Quantum state of an entangled particle is not real

Shubhayan Sarkar[1, *]

[1] *National Institute of Science Education and Research, HBNI, Jatni- 752050, India*

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Is quantum state real or just knowledge of some underlying reality? This question has been asked time and time again but the answer still remains unclear. In the following paper, using the property of the entangled state the author shows that the underlying hidden-variable model for a particle in an entangled state has to be psi-epistemic. This implies that the wavefunction can′t correspond to reality of such a system where the quantum state is entangled. However the result doesn′t contradict the PBR result which says that quantum state is real as those results do not include entangled systems.

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I. INTRODUCTION

The paradox put forward by Einstein, Podolsky and Rosen [1] in 1935 which pointed at one of the spookier aspects of quantum mechanics that two particles can simultaneously affect each other′s quantum state even if they are spatially separated. They argued that existence of such a system points out that quantum mechanics is incomplete [2][3]. Einstein argued that there exist some hidden variables which describe the quantum world correctly and those variables would resolve the problem of locality in such systems. Taking the argument of Einstein of local hidden variables and the simplified entangled state given by Bohm [4], in 1964 Bell [5] based his argument purely on mathematics to show that if Einstein′s argument was correct and the entangled system can be described locally, then it would follow certain statistics and if quantum mechanics is valid, then it would follow a different statistics.

The experiments carried out by Aspect [6][7] clearly showed that quantum mechanics was indeed correct and Einstein′s argument was wrong. Bell showed that even if hidden variables exist then it must be non-local. This meant that nature was non-local in the quantum domains and the properties of systems are affected non-locally, however this can′t be used to transfer signals faster than speed of light [2]. Even if quantum mechanics was proved correct, still there was no comprehensive physical explanation about how the correlation arises and even if the outcomes were being affected non-locally, there was no communication between the entangled partners.

A number of hidden-variable models have been constructed which try to explain the quantum statistics [8][9][10]. However the question still remains whether quantum state is real or projection of some underlying reality (One such model is De Broglie-Bohm pilot wave theory which assumes that wavefunction is real). To answer such questions, the hidden-variable models were classified as Psi-ontic ontological model where the quantum state is real and Psi-epistemic ontological model where the quantum state only represents the knowledge of some underlying reality.

Spekkens and Harrigan [8] gave a comprehensive idea about psi-epistemic and psi-ontic models. They gave a clear distinction between such models, "An ontological model is $\psi-ontic$ if for any pair of preparation procedures,P_ϕ and P_ψ associated with distinct quantum states ϕ and ψ, we have $p(\lambda|P_\phi)p(\lambda|P_\psi) = 0$ for all λ". So for any ψ-epistemic model

$$p(\lambda|P_\phi)p(\lambda|P_\psi) > 0 \tag{1}$$

for $\langle\phi|\psi\rangle \neq 0$.
They also showed that in such a case where the ontic state spaces of the systems are disjoint are psi-ontic ontological models. In such models the quantum state corresponds to reality and is essentially non-local. They showed that "Any psi-ontic models that reproduce quantum statistics violate locality". (An account of ontic variables in Bell's theorem is given in [10].)

Pusey, Barrett and Rudolph showed that under certain assumptions, the wavefunction represents reality. They take two systems which are independently prepared and jointly measure in a particular basis. They show that for quantum statistics to be correct, the ontic model describing the system must be psi-ontic. The major assumptions in [11] are

1. The independently prepared system should have independent ontic spaces.

2. The outcome of the measurement can only depend on the physical states of the two systems at the time of measurement.

There are many other mathematical assumptions in the PBR theorem which are discussed in [10]. However the result given in this paper doesn't contradict the PBR result (This is discussed again in the conclusions where the statement will be made clear).

* subhayan.s@niser.ac.in

The result also doesn't contradict the theorem given by Colbeck and Renner [12]. They show that given the assumption of free choice, independent of the internal structure of the ontic space Λ the quantum state is real. However the theorem excludes one-qubit states as the proof of there theorem requires the initial state of the particle ψ described by Λ decays into two states which requires that the initial state ψ describes atleast two qubits.

II. MAIN RESULT

We show that given the particle is entangled, the ontic state of the particle has to overlap for different states, or the quantum state of the qubit is not real. Though we can't say anything about the internal structure of the ontic space of the entangled system, but we assume that the individual particles have a well defined ontic space. Also we assume free choice for the measurement settings. We look at a scenario where Alice does measurement on her qubit and remotely prepares Bob's qubit in a particular state.

Theorem 1. *Any ψ-ontic ontological model can't describe the ontic state of the individual subsystem or the individual qubits of an entangled system.*

Proof. We take a two qubit entangled state AB. From Bell [5] we know that any correlation $C(a,b)$ between the spins (assuming measurement independence) can be written as

$$C(a,b) = \int C(a,b,\boldsymbol{\lambda})p(\boldsymbol{\lambda})d\boldsymbol{\lambda} \tag{2}$$

where $C(a,b,\boldsymbol{\lambda})$ is the correlation function dependent on λ, and $p(\boldsymbol{\lambda})$ is the probability distribution of the ontic variable $\boldsymbol{\lambda}$ (The $\boldsymbol{\lambda}$ given here is the ontic variable of the composite state AB).

For quantum mechanics when the system is a two qubit state, the correlation function is given by

$$C(a,b) = Tr(\rho\sigma_a \otimes \sigma_b) \tag{3}$$

For the maximally entangled state $C(a,b) = -cos(\theta_{ab})$.

If one of the qubits is measured, then this remotely prepares the other qubit in the same basis as the measured basis of the qubit as given in [8]. Since the measurement a is done which remotely prepared Bob's state, the correlation would now depend only on the choice of the measurement b, and the ontic space of the prepared state of Bob. Therefore the correlation $C(a,b)$ can be written as

$$C(a,b) = \int B(b,\lambda_B)p(\lambda_B|P_a)d\lambda_B \tag{4}$$

The λ_B is the ontic-state space variable for the reduced state B after Alice measures her system A. $B(b,\lambda_B)$ is an arbitrary function dependent on the basis b and λ_B. P_a is the remote preparation of Bob's state, in the basis a, where a is the basis chosen by Alice to measure her qubit. There could be a common misconception looking at the above equation that the correlation is dependent only on b. However the ontic space of Bob's qubit is affected by the outcome of Alice, and since the correlation is dependent on the prepared ontic state $p(\lambda_B|P_a)$ and so is the correlation. As a result the correlation is dependent on Alice's measurement choice a as well as Bob's measurement choice b.(The subscript B will be further dropped for convenience)

Now Alice and Bob share a singlet state. Alice measures her qubit in 01 basis which remotely prepare Bob's state in the 01 basis. These preparations are associated with $|0\rangle$ and $|1\rangle$ states(similarly for any other basis).

If Bob chooses same basis to measure her qubit as Alice, then from quantum mechanics we know

$$C(a,b) = -1 = \int B(b,\lambda)p(\lambda|P_{01})d\lambda \tag{5}$$

(here a represents 01 basis for Alice and b represents 01 basis for Bob)

Similarly Alice chooses her basis as $+-$,but Bob chooses as $-+$ basis.

$$C(a',b') = 1 = \int B'(b',\lambda)p(\lambda|P_{+-})d\lambda \tag{6}$$

(here a' represents $+-$ basis for Alice and b' represents $-+$ basis for Bob).
Also

$$\begin{aligned} p(\lambda|P_{01}) &= \frac{1}{2}p(\lambda|P_0) + \frac{1}{2}p(\lambda|P_1) \\ p(\lambda|P_{+-}) &= \frac{1}{2}p(\lambda|P_+) + \frac{1}{2}p(\lambda|P_-) \end{aligned} \tag{7}$$

Since $(a+b)^2 \geq 4ab$, We have

$$(C(a',b') + C(a,b))^2 \geq 4C(a,b)C(a',b') \tag{8}$$

From 5 and 6 we have

$$0 > C(a,b)C(a',b') \tag{9}$$

Therefore

$$0 > \int B(b,\lambda)p(\lambda|P_{01})d\lambda \int B'(b',\lambda)p(\lambda|P_{+-})d\lambda \tag{10}$$

Since $p(\lambda|P_{+-}) \geq 0$, for 6 to hold $B'(b',\lambda) < 0$ for atleast some values of λ. For B' having the same monotonic nature as B the -ve sign is absorbed in B' $(-B' = B'')$. Therefore

$$0 < \int\int B''(b',\lambda')B(b,\lambda)p(\lambda|P_{01})p(\lambda'|P_{+-})d\lambda d\lambda' \tag{11}$$

Using chebyshev inequality

$$\frac{1}{b-a}\int_a^b f(t)g(t)dt \geq \int_a^b\int_a^b f(t)g(t')dtdt' \tag{12}$$

where f and g are functions of the integarting variable t and t'.

We have

$$0 < \int B''(b', \lambda)B(b, \lambda)p(\lambda|P_{01})p(\lambda|P_{+-})d\lambda \quad (13)$$

Using 7 we have

$$0 < \int B''(b', \lambda)B(b, \lambda)\left(\frac{1}{2}p(\lambda|P_1) + \frac{1}{2}p(\lambda|P_0)\right) \left(\frac{1}{2}p(\lambda|P_+) + \frac{1}{2}p(\lambda|P_-)\right)d\lambda \quad (14)$$

Since we know that B and B'' are greater than 0 for atleast some $\lambda's$. Therefore for 14 to be true

$$\left(\frac{1}{2}p(\lambda|P_1) + \frac{1}{2}p(\lambda|P_0)\right)\left(\frac{1}{2}p(\lambda|P_+) + \frac{1}{2}p(\lambda|P_-)\right) \geq 0 \quad (15)$$

This shows that atleast one of the products has to be graeater than 0.

$$\begin{aligned} p(\lambda|P_1)p(\lambda|P_+) > 0 \\ p(\lambda|P_1)p(\lambda|P_-) > 0 \\ p(\lambda|P_0)p(\lambda|P_+) > 0 \\ p(\lambda|P_0)p(\lambda|P_-) > 0 \end{aligned} \quad (16)$$

This implies that atleast one pair of the four pair of the states $(|00\rangle, |++\rangle)$, $(|11\rangle, |++\rangle)$, $(|00\rangle, |--\rangle)$, $(|11\rangle, |--\rangle)$ which are distinct have overlapping ontic-state space. This means that for entangled system the hidden-variable model has to be psi-epistemic, and the quantum state is just the knowledge or projection of an underlying reality.

This completes the proof.

III. CONCLUSIONS

The proof shown above clearly indicates that for an entangled particle if hidden-variables exist, the models describing such states have to be psi-epistemic. The arguments above also show that the wavefunction can′t be a real entity. The proof require only measurement independence assumption (apart from existence of hidden-variables). The above result shouldn′t be confused with the results of the PBR paper [11] which says that the wavefunction is real, as one of the assumptions of the PBR result is that both the particles are prepared individually and have no knowledge of each other′s state or in other words the ontic state spaces of the particles are independent of each other. However such an argument is not true for entanglement. Also the results doesn't contradict Colbeck and Renner theorem [12] as the theorem is applicable to ontic spaces which describe system of more than one particle. However the result presented above only deals with the ontic space of one-qubit states which are entangled to some other system. The result given in this article along with the PBR results when taken together makes the entire debate of whether nature is Psi-epistemic or Psi-ontic more interesting as well as weird. When independently prepared systems are jointly measured, quantum statistics show that nature is Psi-ontic, however when jointly prepared systems are measured independently, quantum statistics show that nature is Psi-epistemic.

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