Lorentz Symmetry from Multifractal Scaling

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Abstract

We show that relativistic invariance is encoded in the multifractal structure of the Standard Model near the electroweak scale. The approximate scale invariance of this structure accounts for the flavor hierarchy and chiral symmetry breaking in the electroweak sector. Surprisingly, it also accounts for breaking of conformal symmetry in General Relativity and the emergence of a non-vanishing cosmological constant.

Key words: minimal fractal manifold, multifractal scaling, scale invariance, Lorentz group, conformal symmetry, cosmological constant.

1. Introduction

It is well-known that invariance under the Lorentz group (LG) is a fundamental requirement of both Quantum Field Theory (QFT) and the Standard Model (SM). Relativity demands equivalence of inertial frames under Lorentz transformations including rotations, boosts and spacetime inversions. The Poincaré group is a direct extension of LG and accommodates translations in flat spacetime. Lorentz transformations leave the 4-vector inner product invariant and enable the definition of generators associated with the conserved Noether charges of LG.

The object of this work is to explore the unforeseen link between LG and the multifractal structure of SM near the infrared regime set by the electroweak scale ($M_{EW} \approx 250$ GeV). Specifically, exploiting the *minimal fractal manifold* geometry (MFM) of spacetime near

M_{EW}, we show how LG may be naturally mapped to continuous and scale-dependent transformations of four spacetime dimensions described by $\varepsilon = 4 - D << 1$ and referred below to as *dimensional (or scaling) flows* [1-5]. Our findings are consistent with several lines of inquiry where LG, as well as the fundamental gauge structure of the SM, arise as emergent manifestations of low-energy physics [3-8].

The paper is organized in the following way: The second section develops the formal connection between the multifractal description of SM near M_{EW} and the group of Lorentz transformations. Next section examines the implications of this connection, namely,

a) The correspondence between Lorenz generators and the dimensional flow,
b) The interpretation of quantum spin as emerging attribute of the MFM,
c) Understanding of discrete symmetry breaking in the electroweak sector as emerging attribute of the MFM,

The fourth section shows how the large separation of scales induced by the MFM accounts for breaking of conformal symmetry in General Relativity and the non-vanishing magnitude of the cosmological constant. Two Appendix sections are also included to make the paper self-contained and accessible to a large audience.

We caution the reader that this work is entirely provisional. Follow-up research is required to refine, substantiate or challenge our assumptions and findings.

2. MFM and the Lorentz group

MFM is a concept inspired by the Renormalization Group program of QFT and it denotes a spacetime background having arbitrarily small but continuous deviations from four-dimensions ($\varepsilon = 4 - D << 1$). Postulating the MFM is the only sensible way of asymptotically matching all consistency requirements mandated by effective QFT in the conformal limit $\varepsilon = 0$. The underlying rationale, theoretical benefits and implications of the MFM for the development of QFT and SM are extensively detailed in [1-5]. Working in the MFM framework, one finds that:

2.a) Renormalization flow analysis of the generic Landau-Ginzburg-Wilson model reveals the connection between the dimensional parameter ε, low-scale particle masses $m_i = O(m)$ and the far ultraviolet scale $\Lambda_{UV} >> m$ via

$$\varepsilon = O(m^2 / \Lambda_{UV}^2) \tag{1}$$

Renormalization flow consists of continuous variations in scale starting from Λ_{UV} down to $\Lambda < \Lambda_{UV}$. These changes automatically imply that ε is scale-dependent, which means that *the dimensional flow is a consequence of the renormalization process*.

2.b) The MFM geometry of space-time near or above M_{EW} explains the repetitive flavor structure of SM. Its mass spectrum satisfies a "closure" relationship replicating the construction of multifractal sets, namely

$$\sum_{i=1}^{16} (\frac{m_i}{M_{EW}})^2 = 1 \tag{2}$$

2.c) MFM naturally mixes widely separated scales of particle physics and cosmology. The electroweak scale (M_{EW}), the cosmological constant scale ($\Lambda_{cc}^{1/4}$) and the far ultraviolet scale (Λ_{UV}) satisfy the constraint

$$\frac{\Lambda_{cc}^{1/4}}{M_{EW}} = \frac{M_{EW}}{\Lambda_{UV}} \tag{3}$$

Both (2) and (3) agree with experimental observations. It is also worth noting that (1) complies with the latest results from gravitational wave astronomy [23].

If component deviations from classical spacetime coordinates are taken to be independent from each other, the overall deviation ε amounts to the sum of component deviations along the four coordinates ($\mu = 0,1,2,3$), i.e.

$$\varepsilon = 4 - D = \sum_{\mu=0}^{3}(1 - d_{\mu}) = \sum_{\mu=0}^{3}\varepsilon_{\mu} \tag{4}$$

where, following (1),

$$\varepsilon_{\mu} = O\,({m_{\mu}^{2}}/{\Lambda_{UV}^{2}}) \tag{5}$$

For any given SM particle of mass m_i, replacing (1) and (5) in (4) leads to

$$m_i^2 = \sum_{\mu=0}^{3} O(m_{\mu,i}^2) = O(m_{0,i}^2 + m_{1,i}^2 + m_{2,i}^2 + m_{3,i}^2) \tag{6}$$

A glance at (A7) reveals the formal resemblance of (6) to the Euclidean representation of the invariant mass. Based on this comparison, we assume in what follows that the limit of (6) ($O(x) \Rightarrow x$) *replicates exactly the invariant mass associated with the Lorentz transformation of momenta in Euclidean space-time*. Pursuing this interpretation, the four components of the overall deviation ε map to the four translation operators of relativistic QFT.

Let $g_{\mu\nu}$ stand for the components of the metric tensor in Minkowski space-time. The standard definition of the Lorentz boost reads

$$P'^{\mu} = \Lambda^{\mu}_{\nu} P^{\nu} \tag{7}$$

where the set of matrices Λ^{μ}_{ν} are elements of the LG and obey the condition

$$g_{\alpha\beta} = g_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} \tag{8}$$

Previous considerations hint that the analog of (7) applied to the MFM is given by

$$\boxed{(\varepsilon'^{\mu})^{1/2} = \Lambda^{\mu}_{\nu} (\varepsilon^{\nu})^{1/2}} \tag{9}$$

Relation (9) indicates that Lorentz boosts correspond to linear scale transformations on the spacetime components of ε. *On account of (5), these transformations may be understood as continuous changes of the far ultraviolet scale Λ_{UV}, which underlie the dimensional flow*. This interpretation lines up with the view that internal SM symmetries emerge from the MFM structure of spacetime near M_{EW} [3].

3. Implications

We discuss below four key outcomes of our previous analysis.

3.a) First, classical fields are classified based on their transformation properties in response to linear coordinate changes given by:

$$x'^{\mu} = \Lambda^{\mu}_{\alpha} x^{\alpha}$$

Scalars, vectors and tensors transform as

$$\varphi'(x') = \varphi(x)$$

$$V'^{\mu}(x') = \Lambda^{\mu}_{\alpha} V^{\alpha}(x) \tag{10}$$

$$T'^{\mu\nu}(x') = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} T^{\alpha\beta}(x)$$

Following (9), (10) reflects how classical fields are defined by their behavior under dimensional flow. Moreover, since quantum particles are irreducible representations of the LG, whose structure is encoded in (9), it follows that both *fields and their particles emerge as manifestations of the dimensional flow approaching the MFM*.

One can expand this line of reasoning and show that (9) may be used to build the standard classification of quantum and classical fields. To this end, start from the observation that *spinors* are the most basic mathematical objects that can be Lorentz transformed [13]. Interpreting the dimensional parameter $\varepsilon = \{\varepsilon_{\mu}\}$, $\mu = 0,..,3$ as a fundamental four-vector and appealing to (4), (6) and (9), enables the formal construction of Dirac, Weyl, vector and gravitational fields along the path taken by [13-16, 25-26]. However, given the large slew of challenges confronting the SM and General Relativity, the systematic construction of fields from spinors falls short of developing a fully-proven unified field theory [20, 27].

3.b) Secondly, it is known that the infinitesimal expression of a Lorentz transformation takes the form

$$\Lambda^{\mu}_{\nu} = g^{\mu}_{\nu} + \omega^{\mu}_{\nu} \tag{11}$$

where ω^{μ}_{ν} are the six independent parameters of the LG. Relation (11) can be alternatively cast as

$$\Lambda^{\alpha}_{\beta} = g^{\alpha}_{\beta} + \frac{1}{2}\omega_{\mu\nu}(M^{\mu\nu})^{\alpha}_{\beta} \tag{12}$$

or, in matrix form as

$$\Lambda = I + \frac{1}{2}\omega_{\mu\nu}M^{\mu\nu} \tag{13}$$

in which $M^{\mu\nu} = -M^{\nu\mu}$ are the generators of the LG for *rotations* $(\mathbf{J})$ and *boosts* $(\mathbf{K})$ [10, 12]

$$\frac{1}{2}\omega_{\mu\nu}M^{\mu\nu} = i\,(\boldsymbol{\theta}\cdot\mathbf{J} - \boldsymbol{\eta}\cdot\mathbf{K}) \tag{14}$$

Appealing to (4), (6), (9) and (13), (14), hints that the Lorentz generators $M^{\mu\nu}$ arise from the *physical equivalence of all spacetime descriptions of the dimensional flow at a given scale*.

3.c) Third, it is also known that the concept of quantum spin is traditionally linked to the one-particle states representations of the Poincaré group [9-12, 24]. For a massive particle, these representations are labelled by two Casimir operators, namely

$$m^2 = P_{\mu}P^{\mu} \tag{15}$$

and

$$-W_{\mu}W^{\mu} = m^2 j\,(j+1) \tag{16}$$

where j denotes the spin and the Pauli-Lubanski four-vector is given by

$$W^{\mu} = -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}M_{\nu\rho}P_{\sigma} \tag{17}$$

From the above discussion at 2.a), 2.b) and 3.a), one can broadly conclude that the three Wigner invariants of quantum particles (spin, mass and charge) emerge as *observables of dimensional flow*. This conclusion is consistent with the view developed in [1-2], where the connection between MFM and local conformal field theory makes spin a topological property of the MFM.

3.d) Finally, the dimensional flow and the onset of the MFM near M_{EW} strongly supports the *chirality* of fermion coupling to massive gauge bosons [12, Appendix B]. In particular,

1) The high-energy regime of QFT justifies upgrading classical differential operators to fractional operators and the passage to *fractional dynamics*. Breaking of parity and time-reversal symmetries in the electroweak sector follows from these premises [1-2, 17].

2) The high-energy regime of QFT drives physical processes out-of-equilibrium, a setting which naturally fails to comply with LG space-time symmetries [17-18].

3) Breaking of parity and time-reversal symmetries may be also linked to the discrete scale invariance properties of the MFM [3, 5].

Following this line of arguments, one is tempted to speculate that the *1-state neutrino helicity* and the *2-state photon polarization* stem from breaking of parity and time-reversal symmetries near the MFM endpoint of the dimensional flow (Appendix B). Elaborating further, both photon and neutrino, as massless and nearly massless fields respectively, provide the transition border separating baryonic matter from *Cantor Dust* [19, 21-22].

4. MFM and the cosmological constant

Referring to (3) and letting the far ultraviolet scale reach the Planck regime, one finds

$$\Lambda_{UV} = O(M_{Pl}) \Rightarrow \Lambda_{cc}^{1/4} = O(\frac{M_{EW}^2}{M_{Pl}}) = O(G_N^{1/2} M_{EW}^2) \qquad (18)$$

It follows from (18) that the cosmological constant is intrinsically linked to the very existence of classical gravitation and of the Newton constant (G_N).

5. Concluding remarks

We conjectured that Lorentz symmetry arises from the minimal fractal manifold structure of spacetime (MFM) around the electroweak scale. As the Standard Model behaves as a *self-contained multifractal set* near this scale, invariance under the Lorentz group is organically tied with the field content of low-energy physics.

Combining this work with [1-5] leads to the following conclusions on the dimensional flow and its asymptotic approach to scale invariance on the MFM:

1) Transformations leaving the norm of $\varepsilon = 4 - D << 1$ invariant (or nearly-invariant) can be interpreted as "rotations in ε-space" and associated with conventional Lorentz and gauge symmetries of QFT,
2) Transformations that change the norm of $\varepsilon = 4 - D << 1$ embody the nonlinear dynamic aspects of QFT and account for the formation of the observed flavor content of the SM.

APPENDIX A

The formal connection between Euclidean and Minkowski field theories in $D \geq 1$ dimensions is carried out using the technique of *Wick rotation*. To fix ideas, consider the action of a real scalar field

$$S = \int_{-\infty}^{\infty} dt \int d\vec{x} [\frac{1}{2}(\frac{\partial \varphi}{\partial t})^2 - \frac{1}{2}(\frac{\partial \varphi}{\partial x_i})^2 - V(\varphi)] \tag{A1}$$

The change of time variable to $\tau = it$ turns (A1) into the Wick-rotated action

$$S_E = \int_{-\infty}^{\infty} d\tau \int d\vec{x} [\frac{1}{2}(\frac{\partial \varphi}{\partial \tau})^2 + \frac{1}{2}(\frac{\partial \varphi}{\partial x_i})^2 + V(\varphi)] \tag{A2}$$

If the potential is a quadratic function of the scalar field,

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 \tag{A3}$$

(A1) gives the standard Klein-Gordon equation in Minkowski spacetime

$$(\frac{\partial^2}{\partial t^2} - \nabla^2)\varphi + m^2 \varphi = 0 \tag{A4}$$

where

$$m^2 = E^2 - \mathbf{P}^2 \tag{A5}$$

From (A2), the corresponding equation in Euclidean formulation reads

$$(\frac{\partial^2}{\partial \tau^2} + \nabla^2)\varphi + m^2 \varphi = 0 \tag{A6}$$

(A5) and (A6) motivate the definition of *Euclidean invariant mass* as

$$m^2 = -m_E^2 = E_E^2 + \mathbf{P}^2 = E_E^2 + \sum_{i=1}^{3} P_i^2 \tag{A7}$$

APPENDIX B

It is known that only left-handed (L) components of fermions participate in charged current weak processes, that is, the electroweak gauge bosons couple exclusively to the L fermion components. To accommodate this *chiral property* of SM fermions, the L and right-handed (R) fermions are assigned to different representations of the $SU(2)\times U(1)$ group, with the R components being labelled as singlets of $SU(2)$. SM fermions are either *massive* (charged leptons and quarks) or *nearly massless* (neutrinos). When fermions have vanishing masses, the Pauli-Lubanski vector and linear momentum are linearly related via [24]

$$W^2 = P_\mu P^\mu \Rightarrow W_\mu = \lambda P_\mu \tag{B1}$$

λ is known as *helicity* and represents the spin associated with a massless particle such as the Weyl neutrino ($j = |\lambda|$). It can be shown that for each P of a massive particle, there are $2j+1$ linearly independent solutions of the free wave equation characterized as states of definite helicity. Massless particles (photons and gluons) carry *only two* independent helicity states ($\lambda = \pm j = \pm 1$) associated with the (L) and (R) polarizations of the vector field. Neutrinos are nearly-massless and carry *only a single* helicity state ($|\lambda| = 1/2$), which can be either left-handed (L) or right-handed (R). However, only left-handed neutrinos (or right-handed antineutrinos) participate in weak interactions.

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