu.sa

ABSTRACT

We represent each term of Beal's conjecture equation as a number in exponential form with unique prime base-unit as its building block and therefore unique discreteness. It will be proven that any two numbers in exponential form to be added together must have a common prime base-unit due to their discreteness property. We represent the solution of Beal's conjecture equation as an identity that produces the sum of two monomials of common indeterminate. The monomial on the RHS of Beal's equation can be built from the expression on the LHS. Upon factorization of the GCD of the two monomials on the LHS of Beal's conjecture solution it must be combined with the sum of the two coefficients of the terms to yield the monomial on the RHS by the power rules based on the identity solution of the equation.

1.1 Introduction and conclusion

Beal's conjecture states that if $a^x + b^y = c^z$, where a, b, c, x, y and z are positive integers with x, y, $z > 2$, then a, b, and c have a common factor. The conjecture was made by math enthusiast Daniel Andrew Beal in 1997 [1]. So far it has been a challenge to the public as well as to mathematicians to prove the conjecture and no counterexample has been successfully presented to disprove it.

Identifying Beal's equation $a^x + b^y \equiv c^z$ as an identity and the expression on the LHS as the sum of two monomials of common variable allows treating the expression as an independent entity. Taking the GCD of the two monomials on LHS and following exponentiation rules, the summation operation of the two LHS terms may be converted to multiplication to unite the two terms into one that equals the term on the RHS. The LHS terms then necessarily must have a common factor for the conversion process to be successful.

Let's first deductively prove why a successful addition of any two numbers in exponential form must have a common factor. Each monomial term on the LHS of Beal's conjecture solution represents a number in exponential form defined here as a "block-number" with prime base-unit and of modularity evaluated to 0, i.e., Block numbers of mod (prime base-unit) evaluate to 0. In other words, any block-number can only discretely be made of multiples of its prime base-unit, i.e., the block-number 2^{374} can only be made of multiples of 2; the block-number of 3^{473} can only be made of multiples of 3 and the block-number 6^{743} can only be made of multiples of either 3 or 2, and therefore any block-number can be represented by αP, where α is a positive integer and P is a prime number, supported by the fundamental theorem of Arithmetic which asserts that the prime divisors of a given integer are determined uniquely, meaning that $P_1^{k_1} \ldots . P_n^{k_n}$ is only divisible by $P_1 \ldots .\ P_n$. In the case of an integer of the form $P_l^k P_m^n$, then to combine the two terms into a block-number we must follow exponentiation rules. If $l = m$ then the block-number has a unique prime that divides it. If $k = n$ then both primes P_l and P_m uniquely divide the block-number, e.g., $6^{743} = 3^{743} \cdot 2^{743}$.

To add the two block-numbers on the LHS of Beal's solution together, their sum must add to construct the RHS block-number based on multiples of its prime base-units. But let's see why there must be a common prime base-unit for the addition process to be successful. To see that,

we must consider the discreteness constraint in the addition process of proportional block-numbers analogous to the addition process of fractions that have unlike integers in the denominator. To evaluate the expression $\frac{3}{4}+\frac{4}{3}$ we need to multiply the whole expression by unity made of the LCF of the denominators $\left(\frac{12}{12}\right)$ of the two terms in the expression since the two fractions $\frac{3}{4}$ and $\frac{4}{3}$ are made of different fraction-basis of $\frac{1}{4}$ and $\frac{1}{3}$ respectively, i.e., $3\left(\frac{1}{4}\right)$ is made of 3 of fraction-basis of $\frac{1}{4}$ and $4\left(\frac{1}{3}\right)$ is made of 4 of fraction-basis of $\frac{1}{3}$. The expression becomes $\left(\frac{3}{4}+\frac{4}{3}\right)\frac{12}{12}$ and simplifies to $9\left(\frac{1}{12}\right)+16\left(\frac{1}{12}\right)$, respective to the terms in the expression. Proportionally, the first fraction in the expression has 9 of the common fraction-basis proportion of $\frac{1}{12}$ while the other one has 16. Notice that the unity of $\frac{12}{12}$ does not change the value of the fractions in the expression but it only changes the fractions' forms. To add numbers in fraction form then we need to make them in the form αx where the coefficient α is an a positive integer that corresponds to the common fraction proportion x that repeats to form the sum. Adding proportions then, the fraction expression above ads to 25 multiples of the proportion of $\frac{1}{12}$ necessitating that both fractions in the expression must be made of multiples of the proportional quantity $\frac{1}{12}$ for a successful addition, a specific common proportion, with their sum of $25\left(\frac{1}{12}\right)$. The fraction solution $9\left(\frac{1}{12}\right)+16\left(\frac{1}{12}\right)=25\left(\frac{1}{12}\right)$ then requires a common factor of the proportion $\frac{1}{12}$ in fraction form aided by multiplication rules of fractions. Any such equation forms then an identity by simply adding fractions proportionally. Following the same reasoning of addition of discrete proportional numbers, addition of numbers in exponential form, which are identified as discrete numbers of multiples of their prime base-units, needs a common proportion in exponential form as well that repeats to construct the numbers being added together, i.e., the block-number of 3^5 cannot be added to 2^4 because they have different proportional basis of base-units of 2 and 3 and a common proportion in exponential form must be present for a successful addition to yield a number in exponential form. This proportion is the GCD of the two monomials on the LHS of the solution of Beal's equation. In a block-number, the prime-proportion that repeats is the prime base-unit that forms the GCD, which necessitates a common prime base-unit for addition of block-numbers. The following is an example,

Example 1: If we wish to build higher powers of the primitive equation $1 + 2^3 = 3^2$, we may employ a proper CF x^m with higher powers of $m > 0$, e.g. 3^3. This common factor represents the proportion upon which the exponential term on the RHS builds its proportional structure as discussed above. The two terms on the LHS become $(1 + 2^3)3^3$ and can be simplified to $3^2 \cdot 3^3$ and further to 3^5 to yield the identity equation $3^3 + 6^3 = 3^5$ as implied by Beal's conjecture. The new equation simply says that, if 1 proportion of the common proportion of 3^3 is added to 2^3 proportions of the common proportion of 3^3, the sum yields 3^2 proportions of the common proportion 3^3 according to the identity equation $1 \cdot 3^3 + 2^3 \cdot 3^3 = 3^2 \cdot 3^3$. The original equation of $1 + 2^3 = 3^2$ is primitive with a common proportion of 1. Further, since each term of any equation whose terms are block-numbers is made of multiples of prime base-units, we can effectively write it proportionally as a linear relation of $\alpha P_1 + \beta P_2 = \gamma P_3$, where α, β and γ are positive integers and P_1, P_2 and P_3 are prime numbers that must be added only through proportional discreteness approach as discussed above necessitating a common factor and a common prime number. Therefore addition of fractions as well as addition of numbers in exponential form must be made only by relative gain against a common proportion to form an identity. Further, by Bezout's identity, the LHS of the equation should equal 1 if P_1 and P_2 are relatively prime, otherwise the RHS may constitute a GCD different than 1 as clear from the identity as in the case of Beal's conjecture solution. Further, by the addition property of modular arithmetic, we have $(A + B)\ mod\ C = (A\ mod\ C + B\ mod\ C)\ mod\ C$. Since each term in Beal's conjecture equation is a block-number and must evaluate to 0 mod (its own base-unit), it follows that the prime base-unit of C of the term on the RHS of the conjecture's equation must be one of the prime base-units of both of the terms A and B in $(A\ mod\ C + B\ mod\ C)\ mod\ C$ to evaluate the LHS of the equation to 0 of mod C since the equation is identified as identity. For example, $5^3 + 2^3 \neq 3^4$ because neither of the terms on LHS evaluates to 0 mod 3 since 5^3 evaluates to 0 only for mod 5 and 2^3 evaluates to 0 only for mod 2 by the definition of block-numbers, but not necessarily so unless the equation is an identity as the case in Beal's conjecture. This proves that all three block-terms of Beal's conjecture equation must share C as a prime base-unit since the equation is identified as an identity.

The common factor (proportion) allows us to build higher power equations over the positive integers and imposes a rule that all three terms of Beal's equation solution must build their

structure on the common proportion of exponential form of the prime base-unit concept introduced by the GCD. It is claimed here that unless the two terms on the LHS of the higher power equation have a common bundle of elements that is represented by the GCD, it is configurationally impossible to add them together to build the term on the RHS and comply with the constraints laid by Beal conjecture that the terms in Beal's solutions must be in exponential form of single power and unique repeating prime base-unit and hence based on proportional discreteness of the number. This description of the terms of Beal's equation solution is allowable only because the two terms on the LHS are described as numbers in exponential form of single power. Since the values of the coefficients of the monomials have a special relationship with the values of the monomials' indeterminate, it is feasible to represent the sum according to this relationship which remains elementary and follows the rules of exponentiation. Taking the GCD x^m of the two monomial terms on the LHS of Beal's conjecture equation, the monomial expression can be represented as $\alpha x^m + \beta x^m$. By factoring out the GCD the expression becomes $(\alpha + \beta)\, x^m$, where m is positive integer and the coefficients α, β and $(\alpha + \beta)$ are factors such that if multiplied by the GCD of the monomial variable term x^m of exponential form, it results in a product of two terms in exponential form which can be combined by the rules of exponentiation. Only when the two terms of the sum of $(\alpha + \beta)$ and the GCD term of x^m can be manipulated such that they either share the same power or the same base they can combine into a number expressed in exponential form of single power, representing the RHS of Beal's solution. The success of integrating the GCD in all three terms implies that the CF provides the "base-unit" of the basic building block upon which all three terms of Beal's solution build their exponential structure.

The solution in example 1 is an identity of the form $\delta x^m = \delta x^m$, where the two monomials that make the sum are the two terms on the LHS of Beal's solution that share the same variable term x^m and is made of multiples of the prime base-unit 3, where all terms of the solution, $3^3, 6^3$, 3^5 mod 3 must evaluate to 0. The common term x^m must be chosen such that it combines with the three terms by the use of exponential rules to ensure that all three terms are constructed from the same prime base-unit (common proportion). In this example, m can be chosen as 3 and x must be any number from the geometric sequence of 3, 9, 27, 81, to allow the term to be reshaped to combine with both 3^2 and 2^3. Any value of x must abide with the rules of

exponentiation depending on the values of all three coefficients of the equation. In the case of the three terms of the equation having the same exponent, the identity has then infinite number of solutions corresponding to the positive integer values of x and m. This is clear in an identity equation of the form of that of Fermat's Last Theorem, where m must be 2 and x is non-negative integer. It is to be noted that the sum of the coefficients of $(1 + 2^3)$ on the LHS has terms that are elements in the Diophantine quadruple of $\{1,3,8,120\}$ which allows the sum to produce a square that perfectly combines with the chosen CF.

In this article, with a deeper understanding of the mechanism of how numbers in exponential form, **designated here as block-numbers**, behave and the fact that they must be of discrete nature and uniquely constructed from a basic building block of their prime base-unit, and representing the LHS of Beal's conjecture as the sum of two monomials of the same variable, we will prove Beal's conjecture. To be specific, any number in exponential form (block-number) must be built from a specific base-unit made of a prime number, much like a Lego system. All block-numbers then fall in distinct families (groups) of same base-units. In example 1, the base-unit of 2^3 is the prime number 2 and the base-unit of 3^2 is the prime number 3. Any number must be made of a base-unit that repeats. If the number is just the prime number or a composite that is not expressible as a block-number, then the base-unit is the number itself, i.e., 7^1, 349^1.., but the number 42^3 has three unique base-units of 2, 3 and 7, each one can serve as the prime base-unit that repeats to make an exponential number that evaluates to 0 mod (the base unit).

To build block-numbers with higher powers in example 1, which must have prime base-units, from the two terms on the LHS of the primitive equation, the two terms of $1 + 2^3$ must add up to a coefficient of specific number of bundles (proportions), a bundle of base-units that makes a block-number of the GCD, corresponding to those on the RHS and must combine with the GCD by the exponential rules; in the example, the base-unit is the prime number 3 since the sum yields the number 9. There is then the same number of base-units of 3 on the RHS of the equation as those on the LHS of total of 81 of them for the higher power block-numbers of the solution $3^3 + 6^3 = 3^5$. It then all depends on the sum of any primitive two terms to add up to a block-number of specific number of prime base-units. The GCD then necessarily introduces common base-unit. Since Beal's solutions are defined here as identity solutions, we can increase the size of the number on the RHS of a primitive equation keeping it in block-form by multiplying it by a factor

of block-form (GCD) and follow exponentiation rules. For the equality of the identity equation to hold then the factor must multiply the two numbers on the LHS as well and obey the rules of exponentiation. Specifically, all three terms of the solution $3^3 + 6^3 = 3^5$ in example 1 were built with the same bundle of 3^3 and the same base-unit of the prime number 3, introduced by the CF.

The process of simplifying the equation into products of block-numbers imposes a condition that all of the terms must have a GCD of the two monomials and must obey exponential rules satisfying an identity equation with GCD of x^m that must have special relationship with the coefficients of the two terms on the LHS of the equation. The process requires a common factor for a successful transition from addition operation to multiplication to yield a single term with all three terms expressible in exponential form of single power. The process of conversion from addition of two exponential numbers to multiplication of two exponential numbers is similar to that of addition of two fractions. In both you need a common factor of the correct proportion for the process to succeed due to the unlike discreteness of the terms that depends on their prime basis-unit in the case of numbers in exponential form. The success of the conversion process of addition of the two terms on the LHS to multiplication and subsequently the successful combining of the resulting product into one term of exponential form and single power implies that all three terms of Beal's equation solution must have the GCD of their respective monomials.

1.2. The proof of Beal's conjecture

We will now prove Beal's conjecture. We can convert the numbers on the LHS of Beal's conjecture solution from exponential form to fraction form and keep the discreteness and compare the addition of the two terms in exponential form to that in fraction form, proving Beal's conjecture deductively. Since any Block-number is only discretely made of its base, it can be converted to a fraction keeping the base as the denominator to conserve the discreteness of the numbers by multiples of the base by the following formula,

$$x^l = \frac{x^l}{x} \cdot x$$

e.g. $2^4 = \frac{2^4}{2} \cdot 2 = \frac{32}{2}$. Now we follow addition of fractions which confirms that the sum of any two fractions should have a LCF of their denominators for a successful addition as discussed above and therefore the same common factor of their bases in exponential form.

Example 2: To add $3^3 + 6^3$ we need to convert the terms into fractions keeping the bases as the denominators in the fraction form. The expression becomes $81\left(\frac{1}{3}\right) + 1296\left(\frac{1}{6}\right)$. We reduce the second term to like proportion of $\frac{1}{3}$ since 6 is a composite number and 3 is a common factor. The expression becomes $81\left(\frac{1}{3}\right) + 648\left(\frac{1}{3}\right)$. The sum then becomes $\frac{729}{3}$. We convert the fraction to block-form keeping the denominator as the base to get 3^5. The fact that we can convert exponential form to fraction form of the two terms on the LHS of Beal's conjecture solution and that we can keep the same discreteness, it follows that the addition rules of the two fractions must be maintained and a common factor of the terms' denominators necessarily obligates the same common factor of the terms' bases in exponential form since we maintained the bases to remain as the denominators of the two terms. Recognizing that Beal's conjecture solution is an identity, we prove Beal's conjecture.

2. Assertion of mathematical operations and definitions

In this section we will analyze the mathematical operations involved in the process of addition and multiplication of block-numbers.

2.1. Definitions

Block-number: It is a mathematical object defined here by any number a in exponential form of single power $a = b^n$ that is made of specific number of prime base-units, $a \equiv 0 \mod (b)$ and b and n are non-negative integers.

Block-elements: They comprise the block-number. The number of elements is the value of a block-number in the standard form of exponent 1.

Exponent: For a block-number b^n, n is the exponent.

Base: For a block-number b^n, b is the base.

Base-unit: It is an ensemble of elements that constitutes the basic irreducible building prime number that "branches" out to build a block-number. For the block-number 2^n, the base-unit is 2, while for 11^n, the base-unit is 11. Any block-number is unique and can only be made of multiples of its base-units.

Block-bundle: For a block-number b^{n+r}, it is the ensemble b^n of base-units that repeats upon branching when the block-number is increased by a factor of b^r, where r is a positive integer. Alternatively, for a block-number of $(l \cdot b)^n$, it is the ensemble of base-units of b^n that repeats upon branching when the block-number is increased by a factor of l^n, where l is a positive integer.

Exponent branching: It is the degree of branching of the block-bundle with a branching process based on the base's value by increasing the exponent.

Base branching: It is the degree of branching of the block-bundle with a branching process based on the base's value by increasing the base.

Size of a block-number: It is the total number of base-units of a block-number.

Factor of a block-number: When two block-numbers are added or subtracted from each other, a common block-number (block-bundle) may be factored out.

Block equation: a block equation is an equation whose terms are block-numbers.

Size of a block equation: It is the size of the block-number on the RHS of the block equation.

Block-variable: For the block equation $lP^n + kP^n = i(l+k)P^n$, P^n is the block-variable.

Block-coefficient: For the block term lP^n, l is the block-coefficient.

Block-Monomial: For the block equation $lP^n + kP^n = (l+k)P^n$, each term is a block-monomial comprised by the block-variable and its coefficient and share either the power or the base.

Remark The definition of a number in exponential form of a positive integer base and power here as a block-number is an important characterization of the number as it describes the size of the mathematical object depending on the value of its base-unit.

2.2. Analysis of the multiplication process of two block-numbers

When two block-numbers are multiplied together both must have a specific relationship with each other for the multiplication process to yield a single block-number.

LEMMA 1. *Let $b^n \cdot k^l$ be the product of two block-numbers. The product yields a block-number if and only if $n = l$ or $b = k$ or both.*

Proof. If $n = l$, the product becomes $b^l \cdot k^l$. By the power rule of same exponent the product becomes $(bk)^l$, which constitutes a block-number. If $b = k$ the product becomes k^{n+l} by the power rule of same base, which is a block-number. The case of $n = l$ and $b = k$ is satisfied by one of the preceding processes.

Corollary 1. *The two processes of same power or same base to combine the product of two block-numbers by lemma 1 are equivalent.*

Proof. Since the two processes increase the size of the block-number by a specific number of base-units, it follows that they are equivalent.

Example 3: The equivalence of the two multiplication processes of same exponent or same base is obvious from the product of $3^4 \cdot 3^4$ which either equals 3^8 or 9^4. The two block-numbers can also be expressed in the standard form of 6561^1. All three block-numbers have positive integer base and exponent and have base-units of 3 and a size of $6561/3 = 2176$ of base-units.

Remark For the product $b^n \cdot k^l$, when $n = l$, it means that the two terms have the same degree of branching but their bases multiply with each other upon combining and the result is a block-number of larger base, while for $b = k$ it means that the two terms keep the size of their bases unchanged but increase the degree of their power-branching upon unification into a single block-number.

2.3. Reshaping a block-number

For a block-number to be expressed in a different form, it follows the exponent rules. Similar to multiplying a rational number by a common factor both in the numerator and the denominator to obtain the rational number in a different form, we may change the form of a block-number by changing its exponent value as well as its base value.

LEMMA 2. *If we divide or multiply the exponent of a block-number by a factor n we should raise the base to the nth power or take the nth root of the base respectively to keep the size of the block-number unchanged.*

Proof. The proof comes from the definition of the base and the exponent of a block-number. Let a block-number be k^l. Then, dividing the exponent by a factor *n* and raising the base to a power *n* leaves the block number unchanged as $k^{\frac{nl}{n}}$. The other case is accomplished by the same reasoning.

Remark Two block-numbers are equivalent if they have the same number of elements leading to the same number of the same base-unit.

Example 4: The block-number 9^2 equals the block-number 3^4 by taking the square root of the base and multiplying the exponent by 2. Similarly, the block number of 27^2 is equal to 3^6 by taking the third root of the base and multiplying the exponent by 3. By repeating the process we obtain an exponent of 1 and 3^6 equals 729^1. No further up-reshaping can be made if we need to keep the block-number with positive integers.

Remark Reshaping a block-number does not change it into a different block-number since the operation keeps its size unchanged. It follows that the number of base-units of a specific block-number is constant at all times. The number of base-units in any block-number equals the number of elements divided by the base-unit by definition.

Example 5: The block-number 6^5 has a composite base of $3 \cdot 2$ and two base-units of 2 and 3 and two sizes; one is $7776/3 = 2592$ while the other one is $7776/2 = 3888$.

Remark To obtain the base-unit of a block-number we need to down-reshape it until we get the lowest possible integer base. The base is then the base-unit if it is not a composite number. If it is a composite number then it has as many base-units as the number of prime numbers it comprises. In example 2 and from 3^8, the base-unit is 3 and the block-number has 6561 elements and a number of base-units of $6561/3 = 2187$, which is the size of the number.

Remark When reshaping a block-number it is important to keep a positive integer base and power since that is a property of the block-number.

2.4. Increasing the size of a block-number

We can increase the size of a block-number by increasing its degree of power-branching or base-branching. In both processes the block-number increases by multiples of the base-units only and therefore keeps its modularity of value 0 to the prime base-unit.

Corollary 2. *For a block-number, it is only possible to increase its size by multiples of a block-bundle and consequently by multiples of its prime base-unit.*

Proof. The corollary stems from the power rules of numbers. Since to increase the size of a block-number by a factor, the factor itself must be a block-number, and by lemma 1 it is only possible to perform the multiplication process of two block-numbers if they share either the base or the exponent. In both cases the new resulting block-number only increases by multiple of the base-unit. This is obvious since multiplying the block-bundle P^r of base P by a factor of k^r yields $(kP)^r$ and multiplying P^l and P^r yields P^{r+l}; both processes yield a number with multiples of the block-bundle P^r and therefore multiples of the block-base P as well as multiples of the base-units.

2.5. Increasing the size of a block-number by exponent branching process

Increasing the size of a block-number by increasing the value of its exponent can be done by adding multiples of block-bundles and therefore the correct number of multiples of the base-unit every time we increase the power.

Example 6: Fig. 1 illustrates the process of branching by increasing by a block-bundle of 3^2 by multiples of the base-unit of 3.

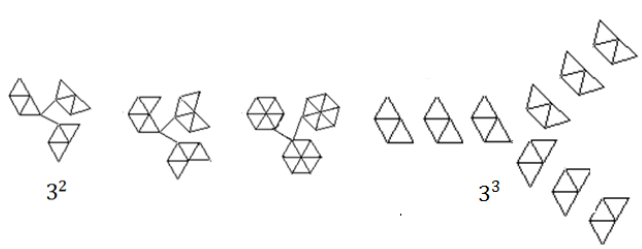

FIGURE 1

The left drawing of groups of block-elements made of triangles represents the block-number 3^2, of base-unit 3, as the block-bundle structure when increasing the exponent of the block-number by one. The right drawing represents the block-number 3^3 by adding three-multiples of the block-bundle structure of 3^2 since it is multiplied by the block-factor of 3^1, but the second from right drawing does not increase the power by one because it only doubles the block-bundle structure to become $2 \cdot 3^2$ instead of tripling it since the base value is 3, three triangles in the figure, and the second from left increases it only by a fraction to become $4/3 \cdot 3^2$. Therefore, the two figures in the middle do not represent a block-number since there is no mechanism to combine the product into one block-term because the multiplication by the factor doesn't allow addition of base-unit multiples of the block-bundle structure using exponentiation rules. To increase the exponent by another factor to make it 3^4, we need to branch one more time by multiplying the whole structure (block-bundle) by a block-factor of 3^1, and the block-number becomes $3^1(3^3)$, which results in 3^4 by the power rules. The new block-bundle structure is then 3^3.

Remark Any increase in size of a block-number by multiplication of a number different than that of a block-bundle that complies with lemma 1 destroys the block-structure of the number.

2.6. Increasing the size of a block-number by base branching process

Increasing the size of a block-number by increasing the value of its base can be done by adding the correct number multiples of the base-units every time we increase the base by a factor.

Example 7: We can keep the exponent of a block-number unchanged by multiplying the block-number by a factor whose exponent is the same as the block-number's exponent. The resulting block-number now has a base value equals the product of the two bases of the original block-number and the multiplied block-factor, hence increasing the base value by multiples of the base-units only. An example is increasing the block-number 3^2 as the block-bundle structure by a factor of 2^2, which changes the block-number to 6^2 of base 6 but keeps exponent of 2 (See Fig. 2). There are then 4 of the original block-bundle that make up the new block-number.

FIGURE 2

We may wish then to multiply the original block-bundle of 3^2 by a factor of 4^2 to increase its size by sixteen times and obtain a new block-number of 12^2. The right side of figure 2 above then will have a base of 12 elements, with two distinct base-units of 2 and 3, branched out twelve times.

Remark Increasing the size of a block-number by repetition of its block-bundle via increase by both exponent and base ensures repetition of the base-unit as well with mod 3 that evaluates to 0 in example 6 and both mod 3 and mod 2 in example 7. Also, increasing the size of a block-number by repetition of the base introduces a new base-unit and a new modularity to the block-number that must evaluate to 0 as a property of block-numbers.

2.7. Families of block-numbers

We can characterize block-numbers as families of the same base-units. Examples of block-numbers originating from the prime number 2 are 2^2, 2^3, 6^2 and the prime number 3 are 3^2, 3^3, 6^2. The block-number 6^2 is constructed from two different base-units of 2 and 3, and therefore is a common block-number of the families of the prime numbers 2 and 3 and can be used in the mathematical block-operations of either of the two sets of block-numbers where the common block-bundle is made of any one of them to construct the sum of two block-numbers. In Example 7 above, block-number of 6^2 equals $2^2 \cdot 3^2$, which means there is 4 of the block-bundles of 3^2 and 9 of the block-bundle of 2^2 to make the block-number of 6^2.

2.8. Addition of block-numbers

It is asserted here that any two block-numbers may be added together only if they belong to the same group of prime base-units to construct the sum based on the same base-unit notion as

reasoned by common proportional discreteness in the introduction. This ensures a common block-factor between the two block-numbers that allows the conversion of addition to multiplication. Further, the total number of base-units of the sum of the two block-numbers must add up to form a block-number complying with exponentiation rules, e.g. the middle two drawings in Figure 1 don't represent a block-number because the total number of base-units doesn't amount to constitute a block-number with the key rule that the resulting product of the two block-factors don't combine by the power rules. *In other words, since any block-number must be made of a unique base-unit, to add two block-numbers to produce a block-number with a unique base-unit, they both must be made from the same common base-unit. So, we cannot add the block number* 17^{900} *with the block-number* 5^{1700}*to produce a block-number simply because the two block-numbers have different prime base-units and different discreteness.*

LEMMA 3. *A common factor is a necessary condition for a successful conversion of the addition process of two block-numbers into a product of two block-numbers.*

Proof. Lemma 1 above places restrictions on addition of two block-numbers such that a proper common factor allows a common block-bundle as a building block to build the sum of the numbers in block-form according to that common discreteness is a must property of both block-numbers.

Example 8: Adding three of the block-numbers of 3^2 yields the product $3 \cdot 3^2$ which can be simplified by the power rules as 3^3. The sum has a block-bundle of 3^2 (See Fig. 3).

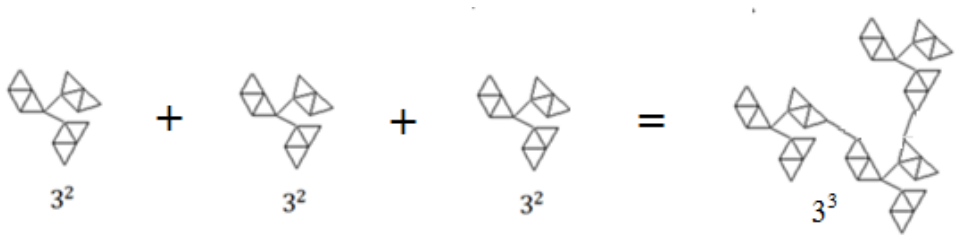

FIGURE 3

Remark Similar to the algebraic addition process of two numbers in the standard form by taking a common factor and adding the "residual" factors to convert the addition to multiplication, we can add block-numbers by factoring out the GCD and sum the residual factors of block-numbers

to convert addition to multiplication. The residual factors must add up to a single block-number that must combine with the GCD complying with power rules to produce a block-number.

Remark As it is asserted here that we can add two block-numbers to each other only if they share a block-bundle and subsequently a base-unit, by lemma 1. The key to the success of the addition process of two block-numbers in yielding a block-number is that one of them must distribute its elements to the other in such a way that it adds up exactly as if the first block-number is being increased in size, which implies that the addition process must be successfully converted to a multiplication one to yield a single term of positive integer power and base, therefore they both must share a base-unit since increasing in size occurs only by multiples of the base-unit. (See Fig. 1 and 2 and the reasoning behind necessary GCD for addition of block-numbers in the introduction).

LEMMA 4. *If a GCD of exponent 0 is not a sufficient factor for a successful conversion of addition process into multiplication process of two block-numbers to yield a block-number of exponent larger than 1, higher exponents of GCD are necessary.*

Proof. Lemma 3 does not place restrictions on the value of the CF.

The GCD can easily be obtained by the process of breaking down the bases to their primes to obtain a greatest common block-factor whose base is the product of the shared primes. If the addition is "primitive", the common factor is unity and the addition is simple algebraic process in the standard form. The process of factoring the GCD out converts the addition process of two block-numbers to a product of two block-numbers. Consequently, if lemma 1 is not applicable a non-integer base or power may be obtained. This is a useful process to check whether the product of two block-numbers can be combined to yield a single block-number to conclude whether the addition process is successful in block-form. Since changing the size of a block-number retains its base-unit value, any successful addition of two block-numbers must ensure as a priori that the block-numbers share the base-unit since increasing the size of one by the other means increasing the size of the other by the first.

Example 9: We can add any two numbers by simply taking GCD as unity. First, up-reshape the two numbers to their standard forms if they are in block-number form. Then, add them together

algebraically and down-reshape the sum to lowest integer base using lemma 2. The solution then is restricted to whether all the terms can be down-reshaped to block-numbers to conform to Beal's or Fermat's equations. Example is the sum of $3^3 + 6^3$. We can up-reshape each block-number to the standard form to get $27 + 216$. The sum is 243, which can be down-reshaped to 3^5. Alternatively, we take 3^3 as the GCD and sum the coefficients to get $9 \cdot 3^3$. By lemma 1 the product produces 3^5.

Example 10: A simple example of the addition of two block-numbers comprised of different prime-bases is the sum of $2^3 + 3^3$. By trying the multiplicative identity 1 as GCD we get a sum in the standard form of 35^1 of power 1. This number cannot be further down-reshaped since 35 is all what we get. Therefore, if higher powers are desired, a GCD of higher power than 0 may be used. If we multiply both terms by a common factor of 35^3 that necessarily complies with lemma 1, the expression becomes $70^3 + 105^3$ which has a sum of 35^4. This is only possible because the new block-numbers share a common block-factor with common base-value of 35 and therefore shame the same discreteness. The factor 35^3 is then the GCD which represents the block-bundle of the sum that repeats by base-value of 35. In other words, the number 35 then repeats and constructs the block-bundle which repeats to construct all three terms of 70^3, 105^3 and 35^4 complying with lemma 1. Note that after multiplication of a proper CF all three terms have the base of the CF as a common unit upon which both can build the sum by adding their own base-units, and the base-size of the resulting sum of 35^4 equals the sum of the base-sizes of the two added numbers of 70^3 and 105^3, which is 42875. Note also that the base of 35 is a composite number made of the product of the prime numbers 7 and 5 and therefore is made of two distinct base-units of the primes 7 and 5 and two distinct sizes corresponding to each base-unit. In other words, the equation $70^3 + 105^3 = 35^4$ can be simplified to $2^3 \cdot 7^3 \cdot 5^3 + 3^3 \cdot 7^3 \cdot 5^3 = 7^4 \cdot 5^4$. This translates to that the first term has a $(2^3 \cdot 7^3)$ of the block-bundle 5^3 and the second term has $(3^3 \cdot 7^3)$ bundles of 5^3 and also the first term has a $(2^3 \cdot 5^3)$ of the block-bundle 7^3 and $(3^3 \cdot 5^3)$ of the block-bundle 7^3.

3. Block-equation

A block-equation is an equation whose terms are block-numbers. It can be represented as,

$$\alpha x^l + \beta x^l = \delta x^l$$

Where $\delta = \alpha + \beta$, x^l is a GCD and each term constitutes a block-number.

Theorem 1. *Let the LHS of the block-equation* $a^x + b^y = c^z$, *where a, b, c, x, y and z are positive integers, represent the sum of two block-monomials with common variable* P. *Further, let the equation be made in the three unique forms,*

Case 1: *Both terms have the same powers but different bases,*

$$l^n P^n + k^n P^n = q^j P^n$$

Case 2: *Both terms have the same bases but different powers,*

$$P^r P^n + P^s P^n = q^j P^n$$

Case 3: *One term has the same base and different powers, the other one has the same power and different bases,*

$$P^r P^n + l^n P^n = q^j P^n$$

where P^n *is the GCD obtained from the expression* $a^x + b^y$; $(l^n + k^n)$, $(P^r + P^s)$, $(P^r + l^n)$ *are the sums of the coefficient factors of the two block-terms on LHS, and* P, l, k, n, r, s *are positive integers. Then there exists a unique method to combine the two terms into a single block-term such that a unique solution* $q^j P^n$ *equals* c^z *exists and shares the common block-number* P^n, *where* q^j *equals one of the sums of the coefficient factors and* q *and* j *are positive integer variables.*

Proof. Building on the facts established by lemmas 1-4, all three forms of Beal's equations are satisfied. The monomial-variable P^n constitutes the block-bundle that repeats to form all three terms of Beal's equation. Factoring the two terms on the LHS of Beal's equation of its block-bundle P^n leaves it as the product of P^n and one of the three sums of the terms' coefficient factors. For positive integer variables in the expressions, each of the coefficient sums must yield the single block-term q^j for the equality to produce a block-term. The sum of the two terms on LHS then becomes $q^j P^n$. After reshaping q^j and P^n by lemma 2 and combining factors

extracted from one to the other if needed, the two terms must combine to a single block-term by lemma 1 to yield the sum of c^z.

Remark The least value of q^j is 2 if each of the coefficient terms yields 1. If it is not possible to extract two terms from any product of two block-numbers and reshape them such that they have either a common positive integer base or a common positive integer exponent, then it is not possible to obtain a single block-number as the result of the multiplication process and the addition process fails.

Corollary 3. *If the sum of any number of block-monomials constitutes part of a block-equation, the equation must abide with theorem 1.*

Proof. Theorem 1 predicts that any solution to any block-equation of any number of terms must conform to the power rules of lemma 1.

Example 11: For the block-equation of the sum of three block-terms of $6^2 + 6^2 + 3^2 = 3^4$, theorem 1 states that there exists a common block-bundle that makes up each term in the solution in the form of P^n. Let's work with the LHS of the equation to produce the RHS. The expression $6^2 + 6^2 + 3^2$ has a GCD of 3^2, leaving out the expression as $(2^2 + 2^2 + 1)3^2$. The expression can be reduced to $3^2 \cdot 3^2$, and by theorem 1 the sum yields the RHS of the block-equation.

3.1. Steps to generate the sum of two block-numbers

If two block-numbers can be added to produce a block-number, the following steps may be taken,

1. Check if 1 is a valid GCD by checking if the bases are coprime and a primitive sum is desired.
2. Obtain a GCD different than 1 of the two block-numbers. If one of the block-numbers is the GCD, a simple check for the GCD is to divide the larger block-term by the smaller one. We can also break down the bases of the two terms to their primes and calculate the GCD.
3. Factor out the common block-bundle represented by the GCD of the two block-numbers to convert the addition process to multiplication.

4. Use the power rules to reduce the product of the two numbers to a single term. Specifically look for how the two block-terms of the product can end up having the same base or the same power.
5. Combine the two terms by either of the two power rules in step 4 to a single term.

Remark One of the main functions of a successful CF is to convert the terms of a primitive equation to block-numbers.

Example 12: In example 11 the coefficients constitute the block-equation $2^2 + 2^2 + 2^0 = 3^2$ where the sum is primitive and the GCD is unity of 2^0. We can choose another proper CF of 3^2, or any block-number of power 2, to generate another valid block-equation as in example 11.

Example 13: For the equation $3^9 + 54^3 = 3^{11}$, theorem 1 states that there exists a GCD in the form of P^n. Let's work with the LHS of the equation to produce the RHS. The expression $3^9 + 54^3$ has a GCD of 3^9, leaving out the expression as $(1 + 8)3^9$. The expression can be reduced to $3^2 \cdot 3^9$. Lastly, the two parts of the expression can be combined by the power rules to 3^{11} since the two terms in the expression have the same base. The block-number 3^9 then is the GCD for the three terms in the equation and 3 is the common base-factor among the three terms. Only when one of the two methods by applying the power rules to combine the product terms is satisfied the combining process succeeds in producing a sum and therefore a common relative base-factor.

Example 14: For the equation $7^6 + 7^7 = 98^3$, one possible common block-factor is 7^3. In this case 7^3 is another block-bundle besides the GCD. The sum of the coefficients yields 2744 which can be shaped down to 14^3 by taking the third root, which produces the RHS of the equation upon combining the terms by the power rule of the product of two numbers having the same power. If we factor out the GCD of 7^6 from the LHS of the equation, the expression becomes $(1 + 7)7^6$ and can further be expressed as $2^3 \cdot 7^6$. Notice that none of the terms in the original primitive expression of $1 + 7$ had a block-form. With the introduction of a CF, the two terms took a block-form and so did their sum. To combine the resulting two terms we divide the power of the second term by 2, to make use of one of the methods to combine two block-terms of same power, and square its base to get $2^3 \cdot 49^3$. This expression yields the same term on the

RHS of the equation. The three terms then share a GCD of 7^6 and therefore a common base and base-unit of 7.

Example 15: For the block-equation $34^5 + 51^4 = 85^4$, an obvious prime common factor of the bases of the two block-numbers on the LHS of the equation is 17. The two numbers are then reduced to $17^5 \cdot 2^5 + 17^4 \cdot 3^4$, which can be further reduced to $(17 \cdot 2^5 + 3^4)17^4$, where 17^4 is the GCD term. The expression becomes then $(625)17^4$ and can be reduced to $5^4 \cdot 17^4$ by fully down-reshaping 625 to the number 5^4. We can see that the original sum was $17 \cdot 2^5 + 3^4$ with the first term expressed in non-block form. The CF transformed the first term into a block-form. The sum becomes 85^4. We conclude that the block-number 17^4 is the GCD-block-bundle of the terms on LHS of the equation and 17 is a common factor of their bases. There are then 625 of the bundle 17^4 that makes up a block-term of 85^4. The base-unit of all three terms is then the prime number 17.

Example 16: The LHS of the block-equation $760^3 + 456^3 = 152^4$ can be broken down to the base's primes and becomes $5^3 \cdot 2^9.19^3 + 3^3 \cdot 2^9.19^3$. The two terms now can be combined to yield $(3^3 + 5^3)\, 2^9.19^3$, and by reshaping 2^9 to 8^3 the expression becomes $(3^3 + 5^3)\, 8^3.19^3$ with a GCD of 152^3 to yield $152 \cdot 152^3$, which yields the RHS of the equation. There are then 152 of the bundle 152^3 that makes up a block-term of 152^4 with the bundle necessarily makes up all three block-terms of the solution; where there is 125 of the bundles that makes up the 760^3 term and there is 27 of them that make up the block-term 456^3. The common base of all three terms is 152 which have two distinct base-units made of the primes 2 and 19.

Remark The process to evaluate a block-equation as described by theorem 1 should always naturally succeed to satisfy the block-equality if we pick the common bundle that builds each term as the GCD of the two terms on LHS of Beal's solution. Block-numbers of prime bases grow in size by branching out by the base-unit. Since block-numbers of different prime-bases grow in size at different branching rates, the only possible way for a successful addition is to build a common bundle based on a base-unit that is made of a composite number of the common primes of the bases of the two terms.

3.2. Uniqueness of the sum term

The sum on the RHS of Beal's equation is unique but it may be expressed in a different form by the process of reshaping.

Example 17: Let's consider the block-equation $27^4 + 162^3 = 9^7$. By factoring out 27^4, or 3^{12} if we reshape it to its lowest base, as the GCD from the LHS of the equation we get $(1+8)27^4$, which becomes $3^2 \cdot 3^{12}$ and produces a final result of 3^{14}, which can be reshaped to produce 9^7; the RHS of the equation. It is important to make sure that the sum-term on the RHS of the block-equation has not been reshaped before we judge whether the resulting equation is identical to the given one.

Example 18: Another example to beware of the end result as deemed different is the equation $33^5 + 66^5 = 1089^3$. By following the steps above we find the sum of the two terms on the LHS of the equation is 33^6, which can easily be reshaped to 1089^3. The same goes with the equation $8^3 + 8^3 = 4^5$. By following the same steps we may get a solution 2^{10} or 32^2 which can be reshaped to 4^5. The last example simply can be simplified by reshaping as $2^9 + 2^9 = 2^{10}$, which simplifies as $2 \cdot 2^9 = 2^{10}$. It is obvious here that both terms on the LHS have the same prime base-unit of 2.

For further exploration we will list more examples and the steps considered to obtain the solution to block-equations.

Example 19: By factoring out the GCD of 19^3 from the LHS of the block-equation $19^4 + 38^3 = 57^3$ we obtain $(19+8)\,19^3$. Simplifying we get $27 \cdot 19^3$ which by reshaping 27 becomes $3^3 \cdot 19^3$ and yields the RHS of the equation. The RHS monomial then is 3^3x^3, with variable value of 19 and coefficient 3^3. It is clear that the primitive solution of the equation of $19 + 8 = 27$ can be reshaped to $19 + 2^3 = 3^3$ with the common bundle of 19^3 used to upgrade the primitive equation to higher powers.

Example 20: By factoring out the GCD of 80^{12} from the LHS of the block-equation $80^{12} + 80^{13} = 1536000^4$ we obtain $(1+80)\,80^{12}$. Obviously the LHS of the primitive equation involved here of $1 + 80 = 3^4$ cannot be reshaped to higher powers to comply with the conditions of Beal's conjecture. Simplifying we get $81 \cdot 80^{12}$ which becomes$3^4 \cdot 80^{12}$, and by

reshaping 80^{12} as 512000^4, the RHS monomial then becomes 3^4x^{12}, with a variable value 80, coefficient 3^4 and degree 12.

Example 21: By factoring out the GCD of 28^3 from the LHS of the block-equation $84^3 + 28^3 = 28^4$ we obtain $(27 + 1)\ 28^3$. Simplifying, we get $28 \cdot\ 28^3$ and the RHS monomial then is $28\ x^3$, where the variable value and the coefficient are 28 and the degree is 3.

Example 22: By factoring out the GCD of 1838^3 from the LHS of the block-equation $1838^3 + 97414^3 = 5514^4$ we obtain $(1 + 148877)\ 1838^3$. By borrowing 1838 factor from the coefficient term and simplifying we get $81 \cdot\ 1838^4$. The 81 can be reshaped to 3^4 and the product yields the RHS.

Remark In examples 9-22 we have produced the RHS of the given equations without a prior consideration of its final form just by applying theorem 1. Also, it is clear from the examples that the role of a common factor of the two terms on LHS of Beal's conjecture solution is to provide the basic base-unit to build the sum onto.

4. Beal's conjecture equation

Beal's conjecture equation is a "block-equation" with the LHS representing the sum of two block-monomials with common variable x. Following properties of addition of block-numbers, the expression on the LHS of the equation deals with addition of two block-numbers followed by multiplication operation to simplify the resulting product to a single block-term. In principle we need to combine the two terms on the LHS of Beal's equation in one term simply by making use of the product rules of exponents as ordained by lemma 1.

5. Beal's conjecture connection to Fermat's last theorem

In this section we will extrapolate the connection between Beal's conjecture and Fermat's Last Theorem. Specifically, we will relate power 2 of the terms of Beal's equation as restricted to Fermat's Last Theorem having no solutions for $n > 2$ of positive integers *A*, *B*, and *C*.

5.1. Beal's conjecture is a generalization of Fermat's Last Theorem

Fermat's Last Theorem states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer n greater than 2. The theorem was rigorously proven by Andrew Wiles [2]. The cases $n = 1$ and $n = 2$ have been known to have infinitely many solutions.

While Pythagorean triples can be generated, for example by Euclid formula, a connection to Beal's conjecture of non-primitive triples has not been made to our knowledge. In the language of block-numbers of monomials, Fermat's Last Theorem says that a successful block-solution of the sum of two block-monomials of same degree to yield same degree monomial is that of degree 2 or 1. Or, no block-equation can be reshaped to the same power for all terms except for powers 1 and 2.

Corollary 4. *Fermat's equation keeps its form of its squared terms only if we multiply it by a CF of power 2 that complies with same exponent rule of lemma 1.*

Proof. If a CF of power different than 0 is desired to evaluate Fermat's equation, only a CF of power 2 may be used, since if it is multiplied by Fermat's equation it combines successfully with all three terms of the equation to yield an exponent of 2, retaining its power of 2 and its block-form.

Example 23: Pythagorean triple of $(3, 4, 5)$ complies with Fermat's equation and are coprime with GCD of unity. We can associate it to another proper GCD of 2^2 and retain Fermat's form. This GCD is carefully chosen such that it combines with the terms on the LHS of the Fermat's equation $3^2 + 4^2 = 5^2$ by the same exponent rule; see lemma 1. Multiplying the proposed GCD of 2^2 converts the triple to non-primitive equation of $6^2 + 8^2 = 10^2$. Any common factor of x^2, where x is integer produces an infinite number of solutions to the non-primitive Pythagorean triples characterizing the block-equation $3^2x^2 + 4^2x^2 = 5^2x^2$.

Example 24: In example 23, the terms of Pythagorean triple of (3, 4, 5) on the LHS of Fermat's equation $3^2 + 4^2 = 5^2$ can be reshaped to $3^2 + 2^4 = 5^2$, which is a "quasi-Beal's equation" Also, if multiplied by a factor of 25^2, the resulting Fermat's equation becomes $75^2 + 100^2 = 125^2$, and reshaping the terms yields a quasi-Beal's equation of $75^2 + 10^4 = 5^6$. A quasi-Beal's equation is described by Fermat-Catalan conjecture.

5.2. The sum of the two terms on the LHS of Fermat's equation

Corollary 5. *The solution of a three-term block-equation of same powers of $n > 2$ is intrinsically unsuccessful.*

Proof. It follows from Fermat's Last Theorem. In the language of theorem 1, a proper CF of power $n > 2$ cannot be admitted to scale up all three terms of Fermat's equation to the same power n by increase in base process since no Fermat's equation exists with same base terms to comply with lemma 1 and a CF of power $n = 2$ keeps the power unchanged. This is a "proof" to Fermat's Last Theorem via theorem 1 since Beal's conjecture is a generalization to Fermat's Last Theorem.

Corollary 6. *Fermat's Last Theorem as a solution to a block-equation and Beal's equation as an equation in block-form suggest that any same power $n > 2$ of the two terms on the LHS of a three term block-equation must yield a sum of power different than n.*

Proof. The proof is implied by Fermat's Last Theorem and Corollaries 4 and 5.

Proposition 1. *Failed Fermat's solution of exponents greater than 2 may be an incomplete Beal's solution.*

It is proposed here that failing Fermat's solutions of powers greater than 2 could be incomplete Beal's solutions, missing a proper CF. We can consider any successful Fermat's equation of either a primitive Pythagorean coprime triples or a scaled one by some factor as a successful quasi-Beal's equation.

Example 25: For the failing Fermat's sum of $6^3 + 21^3$ we have $2^3 \cdot 3^3 + 7^3 \cdot 3^3$. By factoring out the GCD of 3^3 from the two terms we obtain $(351)\ 3^3$. Simplifying the coefficient term to its prime bases we get the expression $(13 \cdot 3^3)3^3$. This expression cannot be combined to a block-form. The wrong GCD has been used. Since the base-units 7 and 2 of the two terms of the original sum of $2^3 + 7^3$ branch at different rates a GCD with composite number of the two terms is necessary to build the block-bundle that builds the sum. A proper one is 351^3, since 351 is the sum of the two terms, and the corresponding proper Beal's block-equation for the sum $2^3 + 7^3$ is $702^3 + 2457^3 = 351^4$. The GCD is the block-bundle and has been chosen carefully such that it combines with all three terms transforming them to block-form, abiding with lemma 1.

Example 26: From example 10, the sum of $3^3 + 2^3$ in the equation $3^3 + 2^3 = 35$ is a failed Fermat's equation of power greater than 2 but it can be made a successful Beal's equation by applying a proper CF of the sum of the two terms to power 3 so it can combine with all three terms of the equation. Multiplying the equation by 35^3 we get the block equation as $105^3 + 70^3 = 35^4$. So, multiplying by a proper CF of a failed Fermat's equation of power greater than 2 yields a valid Beal's equation.

Proposition 2. *Failed Fermat's equation of power 2 is an incomplete quasi-Beal's equation.*

It is proposed here that if you multiply a failed Fermat's equation by a proper CF it becomes a quasi-Beal's equation.

Example 27: The equation $2^2 + 3^2 = 13$ is a failed Fermat's equation of power 2. If we multiply the equation by a CF of 13^2 we obtain the quasi-Beal's equation of $26^2 + 39^2 = 13^3$. See also examples 23 and 24.

6. The proof of Beal's conjecture and summary of the main assertion

Beal's conjecture proof stems from that the conjecture equation is an identity and the assertion that block-numbers are discrete and their discreteness arises from the fact that they are made of a specific repeating prime number designated here as the base-unit. Block-solution to the summation of any two block-numbers of powers greater than 2 is only successful if the block-numbers share a block-bundle to build the sum upon and necessarily share a prime base-unit. The common block-bundle is the GCD whose existence in the equation is a priori condition for a successful addition of two block-numbers due to the discreteness constraint in the addition process of proportional block-numbers. In the introduction, this was reasoned by the fact that the discreteness constraint of a block-number necessitates that any two block-numbers to be added together with coprime bases and therefore unlike discreteness must be adjusted such that they have the same discreteness by multiplying them by a CF. This is analogous to addition of two fractions of different integers in the denominator by multiplying the two fractions with a unity made by the LCF of the two fraction's denominators transforming them to fractions with same discreteness and allowing adding them proportionally. Also, it was reasoned in the introduction that by the addition property of modular arithmetic both of the terms on the LHS of Beal's conjecture equation must evaluate to 0 mod the prime base-unit of the term on the RHS.

The process of converting the summation of two block-numbers to a product of two block-numbers and the successful process of combining the two terms of the product by lemma 1 to yield a single block-number ensures that the total number of base-units is equal on both sides of a block-equation and therefore the sum ensures equal block-sizes on both sides of the equation.

The following table lists possible sums and the description of their expressions on the LHS of the solutions of Beal's equation $lP^n + kP^n = (l+k)P^n$ starting from $1+1$; some with no solution.

Eqn.	lP^n	kP^n	$(l+k)$	P^n	$q^s P^n$	c^z Block-solution	Block-solution of power > 2	Description
1	1	1	$1+1$	1	$2 \cdot 1$	2^1	No	Conjecture unsatisfied
2	1	2	$1+2$	1	$3 \cdot 1$	3^1	No	Conjecture unsatisfied
3	2^1	2^1	$1+1$	2^1	$2^1 \cdot 2^1$	2^2	No	Conjecture unsatisfied
4	2^2 4	2^2 4	$1+1$	2^2	$2^1 \cdot 2^2$	2^3	yes	Conjecture unsatisfied
5	5	5	$1+1$	5^1	$2^1 \cdot 5^1$	10^1	NO	Conjecture unsatisfied
6	2^3 8	7^1 7	$2^3 + 7^1$	1	$15 \cdot 1$	15^1	NO	Conjecture unsatisfied
7	2^3 8	2^3 8	$1+1$	2^3	$2^1 \cdot 2^3$	2^4	yes	Conjecture satisfied
8	2^4 16	2^4 16	$1+1$	2^4	$2^1 \cdot 2^4$	2^5	yes	Conjecture satisfied
9	3^3 27	3^5 243	$1+9$	3^3	$10 \cdot 3^3$	$-$	$-$	Unsuccessful block-simplification

The key to which of the next block-equation in the table would satisfy Beal's conjecture depends on whether each term of the block-equation $lP^n + kP^n = (l + k)P^n$ individually can be shaped in a block-form such that it satisfies Beal's restrictions which can be checked by lemma 1 by an iterative algorithm.

The process as described in this article to obtain the RHS of any block-equation from the LHS describes an identity equation. This is confirmed by the evaluation of the sum on the LHS of Beal's equation as a stand-alone expression which justifies representing it as the sum of two block-monomials of common block-variable that constitutes a common base of the GCD and suggests a proof of Beal's conjecture.

Finally, the theme for the proof of Beal's conjecture is that we must increase the size of the block-terms of Beal's solution or a primitive equation if we wish to upgrade the solution to higher size by Corollary 2, which is done by multiplying the terms by a proper block-factor. Starting with the RHS term, increasing its size by multiplying it by a proper block-factor, and for the equality to hold, the block-factor must be used to upgrade the terms on the LHS of the equation as well, which necessitates that all terms must have a common factor and must follow Lemma 1. For non-Fermat block-equations of low powers, employing a block-factor to the equation successfully upgrades it to Beal's equation of higher powers by theorem 1; see the table above for some block-equations of low powers. To illustrate the method, let's revisit example 1. The primitive equation $1 + 2^3 = 3^2$ can be upgraded to higher powers by multiplying the RHS by a proper block-bundle of 3^3 to become 3^5. For the equality to hold, the common factor has to be used to upgrade the terms on the LHS of the equation as well on the basis of an identity solution and must follow theorem 1 to produce the equation $3^3 + 6^3 = 3^5$. The block-bundle then is the GCD that is needed to upgrade the equation. Any other non-primitive solution must comply with theorem 1. A successful upgrading process then necessitates a common base-factor among the three terms of any three-term block-equation satisfying Beal's conjecture.

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[2] Wiles, Andrew (1995) "Modular elliptic curves and Fermat's Last Theorem" Annals of Mathematics. **141** (3): 443–551.