The algebra of non local quantum gravity

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Abstract

The elementary algebra underlying the non local neutrino hypothesis is used to compute the low z value of Hubble's constant in terms of other physical constants, using the $R_h = ct$ semiclassical cosmology. It is assumed that quantum gravity breaks the equivalence principle, in conjunction with a quantum Higgs mechanism. This new view of the electroweak vacuum indicates an absence of dark matter and dark energy. Mass quantisation uses the Brannen-Koide scheme.

Contents

1 A new cosmology

The failure of the ΛCDM paradigm, for both dark energy and dark matter, forces theorists to consider gravity beyond general relativity. General relativity occurs as an emergent property of a deeper quantum theory of gravity, which presumably has applications that break the restrictions of the classical theory. Since Lorentz invariance underpins all the successes of 20th century physics, it is clear that we must instead break the equivalence principle, mildly. The dualities of M theory suggest a search for a *maximally non local* cosmological principle, closely associated to the quantisation of rest mass.

The hypothesis of non local neutrinos is sufficient to solve all cosmological conundrums [1]. The history of this discovery goes as follows. The cosmology for the mirror neutrino hypothesis [1] was developed immediately after the 2010 discovery [2][3][4] of the exact correspondence between a (mirror) neutrino rest mass and T_C, the present day temperature of the CMB. Successful predictions included a computation of the observable mass M_U of our universe. The possible identification of non local mirror neutrinos and antineutrinos remained unclear until recently, when connections to holography were discussed in [5], based on older work on quantum inertia [6]. Non local neutrinos were originally designed to recover the successful MOND paradigm [7] for galactic rotation curves. The concept of quantum inertia has been properly developed by McCulloch et al [8][9][10][11] and the specialness of neutrinos follows from the ribbon spectrum [12][13] for Standard Model states, which is justified by applications of category theory in quantum gravity. In 2006, the author used category theory to invent polytopes for scattering amplitudes [14] in QCD, and Brannen [15][16] showed how Koide's formula [17][18] for the low energy charged lepton masses could be applied to neutrino masses. Over this decade, there were many online discussions about the cosmology between the author, Brannen and the originator of the $R_h = ct$ solution, Riofrio [19]. Overwhelming observational evidence selecting $R_h = ct$ over ΛCDM is summarised in [20].

In all of this, twistor [21][22] geometry $\mathbb{T} = \mathbb{CP}^3$ is the proper setting for compactified Minkowski space $\mathbb{M} = S^3 \times S^1 = SU(2) \times U(1)$. The gauge group $SU(2)$ only emerges with non local effects, crucial to electroweak CP violation. This is well motivated by universality in quantum computation. Similarly,a $U(1)$ emerges in the string net condensation picture [23].

In contrast to the common habit of tripling sets of solutions to the Dirac equation, we insist there is only one copy of the Dirac and Klein-Gordon equations. As is well known, massless solutions are H^1 cohomology classes on a suitable restriction of $\mathbb{T}$. A massive solution was first defined as an H^2 class $\psi_1 \otimes \psi_2$ in [24], the ψ_i being massless Dirac spinors. Over the last decade, the massless solutions have been successfully employed in scattering amplitudes in QFT, under the assumption that locality is central. For mass, however, the non local inversion of operators is absolutely essential, as we will show. This inversion, solving the so called *googly problem*, is rooted in a T duality for M theory, which discards the supersymmetry of strings in favour of the supersymmetry of the quantum Fourier transform [13]. Such dualities are well studied in the context of entanglement measures for Jordan algebras [25][26][27]. The categorical language is well developed for spin foam models [28][29][30] of quantum gravity.

By definition, an operator is *maximally non local* if it can be described by two distinct local operators: a standard UV local operator and a cosmological scale IR operator. This is the key to dual quantum inertia, which associates non local states to cosmological boundaries in a new form of the holographic principle [31]. The T_C correspondence identifies non local (once mistakenly thought to be mirror states) neutrinos and CMB photons, applying the Fourier supersymmetry to map neutrinos to photons. This identification requires nothing more than Wien's constant and T duality, where T_C is the Hawking temperature for a universal mass M_U dual to the minimal neutrino mass.

Astonishingly, we find that *non local neutrinos are antineutrinos*, implying that the non locality of neutrino mass underpins the true electroweak vacuum, for which the Higgs mass is roughly $m_H \sim \sqrt{m_\nu m_P}$ with m_P a Planck (or GUT) scale mass. In other words, although our semiclassical intuition puts the cosmological boundary far away, pair production everywhere is a form of Hawking radiation. Every time we observe an antineutrino in the laboratory, its true state is the cosmological right handed state. The remaining task is to see how charged lepton pair production e^-e^+ also arises from this mechanism, and how color is included in the ribbon scheme.

On the local side, the Hawking event horizon is replaced with a causal Rindler horizon, whose Unruh temperature [9] decreases an inertial mass m_i by a factor of $\lambda/4R_h$. That is, Unruh wavelengths only fit into twice the Hubble diameter at any cosmological time t. Quantum inertia easily recovers the MOND description [7] of galactic rotation curves, and also gets rid of dark energy, which is eliminated anyway by the FLRW $R_h = ct$ theory (for which the ficticious black hole dark matter approximates the effect of mirror states).

It is impossible to overestimate the importance of maximal non locality. Planck's black body spectrum is based on Wien's law, and it now follows that energies away from Wien's peak are associated to the uncertainty of non local mass energy, for short lived virtual states in the true vacuum. This manifestation of the uncertainty principle, like the Higgs mechanism, is deeply rooted in the arithmetic properties of the quantum Fourier transform. Bosonic and fermionic statistics are related by Fourier supersymmetry, and the T dual confinement of fermions to our observable universe resolves the black hole information paradox, improving on Penrose's classical cyclic cosmology [32].

Bulk boundary correspondences for general relativity are known to come from entanglement renormalisation in tensor networks. Mathematically, category theory is the correct setting for discussing multistate entanglement and quantum computation, since algorithmic processes transcend the limitations of set theory. The ribbon particle spectrum [12][13] of Standard Model states has a natural setting in modular tensor categories, which arise as representation categories for quantum Hopf algebras, and notably underlie the rational conformal field theories that one expects to appear in a finite, arithmetic quantum gravity theory.

Here a ribbon twist represents electromagnetic charge, which naturally appears when we attempt to localise a neutral neutrino ribbon strand. Each particle is a three stranded ribbon diagram. For charged leptons, all three strands are twisted, while quarks have some twisted and some untwisted strands, leaving quarks in the non local domain. Quark confinement is a property of three dimensional categories [33][14], which allow the cyclic $3+1$D ribbon pictures used for amplitudes. Now leptons and quarks are transformed into one another through composition of a particle with a triplet of ribbon twists. Fermions are braided in B_3, while bosons represent the identity in B_3. (Recall that B_3 covers the modular group $PSL(2,\mathbb{Z})$).

The right handed singlets required for Yukawa couplings are *not* mirror states, but occur already in the SM ribbon spectrum, along with all the antiparticle states, including the neutrinos and antineutrinos. Only the neutrinos have the property that (i) no right handed local states exist and (ii) $\overline{\nu}$ masses are slightly different from ν masses, due to the presence of a fundamental arithmetic $\pi/12$ phase in the Brannen-Koide rest mass phenomenology. Mirror fermion braids, namely the crossing swapped B_3 braids, do not occur in nature except as information on the cosmological boundary, ensuring strict chirality for all Standard Model particles. Photons are the only massless electroweak bosons because every strand in the ribbon diagram is neutral, whereas the Z boson utilises a more subtle color structure, dictated by color gravity [34][35].

Here, IR (and UV) unification will rely crucially on the monster moonshine of bosonic string theory, but we do not approach this using stringy techniques. Rather, the Leech lattice should come from color gravity, with its automorphisms neatly wrapped up in the 27 dimensions of the exceptional Jordan algebra over the octonions.

2 Basic algebra

2.1 Mutually unbiased information

The quantum measurement of spin is characterised not so much by the three Pauli matrices

$$X = i\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad Y = i\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}, \quad Z = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{1}$$

but by the unitary sets of normed eigenvectors of these operators,

$$F_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \quad R_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}, \quad I_2 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{2}$$

Here F_2 is the 2×2 quantum Fourier transform. Let ω be the primitive cubed root of unity $\exp(2\pi i/3)$. For three outcome measurements, the four bases are

$$F = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \overline{\omega}\\ 1 & \overline{\omega} & \omega\end{pmatrix}, \ R = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & \omega & 1\\ 1 & 1 & \omega\\ \omega & 1 & 1\end{pmatrix}, \ R^{-1} = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & \overline{\omega}\\ \overline{\omega} & 1 & 1\\ 1 & \overline{\omega} & 1\end{pmatrix}, \ I. \tag{3}$$

In these sets, every pair of eigenvectors v and w from two distinct bases is *mutually unbiased* [36][37], meaning that $|v \cdot w| = 1/\sqrt{d}$ for d the dimension. Observe that R_2 is a representation of the eigth root of unity $\pi/4$, and R of the twelfth root $\pi/6$, so that a six dimensional system that combines one qubit with one qutrit picks up a phase factor of $\pi/12$, the difference between $\pi/6$ and $\pi/4$. This essential arithmetic phase is crucial in everything that follows, appearing in the neutrino mass matrices of section 2.3.

In any prime power dimension $d = p^r$ there exist $d + 1$ mutually unbiased bases, and these may be represented by the matrices F_d, I_d and R_d^j ($j \in 1, \cdots, d-1$) [38]. The matrix R_d is always a 1-circulant, for which each row is a one step cyclic permutation of the row above. 1-circulants are diagonalised [39] by the Fourier transform F_d, as in the example

$$p_1 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix} = F \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} F^{\dagger}. \tag{4}$$

Omitting F_d from the basis set, the remaining d circulant mutually unbiased bases represent the abelian cyclic group C_d. Thus any finite abelian group is built out of such bases. For $d = 3$, we define three spatial points

$$x_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad x_3 = I, \tag{5}$$

in the fundamental representation of $C_3 \subset S_3$. The Fourier transform F gives the momentum points p_1, p_2 and p_3. These six points will play a key role in section 4.1.

In $d = 3$ there are only 1-circulants and 2-circulants, which are the two step permutations. A general 3×3 circulant matrix over $\mathbb{C}$ is a linear combination of the six permutations in S_3. Section 4.2 defines the Hopf algebra structure for such circulants. Circulant parameterisations for unitary matrices are used below to analyse the CKM and PMNS mixing matrices [6][16].

2.2 Spacetime and Jordan algebras

A study of black holes and entanglement in M theory [27] relates division algebras to the symmetries of supersymmetric Yang-Mills theories. We are interested in the application of 3×3 matrix algebras to canonical mass energy spectra. There are only four division algebras, namely $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ and the octonions $\mathbb{O}$, which are all used to define matrix algebras.

In order to generalise to higher dimensions, a complex number is written in the form $Z = z_0 + z_1 e_1$, where $e_1 = i$ satisfies ${e_1}^2 = -1$. A quaternion $h_0 + h_1 e_1 + h_2 e_2 + h_3 e_3$ in $\mathbb{H}$ is instead written in terms of the Pauli matrices of (1),

$$Q = tI + xX + yY + zZ = i \begin{pmatrix} t + z & x + iy \\ x - iy & t - z \end{pmatrix}, \tag{6}$$

because the matrix Q encodes the Lorentzian metric of spacetime

$$\det(Q) = -t^2 + x^2 + y^2 + z^2. \tag{7}$$

A vector Q in spacetime transforms to a 2×2 circulant under the conjugation action of the Lorentz group $SL(2, \mathbb{C})$,

$$-iQ = \begin{pmatrix} n & m \\ p & q \end{pmatrix} \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} q & -m \\ -p & n \end{pmatrix} \tag{8}$$

with

$$t = a, \; z = b(qm - pn), \; x = \frac{b(q^2 + n^2 - p^2 - m^2)}{2}, \; y = \frac{b(p^2 + n^2 - q^2 - m^2)}{2}. \tag{9}$$

In twistor geometry one usually complexifies spacetime, but we see here that this won't put three degrees of freedom into a circulant, for which x, y and z are all multiples of b. What we need are three copies of Q, embedded in three ways in a 3×3 matrix. This is the natural way to account for three dimensions of space.

The octonions are a little like two copies of $\mathbb{H}$. An octonion [40] in $\mathbb{O}$ requires 7 units e_i for $i \in 1, \cdots, 7$, such that ${e_i}^2 = -1$. If we write

$$K = k_0 + k_1 e_1 + k_2 e_2 + k_3 e_3 + \cdots + k_7 e_7, \tag{10}$$

then conjugation in $\mathbb{O}$ is defined by

$$\overline{K} \equiv k_0 - k_1 e_1 - k_2 e_2 - k_3 e_3 - \cdots - k_7 e_7, \tag{11}$$

so that the norm is given by

$$N(K) \equiv K\overline{K} = \sum_{i=0}^{7} {k_i}^2. \tag{12}$$

The seven units e_i are related by the Fano plane diagram, which we omit for now.

By definition, a commutative but not necessarily associative Jordan matrix algebra [41][42] has a product

$$x \circ y = \frac{1}{2}(AB + BA) \tag{13}$$

for two matrices A and B. We are especially interested in the algebras $J_3(A)$ of 3×3 Hermitian matrices over a division algebra A. The *exceptional Jordan* algebra is the case $J_3(\mathbb{O})$. Let a, b and c be real variables and X, Y, Z three octonions. A Hermitian element H of $J_3(\mathbb{O})$ is written

$$H = \begin{pmatrix} a & X & \overline{Y} \\ \overline{X} & b & Z \\ Y & \overline{Z} & c \end{pmatrix}. \tag{14}$$

As with $J_3(\mathbb{C})$, which we will use often, H is a 1-circulant when $X = Y = Z$. Three directions appear properly when 2-circulants are included.

Since $\mathbb{O}$ is often used to analyse color symmetry, one expects matrix Jordan algebras to feature prominently in color gravity [34][35]. For now, we focus on the complex case $J_3(\mathbb{C})$.

2.3 Koide rest mass triplets

In QFT one often studies running masses at special pole values away from the expected low energy value for the rest mass. However, the Standard Model has nothing to say about the actual rest masses that we observe, and the behaviour of electroweak couplings suggests a link between low energy and pole masses. If rest mass is generated by a duality between the smallest and the largest possible scales, it would not be surprising if the low energy masses took a simple form. Indeed, they do.

Observe that a 3×3 complex Hermitian circulant requires only two real parameters. Our basic physical degree of freedom is the square root of mass energy $\sqrt{M}$, which introduces a third parameter, the mass scale. Such a circulant is conveniently expressed in the form

$$\sqrt{M} = \frac{\sqrt{\mu}}{r} \begin{pmatrix} r & \delta & \overline{\delta} \\ \overline{\delta} & r & \delta \\ \delta & \overline{\delta} & r \end{pmatrix} \tag{15}$$

for a dimensionful scale μ and complex phase δ. In the 1980s, Koide [17][18] discovered a relation between the three charged lepton masses at low energy, which later turned out to be remarkably accurate. It corresponds to the choice $r = \sqrt{2}$. Brannen [15] found in 2006 that $r = \sqrt{2}$ also works for the conjectured neutrino masses of the normal hierarchy.

Why is $\sqrt{2}$ special? As usual, ω is the primitive cubed root of unity. Setting $\mu = 1$ for the moment, let λ_j for $j = 1, 2, 3$ be the three eigenvalues

$$\lambda_j = 1 + \frac{2}{r}\cos(\delta + \omega^j) \tag{16}$$

of $\sqrt{M}$. Since the Fourier transform F diagonalises $\sqrt{M}$, the determinant is defined by the cubic

$$\det(\sqrt{M}) = \lambda_1\lambda_2\lambda_3 = r^3 - 3r + 2\cos(3\delta). \tag{17}$$

At $\delta = \pi/6$, the determinant vanishes at $r = \sqrt{3}$. When $\delta = \pi/12$, it vanishes at $r = \sqrt{2}$. These are our basic two values of r. All lepton masses are of the form

$$m_i = \mu\lambda_i^{\,2}. \tag{18}$$

Empirically, for the charged leptons, $\mu_l = m_p/3$, where m_p is the proton mass, and $\mu_\nu \simeq 0.01$eV for the neutrinos and antineutrinos. The charged lepton phase is $\delta_l = 2/9 + \epsilon$ for some very small quantity ϵ, while the neutrino phase $\delta_\nu =$

$2/9+\pi/12$. We associate this special $\pi/12$ phase with the improper localisation of neutrinos in the charged lepton spacetime, because it turns out that the $\overline{\nu}$ phase is $\delta_{\overline{\nu}} = 2/9 - \pi/12$. It is the charged leptons that are fundamental, in the sense that the adjoint Cayley permutation representation of S_3 in S_6 looks like a three stranded ribbon diagram with *twisted* strands, which is the $e^{\pm}$ diagram.

At $\pi/6 = \delta_\nu - \delta_{\overline{\nu}}$ and $r = \sqrt{2}$, we find a new rule

$$\sqrt{m_1 m_2 m_3} = \frac{1}{27}(\sqrt{m_1} + \sqrt{m_2} \pm \sqrt{m_3})^3 \tag{19}$$

for the triplet of masses. A similar rule holds in general. The rest masses within a triplet may be expressed in terms of a representative dimensionless mass m_0 using the differences $m_i - m_j$ as a function of δ. We compute

$$\Delta_\omega = \frac{3}{2}\cos^2\delta - \frac{3}{2}\sin^2\delta - \sqrt{3}\cos\delta\sin\delta + 3\sqrt{2}\cos\delta + \sqrt{6}\sin\delta \tag{20}$$

$$\Delta_{\overline{\omega}} = \frac{3}{2}\cos^2\delta - \frac{3}{2}\sin^2\delta + \sqrt{3}\cos\delta\sin\delta + 3\sqrt{2}\cos\delta - \sqrt{6}\sin\delta$$

so that

$$\begin{aligned} \det(M) &= m_0(m_0 - \Delta_\omega)(m_0 - \Delta_{\overline{\omega}}) \\ &= {m_0}^3 - {m_0}^2(\Delta_\omega + \Delta_{\overline{\omega}}) + m_0(\Delta_\omega\Delta_{\overline{\omega}}). \end{aligned} \tag{21}$$

Weierstrass cubic coefficients for a mass triplet are defined as usual by

$$g_3 = 0, \quad g_2 = -\Delta_\omega\Delta_{\overline{\omega}}, \tag{22}$$

in which case we define

$$y^2 = x^3 - \Delta_\omega\Delta_{\overline{\omega}}x. \tag{23}$$

At $r = \sqrt{2}$, a mass cubic is only in this form for $\delta = 45^\circ + 38.71^\circ$, but we now consider arbitrary triplets. For pair production $\gamma \to e^- e^+$ the basic triplet is $(0, \lambda, -\lambda)$. The Weierstrass coefficient g_3 in a cubic is a multiple of the Eisenstein series E_6 from (69). When $E_6 = 0$ in (22), g_3 is fixed by the phase $z = i$. Taking $E_4(i)$ and solving $g_2 = 4\Delta_\omega\Delta_{\overline{\omega}}$, we obtain the special phase $30^\circ - 8.49^\circ$, which we will see later on.

Consider the $\overline{\nu}$ phase $\delta_{\overline{\nu}} = 2/9 - \pi/12$. In this case, the dimensionless eigenvalues are written as

$$m_1 = 0.1169 + 5.7062, \quad m_0 \equiv m_2 = 0.1169, \quad m_3 = 0.1169 - 0.0569, \tag{24}$$

where $\mu_\nu m_0 = 0.00117$eV is the present day CMB temperature.

A value of $r = \sqrt{3}$ works for the down quark (d, s, b) triplet, with $\delta_d = 4/27$ [6]. When the lepton triangle is inscribed inside the quark triangle, simple trigonometry suggests a value of $r = 1.76$ for (u, c, t), when $\delta_u = 2/27$. Clearly, these rational phases have something to do with charge! Neutrinos lose their charge because ν and $\overline{\nu}$ cannot combine to form a standard Dirac spinor, since they have distinct masses.

Viewing the complex phase δ as a restricted choice of norm 1 octonion, M belongs to $J_3(\mathbb{O})$. Since 1-circulants commute, the Jordan product is simply the ordinary matrix product.

2.4 The cosmological Higgs mass

Our Higgs particle has a mass $m_H \sim \sqrt{\mu_\nu m_P}$ for μ_ν a neutrino scale and m_P the Planck mass. This is written in the form

$$\mu_\nu = \frac{m_H^2}{m_P}, \tag{25}$$

known as a see-saw relation. The local neutrino see-saw traditionally uses a right handed neutrino mass in place of m_P, but we don't have right handed neutrinos. The Higgs mass replaces the Dirac mass of a neutrino. Given a formula for μ_ν, along with the definition

$$m_P = \sqrt{\frac{ch}{2\pi G}}, \tag{26}$$

we would have an estimate of m_H from quantum gravity.

3 Non local structure

3.1 Note on the Dirac equation

The smallest simple, nonabelian finite group is $A_4 \subset S_4$ in dimension 4. Unlike for $d = 3$, where there were only *left* and *right* non identity 1-circulant permutations, the subgroup C_4 contains a third one, known as γ_5 [43]. In $d = 4$, the Fourier transform

$$F_2 \otimes F_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \tag{27}$$

lists the eigenvectors of

$$\gamma_5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{28}$$

and the Dirac representation gives the other four measurement matrices,

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}. \tag{29}$$

We consider the measurement perspective to be more fundamental than the Dirac representation, which hinges largely on a pre-existing Lorentz group. In quantum gravity, classical spacetime should emerge from the underlying topological information.

B_3 representations are used in the theory of quantum computation to generate a dense subset of $SU(2)$, with a finite set of operators that are computationally universal. The canonical example is the system of three Fibonacci anyons (to appear in the next paper) with precisely the charge 1/3 that we need for a ribbon strand. A qutrit uses four anyons, and the minimal naive Hopf algebra dimension for both together is of course 24.

Our version of supersymmetry uses the six real dimensions of B_3 in its adjoint Cayley representation. Section 4.1 gives the Standard Model particle spectrum in terms of 3×3 complex matrices, while section 2.2 suggests that 27 dimensions can be packaged into the 3×3 matrices over $\mathbb{O}$. In $J_3(\mathbb{O})$, the 24 dimensions sit in the triplet of off diagonal octonion entries. An interesting duality between (the real part of) $\mathbb{C}$ and $\mathbb{O}$ is studied using ribbon graphs in [44]. Note also that the integral octonions $\mathbb{Z}_{\mathbb{O}} \sim \mathbb{Z}^8$ can be embedded densely in $\mathbb{C}$ using golden ratio parameters associated to a pentagon tiling. That is, if we do not require nonassociativity for the leptons, the algebra over $\mathbb{C}$ should do everything for us.

3.2 Quantum inertia

If quantum gravity subsumes general relativity, one should not be afraid to break the equivalence principle in domains outside solar system physics, where GR is a stunning success. The hypothesis of quantum inertia will immediately rid us of dark matter and dark energy, which have failed every observational test.

To every energy is associated a wavelength,

$$E = \frac{hc}{2\pi\lambda}. \tag{30}$$

A minimal uncertainty satisfies the constraint

$$Et = \frac{h}{4\pi}. \tag{31}$$

The maximum possible interval of time t is t_0, the present age of the universe, which defines a radius $R_h = ct_0$ [19][20] called the Hubble radius. Wavelengths are restricted to $\lambda \leq 4R_h$, since a quarter wavelength can fit inside the observable universe [8][9]. The uncertainty principle defines a complementary energy $2E_0 = h/4\pi t_0$ so that

$$\frac{h}{8\pi E_0 t_0} = 1 = \frac{\lambda_0}{4ct_0} = \frac{\lambda_0}{4R_h}. \tag{32}$$

In other words, $\lambda_0 = 4R_h$ is the limiting wavelength, for which an inertial mass $m_i \to 0$ in the conformal boundary. In general, we write [8][9]

$$m_i = m_g\left(1 - \frac{\lambda}{4R_h}\right). \tag{33}$$

Black hole physics teaches us that quantum mass energy is thermodynamic in nature. Each energy is associated to a temperature through Wien's displacement law

$$E \equiv \frac{hc}{2\pi\lambda} = \beta kT, \tag{34}$$

where Wien's constant $\beta = 4.965$ was originally used by Planck to derive the black body spectrum. For a local, accelerating mass, the Unruh temperature T_U depends on the magnitude a_U of the acceleration,

$$kT_U = \frac{ha_U}{4\pi^2 c}. \tag{35}$$

Using Wien's law to define E_U and λ_U, it follows that a maximal Unruh wavelength in (33) corresponds to a minimal acceleration a_0, which agrees with the MOND hypothesis [7] for galactic rotation curves. The connection between quantum inertia and holography is discussed in [10] [11].

Employing T and S dualities, we now take seriously [6] the inversion of mass in the Hawking temperature

$$kT_H = \frac{hc^3}{16\pi^2 GM}. \tag{36}$$

When $T_H = 1\text{eV}$, which is the CMB temperature T_C at a redshift of $z_0 = 1090$, the Hawking mass M equals 10^{52}kg, the correct mass M_U for the observed universe. CMB photons are Hawking radiation for the universe. Using (34), we find a mass $m = E/c^2$ for a CMB photon, $m_0 = 0.00117\text{eV}$, which is precisely an $\overline{\nu}$ rest mass [2], so that T_C is derived from quantum information theory. The CMB is the observable boundary to the universe, unifying the long range mediators: photons, neutrinos and gravitons. Section 4.1 defines the supersymmetry that turns Standard Model fermions into Standard Model bosons. With an Unruh wavelength for T_C, a CMB fermion has a characteristic acceleration a_C related to the Planck length L,

$$m = \frac{L^2 \beta a_C}{G}. \tag{37}$$

The minimal temperature at the maximum wavelength defines a minimal mass of 10^{-54}kg, or minimal energy of 10^{-37}J. When z_0 is the universal CMB redshift, the Hawking temperature for M_U is associated to $z_0 m_0 = 1.29\text{eV}$, which is an effective sterile $\overline{\nu}$ mass. The resulting derivation of the non local speed of light gives

$$c = \frac{h\beta}{32\pi^2 m_0 z_0 R_h} \tag{38}$$

in terms of other fundamental constants. Observe that c grows smaller as the universe expands, starting at a large value in the CMB epoch. This cosmologically varying c was used by Riofrio [19] to explain the horizon problem. Rearranging for R_h, we have a derivation of Hubble's expansion parameter

$$H \equiv \frac{1}{t_0} = \frac{c}{R_h}. \tag{39}$$

Future astrophysical measurements of H should always be compared to neutrino masses.

Table 1: neutrino masses (eV)

ν	0.0507	0.0089	0.0004
$\overline{\nu}$	0.0582	0.00117	0.0006

Table 2: mixing phases (°)

	$2(\pi/12 - 4/27)$	$1/24 - 1/27$	$1/24$
0	13.02	0.26	2.40
	$\pi/4 - 5/27$	$4/27$	$\pi/4 - 2/27$
0	34.4	8.5	40.8

3.3 Quantitative results

Table 1 lists both $\pm\pi/12$ neutrino masses, assuming that the $\delta_{\overline{\nu}}$ phase does in fact correspond to $\overline{\nu}$. There are no right handed neutrinos. The CMB temperature $T_C = 2.725\text{K}$ was found to coincide with the central 0.00117eV in 2010, when the MINOS experiment [45] first observed a difference in ν and $\overline{\nu}$ masses through neutrino oscillations. Resulting initial developments in the new cosmology appeared in [2][3][4].

Anomalies in oscillation experiments remain to be clarified, but a local $3 + s$ scenario with steriles now seems highly unlikely [46], because active sterile mixing is tightly constrained. With non local neutrinos, there is the option of ignoring the old LSND result while still taking seriously the reactor $\overline{\nu}$ anomaly and MiniBooNe low energy excess.

The Koide phases are closely related to the Euler angles of the PMNS [47][48] and CKM [49][50] mixing matrices for leptons and quarks. Table 2 shows the best bet to date for mixing angles, all close to the experimental values. Let us start with the 4/27. Triality acts on the X, Y and Z octonions of (14) with a phase θ in such a way that the complex circulant

$$\begin{pmatrix} a & X & \overline{X} \\ \overline{X} & a & X \\ X & \overline{X} & a \end{pmatrix} \tag{40}$$

satisfies $\overline{\theta} X = \theta^2 \overline{X}$. For the charged lepton phase $X = 2/9$, the solution is $\theta = 4/27$. The phase $\pi/6 - 4/27$ also answers a second question: which δ solves $g_2 = 4\Delta_\omega \Delta_{\overline{\omega}}$ when $g_2(q)$ depends on the cubic q parameter for $m_1 m_2 m_3 = g_3 = 0$.

A unitary mixing matrix U of the form (55) has a maximal CP phase of $\sim 3\pi/2$, in rough agreement with current experimental hints [46].

To summarise, the basic physical consequences of non local quantum gravity include

1. distinct ν and $\overline{\nu}$ masses in the Brannen-Koide scheme
2. no dark matter: a minimal acceleration accounts for MOND

3. no dark energy: $\sqrt{\Lambda} \sim 10^{60}$ is the ratio M_U/m_P of maximal and minimal spacetime masses

4. a calculation of $M_U \sim 10^{52}$kg [6]

5. more astrophysical structure at high z due to an increase in t_0

6. an effective sterile $\overline{\nu}$ mass of $1.29\text{eV} = m_0 z_0$

7. a derivation of c in terms of other constants, (38)

8. a derivation of the Hubble constant H in terms of fundamental constants

9. a possible e^+ excess at high energy due to non local vacuum creation

3.4 The mass gap and other questions

When observation is bounded by the limits of measurement, the singularities of classical cosmology are removed. Cosmic censorship follows from T duality and quantum inertia. In fact, one can view quantum inertia as a result of a cosmic censorship principle.

The natural neutrino mass gap, and related cosmological masses, introduces a quantisation of mass energy to supersymmetric Yang-Mills theories. Given the importance of Fourier supersymmetry in this construction, it seems clear that a rigorous definition of a field theory only exists in the case of massive, supersymmetric Yang-Mills. This is one of the famous Millenium Problems, asking for an axiomatic scheme for SYM, which we presume to be fully categorical, like the polytopes of scattering theory [14].

Quantum gravity is expected to illuminate a number of Millenium Problems, a second one of note being the Riemann hypothesis. The zeta function $\zeta(s)$ is often interpreted as a partition function summing over a canonical energy spectrum. Looking at the terms of $\zeta^2(s)$, which are written as divisor functions, we see that the underlying q-numbers all lie on the unit circle precisely when the real part of s equals $1/2$. Such q-numbers are a part of the structure of modular ribbon categories, which we use here to analyse Standard Model states. They occur also for rational conformal field theories, which we imagine being related to the true fractal dimensions of ribbons in the quantum chaos of the spacetime foam.

The geometry of cubics underlies modern methods in classical cryptography, while protocols for quantum computation are most easily described using diagram techniques. Entanglement entropy in tensor networks is used to study general relativity. Surely the dualities and trialities of quantum gravity will extend such successes to new heights.

4 Modular tensor categories

4.1 Chiral ribbon states and supersymmetry

The next version of this paper will have pictures, but for now we stick to the associated matrices. Supersymmetry in non local quantum gravity transforms Standard Model fermions into Standard Model bosons, without exception. A fully chiral spectrum uses braid crossings, but every state is distinguished on projection to the underlying permutation groups. In other words, there is a 3×3 matrix representation for each state.

In this scheme [12][13], there is only one generation and the neutrinos are massless, so that ν and $\overline{\nu}$ annihilate to a photon identity

$$\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{41}$$

It is important to understand that we look at electroweak symmetry breaking in reverse. Physical states are natural in the ribbon scheme, while gauge states are not. The neutrinos are the cyclic permuations

$$\nu = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \overline{\nu} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{42}$$

Electromagnetic charge is a set of twists in the three ribbon strands, where distinct twists may be assigned to each strand. We can represent this by replacing the ones in the matrices by one of three phases: 1 for neutral, ω for $+1/3$, or $\overline{\omega}$ for $-1/3$. Then the charged leptons are

$$e_L^- = \overline{\omega} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^+ = \omega \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{43}$$

which indeed compose to the identity. Similarly,

$$e_L^+ = \omega \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^- = \overline{\omega} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{44}$$

For the quarks, just put the charges onto individual strands, as in the colored matrices

$$u_L(1) \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix}, \; u_L(2) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega & 0 & 0 \end{pmatrix}, \; u_L(3) \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & 1 \\ \omega & 0 & 0 \end{pmatrix} \tag{45}$$

for left handed up quarks. We leave it to you to write out the remaining fermion states. Observe that leptons are circulants while quarks are not. The $W^{\pm}$ bosons are simply

$$W^- = \overline{\omega} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad W^+ = \omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{46}$$

The only tricky particle is the Z boson. There are six remaining possible neutral boson matrices, namely

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix}, \quad \begin{pmatrix} \overline{\omega} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} \omega & 0 & 0 \\ 0 & \overline{\omega} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{47}$$

and its three conjugates. In order to be color neutral, the low energy Z boson must be a mixture of these states.

Although simple, this scheme permits [13] a natural supersymmetry, including also a triality. The *twisted Fourier* transform $\mathbf{F}$ is defined on e_L^- by

$$\mathbf{F}(e_L^-) \equiv \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \overline{\omega} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix} \begin{pmatrix} 0 & \overline{\omega} & 0 \\ 0 & 0 & \overline{\omega} \\ \overline{\omega} & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \overline{\omega} & \omega \\ 1 & \omega & \overline{\omega} \end{pmatrix} = W^-. \tag{48}$$

For right handed states we replace the mixed diagonal by its complex conjugate,

$$\mathbf{F}(e_R^-) \equiv \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \overline{\omega} & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix} \begin{pmatrix} 0 & 0 & \overline{\omega} \\ \overline{\omega} & 0 & 0 \\ 0 & \overline{\omega} & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \overline{\omega} & \omega \\ 1 & \omega & \overline{\omega} \end{pmatrix} = W^-. \tag{49}$$

In this way, circulant leptons map to electroweak bosons

$$e^\pm \mapsto W^\pm, \qquad \nu, \overline{\nu} \mapsto \gamma. \tag{50}$$

Naming the diagonals in $\mathbf{F}$, respectively, Z_L and Z_R, there are many relations like

$$\mathbf{F}(e^-) = W^- = \mathbf{F}(Z_L)\mathbf{F}(Z_R) \tag{51}$$

that construct all particles out of pieces of the Z boson.

Since quarks are not circulant, the corresponding bosons are mixed phase non diagonals, presumably associated to the confinement of color. If we had chosen a different Fourier matrix F, another Z boson diagonal would be needed in $\mathbf{F}$.

The braid group B_3 extends the underlying permutations of S_3, and this important group, which covers the modular group, turns up in the construction of gates that are universal for quantum computation [51], particularly with Fibonacci anyons. It is also well known that string net condensation [23], using modular tensor categories, can recover QED and QCD. Here $SU(2)$ emerges from the B_3 braids in a 2×2 representation, ignoring ribbon twists, while $U(1)$ emerges from the B_2 information. Wen et al [23] have described the Anderson-Higgs mechanism from this point of view.

4.2 Quantum Hopf algebras

Let G be a finite group, such as the permutation group S_3. Its *group algebra* $\mathcal{A}(G)$ [52][53] over a field $\mathbb{F}$ is the set of all sums $\sum_{g \in G} c_g g$ over the group, where $c_g \in \mathbb{F}$. For example, the set of 3×3 complex circulant matrices is the group algebra for S_3 over $\mathbb{C}$, with a basis of permutations.

The product in $\mathcal{A}(G)$ is the usual product of polynomials, where we are careful not to mix up the ordering since G may be noncommutative. It is clear that $1 \in G$ acts as a unit in $\mathcal{A}(G)$. There is also a *coalgebra* structure on the group algebra, given by a canonical coproduct Δ, which on group elements is given by

$$\Delta(g) = g \otimes g. \tag{52}$$

This coproduct is obviously cocommutative. The counit map $g \mapsto 1$ sends $g \in G$ to $1 \in \mathbb{F}$. Finally, $\mathcal{A}(G)$ is actually a Hopf algebra, with an *antipode* map $S : \mathcal{A} \to \mathcal{A}$ that uses the group inverse,

$$S(g) \equiv g^{-1}. \tag{53}$$

There is one natural Hopf axiom relating all these maps, but we won't worry about it here.

Take $\mathcal{A}(S_3)$. An element has the form

$$C = \begin{pmatrix} a+x & b+y & c+z \\ c+y & a+z & b+x \\ b+z & c+x & a+y \end{pmatrix} \tag{54}$$

for six complex variables. A unitary matrix in the algebra must look like

$$U = \begin{pmatrix} r_1 & i & 0 \\ i & r_1 & 0 \\ 0 & 0 & r+i \end{pmatrix} \begin{pmatrix} r_2+i & 0 & 0 \\ 0 & r_2 & i \\ 0 & i & r_2 \end{pmatrix} \begin{pmatrix} r_3 & 0 & i \\ 0 & r_3+i & 0 \\ i & 0 & r_3 \end{pmatrix} \tag{55}$$

for three real variables r_i, up to normalisation. Each factor defines an Euler angle for a 3×3 mixing matrix. The variables in (54) are recovered under the change of variables

$$-ia = r_1 r_2, \quad -ib = r_2 r_3, \quad -ic = r_1 r_3 - 1, \tag{56}$$

$$x = -r_1 - r_3, \quad y = -r_2, \quad z = r_1 r_2 r_3.$$

A quantum Hopf algebra for $\mathcal{A}(G)$, which is neither commutative nor cocommutative, is built from two factors: the cocommutative group algebra and the Hopf algebra $\mathcal{F}(G)$ of $\mathbb{F}$ valued functions on G. The latter is commutative for pointwise multiplication of function values, but not necessarily cocommutative, since

$$\Delta f(g,h) \equiv f(gh) \tag{57}$$

depends on the order of g and h. However, for the abelian group $C_3 \subset S_3$, everything is cocommutative. By construction, these Hopf algebras are associative and coassociative, but in general we are interested in breaking the associativity in three dimensional categories.

The *quantum double* algebra $\mathcal{A}(G) \otimes \mathcal{F}(G)$ is defined as follows. Let $\delta_{g,k}$ denote the delta function on G, and f_g the function that is 1 on g and otherwise zero. The product is

$$(f_g \otimes u)(f_h \otimes v) = \delta_{gu,uh} f_g \otimes uv, \tag{58}$$

the coproduct

$$\Delta(f_g \otimes u) = \sum_{k \in G} (f_k \otimes u) \otimes (f_{k^{-1}g} \otimes u), \tag{59}$$

and the counit map is given by $f_g \mapsto \delta_{g,1}$. This double algebra comes with a canonical braiding $\mathbf{R}$-matrix,

$$\mathbf{R} \equiv \sum_{g \in G} (f_g \otimes 1) \otimes (1 \otimes g). \tag{60}$$

Most importantly, the representation category for the double is a modular braided tensor category of ribbon diagrams.

4.3 The modular group and Leech lattice

The modular group $\Gamma = PSL(2, \mathbb{Z})$ is the set of 2×2 integer matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{61}$$

of determinant 1, up to a factor of ± 1. Γ acts on a complex number z in the upper half plane, as

$$z \mapsto gz \equiv \frac{az+b}{cz+d}. \tag{62}$$

A modular form is a function $f(z)$ that is almost invariant under the action of Γ. We consider the general case of automorphic modular forms, which may acquire a phase factor under the action. For example, the Dedekind η function

$$\eta(z) = e^{\pi i/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i z n}) \tag{63}$$

is automorphic of weight 1/2, since

$$\eta(gz) = \phi(a,b,c,d)(cz+d)^{1/2} \eta(z), \tag{64}$$

where $\phi(a,b,c,d)$ is a known set of phases. The theta function

$$\theta(z) = \sum_{n \in \mathbb{Z}} e^{2\pi i n^2 z} \tag{65}$$

is closely related to $\eta(z)$ by

$$\theta(2z-1) = \frac{\eta^2(z)}{\eta(2z)}. \tag{66}$$

On the two generators of Γ, θ picks up the phases

$$\theta\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} = e^{\pi i/12}, \quad \theta\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} = e^{\pi i/4}. \tag{67}$$

The *modular function*, or j invariant [54][55], is the essentially unique modular form of weight zero for Γ. It is given by the ratio of two weight 12 forms, starting with the denominator

$$\Delta(z) = \eta^{24}(z) = q\prod_{n=1}^{\infty}(1-e^{2\pi izn})^{24}. \tag{68}$$

Forms are most easily expressed in terms of the variable $q = \exp(2\pi iz)$, so that large powers of q vanish when $z \to i\infty$. Both numerator and denominator are integral polynomials in the Eisenstein forms $G_n(z) = 2\zeta(n)E_n(z)$, where

$$E_4 = \frac{1}{240} + q + 9q^2 + 28q^3 + \cdots \tag{69}$$

$$E_6 = -\frac{1}{504} + q + 33q^3 + 1057q^4 + \cdots$$

have integer coefficients coming from, respectively, the divisor functions $\sigma_3(n)$ and $\sigma_5(n)$. The canonical Weierstrass form for a cubic is

$$y^2 = 4x^3 - 60G_4x - 140G_6 = 4(x-r_1)(x-r_2)(x-r_3). \tag{70}$$

Then

$$j(q) \equiv 1728\frac{(60G_4)^3}{(60G_4^3 - 27\cdot 140G_6^2)} \tag{71}$$

or more conveniently,

$$j(\lambda) = 1728(\frac{4}{27})\frac{(\lambda^2-\lambda+1)^3}{\lambda^2(\lambda-1)^2} \tag{72}$$

where $\lambda = (r_3-r_2)/(r_3-r_1)$ is a representative for z in the equivalence class of cubics with the same value of j, such that $(r_1, r_2, r_3, \infty) \mapsto (\infty, 0, \lambda, 1)$. The q series for j

$$j(q) = q^{-1} + 744 + 196884q + 21493760q^2 + \cdots \tag{73}$$

points to *monstrous moonshine* [56][57][58], which is the observation that the integer coefficents of $j(q)$ come from an infinite dimensional module for the large, finite group known as the monster. Moonshine mathematics is the construction of such modules from an algebra associated to stringy tree diagrams, which are copies of $\mathbb{CP}^1$ (the genus zero case) with marked points. This is a complexification of the usual particle trees, built out of one dimensional edges. But we would like instead to construct $j(q)$ from quantum gravity, hoping eventually to see how the classification of simple finite groups (or rather, their Hopf algebras) dictates the quantisation of mass energy.

To this end, the 24 dimensional Leech lattice is neatly defined using octonion triplets [59][60]. We start with the 8 dimensional root lattice L_8 of the Lie group E_8, generated by a set of 240 unit octonions. These are the 112 octonions of the form $\pm e_i \pm e_j$ for any distinct units e_i and e_j of $\mathbb{O}$, and the 128 octonions of the form $(\pm 1 \pm e_1 \pm \cdots \pm e_7)/2$ for an odd number of minus signs. E_4 is the lattice function for L_8, with the q series counting the vectors of length $2n$. Let $s = (-1 + e_1 + e_2 + \cdots + e_7)/2$. The Leech lattice [59] is the set of all triplets $(X, Y, Z) \in \mathbb{O}^3$ such that

1. X, Y, $Z \in L_8$
2. $X + Y$, $Y + Z$, $Z + X \in L_8 \overline{s}$
3. $X + Y + Z \in L_8 s$.

5 Future directions

Topological approaches to quantum gravity have long considered knot invariants in $2 + 1$D theories [61]. In $2 + 1$D, mass often acts like a conical singularity, reducing the 2π shift around a loop. Chern-Simons field theory [62] recovers the Jones link polynomials, and this theory was used in 2007 [63] to define the Bekenstein-Hawking black hole entropy in terms of the j invariant (see section 4.3).

The full emergence of general relativity is expected to come from entanglement in tensor networks, which we would like to improve with categorical, non local methods. Also relevant is the study of smooth structures for 4-manifolds and the use of topology change in physics [64].

But perhaps the most startling aspect of non local cosmology is its potential for material and condensed matter physics, once the non local nature of the vacuum becomes clear. Superconductors, for instance, have long lacked a good mathematical model, and display notable anomalies.

6 Conclusions

From the single hypothesis of maximally non local mass generation, many observational dilemmas are effortlessly solved. There is no dark matter, no dark energy and no unobservable early universe. Moreover, immediate predictions of the theory, such as the mass M_U of the universe [6] or accurate values for the neutrino masses, were *later* shown to be in good agreement with experiment.

In the search for a rigorous theory of quantum gravity using twistor methods, nothing rivals the economy of these results.

For many years, the correspondence between $\overline{\nu}$ masses and the CMB temperature [1][3][4] was widely considered lunacy, but the time for dual quantum inertia has come. Our intuition puts the Higgs mechanism back into the realm of condensed matter physics, where it always belonged.

In the future, quantum inertia is expected to play an important role outside of particle physics, particularly in the design of new materials and the development of quantum computers, where we crave a deeper understanding of thermodynamic issues and protocol semantics. The ribbon scheme [12][13] already fits neatly into the categorical workings of quantum information theory.

On the particle physics side, it is now well known that scattering amplitudes in the Standard Model are computed using categorical diagram techniques. Motivic mathematics aims to compute every possible Feynman amplitude in terms of cohomological combinatorics, based on Hopf algebras. But to date, such work has neglected the problem of mass generation. Insights from non local quantum gravity will certainly aid this program, bringing together categorical diagrams and number theory.

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Declaration

A proposal for this work was rejected by the University of Auckland in 2016. The author was banned from the arxiv in 2004, ostracised from the community in 2009, and has difficulty accessing papers online, due to both wide scale abuse and her present circumstances of homelessness and poverty. This original work was written on an ipad in her tent in West Auckland.

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