A time vector and simultaneity in TSR (v7, 2018-06-22)

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Abstract. We specify a 'time vector' for any event in the theory of special relativity (TSR). This vector is well suited to specify various types of simultaneity. (Imagined) moving clocks, being synchronized at a common 'point of initiation' play a crucial role. We may present the time vector as a complex variable, and there is a relation to the Minkowski distance. We exemplify the approach by including a short discussion of the 'travelling twin'.

Key words: Time dilation, simultaneity, Lorentz transformation, time vector, Minkowski distance, travelling twin.

1 Introduction

The concept of *simultaneity* becomes crucial when inertial reference frames (RFs) are moving relative to each other. Of course, we have the 'basic simultaneity'; *i.e.* simultaneity of events occurring at the same instant *and* same location, but these are rather the same event, just seen in the perspective of two different RFs. For events at a distance, we can essentially observe simultaneity from the 'perspective' of a certain RF: When the synchronized clocks of a specific RF show the same readings, we have simultaneous events *in the perspective of* this RF. The literature further refers to the 'relativity of simultaneity'. Here we present an approach, suggesting an alternative definition of simultaneity also 'at a distance'. Some further background is given in Hokstad (2017, 2018).

We introduce 'basic clocks' (BC), which at time 0 are (imagined to be) located at the common origin, and then are synchronized. The readings of these BCs provide a basis for specifying a two-dimensional 'time vector' related to any event. This vector proves useful for defining simultaneity also 'at a distance'.

This paper gives a purely mathematical description of the phenomenon, investigating implications of the Lorentz transformation (LT), and there is no attempt of a physical interpretation.

2 Foundation

As a background we here present some basic results related to the LT.

2.1 Basic notation

We start out with a RF, K_0, where the position along the x-axis is denoted x_0. At virtually any position there are synchronized clocks with clock reading denoted, t_0. We will simply refer to (t_0, x_0) as an event. Further, there is a RF, K_v, moving along the x-axis of K_0 at velocity v. On K_v we have

$x_v =$ The position on K_v, being identical to the location x_0 at a time t_0 on K_0

$t_v =$ Clock reading at position x_v on K_v, when x_v corresponds to x_0, and the clock on K_0 reads t_0.

Observers (observational equipment) on both of these two RFs agree on these four observations. Further,

- There is a complete *symmetry* between the two RFs K_0 and K_v; these being identical in all respects.
- We provide precise initial conditions: The clock at $x_v = 0$ and the clock at $x_0 = 0$ will when $t_v = t_0 = 0$ be at the same location, and they are then synchronized. We refer to this as the 'point of initiation', and these clocks as 'basic clocks' (BCs).

2.2 The Lorentz transformation (LT) and time dilation

The LT represents the fundament for our discussions. In the above notation the LT takes the form

$$t_v = \frac{t_0 - (v/c^2)x_0}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x_0 - vt_0}{\sqrt{1-(v/c)^2}} \tag{2}$$

We prefer a modified version of the LT. At any time, t_v and position, x_v we introduce w_v equal to $w_v = x_v/t_v$, (and therefore also $w_0 = x_0/t_0$). Then we insert $x_0 = w_0 t_0$, and (1) directly gives that the clock reading on the RF, K_v at this position equals:

$$t_v = t_v(w_0) = \frac{1 - vw_0/c^2}{\sqrt{1-(v/c)^2}} t_0 \tag{3}$$

So equation (3) express the LT by parameters (*t, w*) rather than (*t, x*). Note that we – when appropriate – will write $t_v(w_0)$ rather than t_v to pinpoint its dependence on w_0, The new time dilation formula (3) will – for a given clock reading, t_0 on the primary system, K_0 – give the clock reading, $t_v(w_0)$ on the secondary system, K_v, as a linear, decreasing function of w_0. Observe that (3) shows that we can write t_v in the form

$$t_v = t_v(w_0) = \gamma_v(w_0) t_0.$$

Fig.1 provides an illustration of this time dilation formula. Here we give clock reading ('time') both on K_0 and K_v *in the perspective of* K_0; (*i.e.* all clocks on K_0 reading the same time). Therefore, the figure illustrates an instant when time equals t_0 all over this reference frame. The horizontal axis gives the 'position' $w_0 = x_0/t_0$ on K_0 at which the clock measurements are carried out. The vertical axis gives the actual clock readings. So as time on K_0 equals t_0 at any 'position', w_0, the clock readings on K_v at this instant, $t_v = t_v(w_0)$, is a linear function of w_0. Fig. 1 also shows the value $w = w^*$, giving $t_0 = t_v$; (*cf.* discussion in Hokstad (2017).

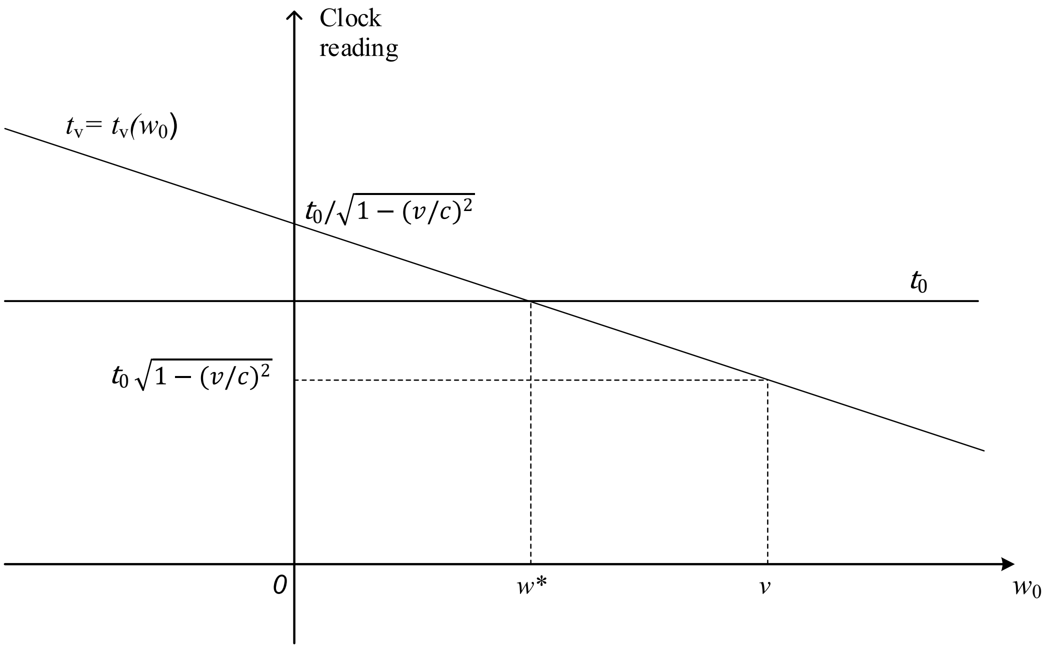

Figure 1. Clock readings in the perspective of K_0. Thus, 'time' all over K_0 equals t_0, while clock readings, $t_v(w_0)$ on the other RF is given as a function of w_0, where $w_0 = x_0/t_0$ provides the 'position' on K_0; *cf.* (3).

2.3 Two standard special cases (observational principles)

Two special cases are of particular interest. Recall that the first clock comparison is carried out at the origins $x_v = x_0 = 0$ when $t_v = t_0 = 0$. Now repeating some essential (and well-known) arguments given in Hokstad (2016, 2018), we specify two choices for the second comparison of clock readings.

First we compare the clock located at $x_v = 0$ on K_v (with the passing clocks on K_0, showing time t_0). Thus, also $w_v = 0$, and (4) implies $w_0 = v$, and (3) gives the relation between the two clock readings at this position, (*cf.* Fig. 1):

$$t_v = t_v(v) = t_0\sqrt{1-(v/c)^2} \tag{4}$$

This equals the standard 'time dilation formula'. Secondly, we can compare the clock located at $x_0 = 0$ on K_0 with a passing clock on K_v. For $x_0 = w_0 = 0$, (*i.e.* following the basic clock at the origin of K_0), *eq.* (3) gives the following relation, (again see Fig. 1):

$$t_v = t_v(0) = t_0/\sqrt{1-(v/c)^2} \tag{5}$$

Apparently, the relations, (4), (5) are contradictory; *eq.* (4) tells that the clock on K_v goes slower, and (5) tells that the clock on K_0 goes slower; *cf.* the Dingle's question, (McCausland 2008, 2012). Thus, the time dilation is not a feature of the RF, but follows from which single clock we choose to follow when we perform the second clock comparisons. Therefore, we prefer to formulate the time dilation formulas (4), (5) in compact form as

$$t^{BC} = t^{MC}\sqrt{1-(v/c)^2} \tag{6}$$

Here we have introduced the notation

t^{BC} = The clock reading of a basic clock (BC), *i.e.* clock located at the origin of a RF[1].

t^{MC} = The clock reading at the same location but on the other RF; *i.e.* the clock reading on a RF using multiple clocks (MC) for clock comparisons with the basic clock.

Therefore, both of the RFs can apply a BC for a certain clock comparison, and then conclude that 'time goes slower' on the RF which use BC. However, the same RF would also apply MC for a clock comparison with a BC on the other RF; and we would then conclude that 'time goes slower' on this other RF. Thus, it is the observational principle, *i.e.* choice of clocks for the clock comparisons that matters; *cf.* discussion in Hokstad (2018). This is a well-known result. According to Petkov (2012) already Minkowski referred to proper time and coordinate time, corresponding the above two concepts of time. However, the underlying duality has perhaps not received the attention it deserves in standard literature.

3 Concepts of simultaneity

Within a single RF simultaneity is easily established by the synchronization of clocks, *e.g.* using light rays, for instance see Einstein (1924), Giulini (2005), Mermin (2005). Further, a specific event, (t, x) will be specified differently by the two RFs. However, this is rather the same event; described by different (time, space) parameters.

However, for moving reference frames there is within the TSR no unique definition of simultaneity at a distance. Rather, one refers to *relativity of simultaneity*, *e.g.* see the discussion in Debs and Redhead (1996). In particular they argue for the *conventionality of simultaneity.* That is, when establishing simultaneity at a distance by the use of light signals, the definition of simultaneity is essentially a matter on convention; any time in a certain interval can be seen as simultaneous with a specified distant event.

For a single RF the task is simpler. Events with the same clock reading (t) on a specific RF, are *simultaneous in the perspective* of this frame. So this simultaneity depends on the chosen RF.

Hokstad (2018) introduced an auxiliary reference frame as a tool to obtain simultaneity at a distance. We simply postulated an auxiliary RF with origin always located at the midpoint between our two main RFs. Further, we utilized the symmetry of this model, so that simultaneous clock readings at the auxiliary RF implies a certain simultaneity at a distance for the two main RFs.

In the present paper we will pursue a slightly different approach. First we point out that an essential requirement for the use of the LT is that we start out with three sets of synchronizations.

1. All clocks on the first RF, K_0;
2. All clocks on the second RF, K_v;

[1] We have previously also used t^{SC} (where SC = Single Clock) to denote this clock reading

3. The two clocks at the origins of K_0 and K_v at time 0, this represent a *basic simultaneity*, and we refer to these as the *basic clocks* (BCs).

Usually one will here implicitly assume that all clocks on K_0 *remain* synchronized; as also do the clocks on K_v. We will now further argue that also the two BCs at the origins of K_0 and K_v - being synchronized at time 0 - will remain synchronized. They are moving away from each other at speed, *v,* but in a symmetric situation, there is no way to claim that one of the two clocks goes faster than the other.

So our claim is that when the two 'basic clocks' at the origins of the two RFs show the same time, this corresponds (in some sense) to simultaneous events 'at a distance'. (We consider this rather to be a consequence of our assumption of symmetry between the RFs.) This leads to a rather strong form of simultaneity, as all observers can agree on this, see our discussions in the next chapter.

4 The time vector and simultaneity

We now introduce a two-dimensional state vector. We refer to this as a time vector and will utilize it to define simultaneity. We first introduce the time vector of a single RF; next look at two RFs related by the LT.

4.1 Time vector of a single RF

Since we now consider just one RF, we make a slight change in the notation, by dropping all subscripts *v* (and 0). Thus, we consider a RF, *K,* and consider an arbitrary event (*t, x*); *i.e* having clock reading, *t* at position, *x.* We have previously (Hokstad (2017)) suggested the following time vector for this event

$$\vec{t}(t,x) = \binom{\sqrt{t^2-(x/c)^2}}{x/c} \tag{7}$$

As above we also introduce $w = x/t,$ and can then write this time vector as

$$\vec{t}(t,w) = \binom{\sqrt{1-(w/c)^2}}{w/c} t \tag{8}$$

We note that the absolute value of the time vector equals the clock reading, *t* of the event. The first component of this vector seems particularly interesting, as this equals what we have referred to as the basic clock (BC) reading of this event. Recalling (6), we have that the first component equals

$$t^{BC} = t\sqrt{1-(w/c)^2} \tag{9}$$

where t^{BC} equals the clock reading of the BC at this position (that is the clock located at the origin of a RF, K_w). (Of course, t^{BC} depends on the event (*t, x*), and we could write $t^{BC}(t,x)$.) Now we have the following alternative expression for the time vector

$$\vec{t}(t,x) = \binom{t^{BC}}{x/c} \tag{10}$$

We note that the first component of the time vector, (see (9)) is a valid expression only when $x/c < t,$ (that is $w<c$). So at a given position, *x* we must have time $t > x/c$, were x/c is the time required for a light flash initiated at the point of initiation to reach this position, *x*. So it is only after this time that (10) defines the time vector at a fixed position *x*.

Thus, we extend the above definition to also cover $t < x/c$. We simply define

$$\vec{t}(t,w) = \begin{cases} \binom{0}{1} t\,; & t < x/c \\ \binom{\sqrt{1-(w/c)^2}}{w/c} t; & t > x/c \end{cases} \tag{11}$$

Here we recall that *w* equals the speed at which the BC has 'arrived', at the event, (*t, x*). Obviously $w<c,$ and we introduce

$$w_- = \min(w,c) \tag{12}$$

Thus, we can in general write $\vec{t}(t,w)$ as

$$\vec{t}(t,w) = \binom{t_1}{t_2} = \binom{\sqrt{1-(w\_/c)^2}}{w\_/c} t \tag{13}$$

However, for the (main) case, $t > x/c$, we recall that (7), (8) and (10) are all valid expression for the time vector:

$$\vec{t} = \binom{\sqrt{t^2-(x/c)^2}}{x/c} = \binom{\sqrt{1-(w/c)^2}}{w/c} t = \binom{t^{BC}}{x/c}; \text{ for } t>x/c$$

Thus, for $t > x/c$ we can present the time vector, $\vec{t} = \vec{t}(t,w) = \vec{t}(t,x)$ as a point on the semicircle with radius, t in the (t^{BC}, x/c) space; see Fig. 2. In summary, for a specific clock time, t and position, $x = wt$ on K we interpret the two components of this time vector as follows:

- The first component, $t^{BC} = t\sqrt{1-(w/c)^2}$ equals the clock reading of the BC of the (possibly imagined) RF, K_w which by now is located at the position $x = w\,t$ on K. We call this the 'basic time' ('BC reading') at this position. (Only for $x = w = 0$, this BC refers to the BC at K itself.)
- The second component, x/c, equals the distance, x from the origin (and thus from the BC) of K to the given position, measured as the time $t \cdot w/c = x/c$ required for a light flash to go to this distance. Note that this also is the distance between the BC 'on location' and the BC at the origin of K itself.

Thus, both components refer to a distance from the 'point of initiation', ($x = t = 0$. We further repeat that the absolute value of our time vector is independent of w (and x):

$$\left|\vec{t}(t,w)\right| = t \tag{14}$$

cf. the semicircle of Fig. 2 which represent all time vectors on K with absolute value, t. Thus, events on K, which have time vector with the same absolute value, have the same clock time on K, and are simultaneous '*in the perspective of K'*. We will refer to this as Simultaneity Type I.

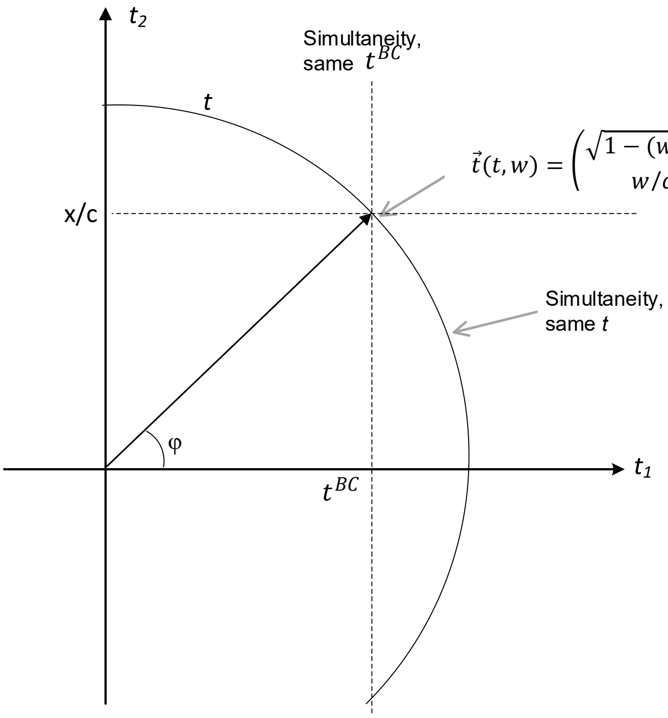

Figure 2 One specific time vector, $\vec{t}(t,w) = \binom{t_1}{t_2} = \binom{\sqrt{1-(w/c)^2}}{w/c} t = \binom{t^{BC}}{wt/c}$ for a specific clock reading t at the position, $x = wt$, when $t>x/c$; (here $\sin\varphi = w/c$).

However, this can be seen as a rather weak form of simultaneity: Events that are simultaneous in the perspective of one RF, are usually *not* simultaneous in the perspective of another RF. As suggested in Chapter 3 the BC of the event can provide an alternative form of simultaneity. Thus, we point out that events with the same BC reading, t^{BC} exhibit a form of simultaneity. Therefore, events having time vectors with identical first components are simultaneous in this sense; *cf.* stipled vertical line in Fig. 2. We will refer to this as Simultaneity, Type II.

We here observe that there is a strong link between this approach and Minkowski's approach to space-time; *cf.* space-time distance as $\sqrt{c^2t^2 - x^2 - y^2 - z^2}$ in his four-dimensional space, Minkowski (1909). As stated in Petkov (2012), Minkowski refers to the time of the basic clock, (*cf.* $\sqrt{t^2 - (x/c)^2}$ of *eq.* (7)), as 'proper time, and our *t* as 'coordinate time'. However, to my knowledge this has not been applied in the discussion of simultaneity.

Finally, we note that the BC reading t^{BC} is not just the BC reading of a specific, RF, *K*. It must be the BC reading at this position for *any* RF. Thus, the component, t^{BC}, of the time vector is also most relevant when we discuss simultaneity 'across RFs'. We return to this in Section 4.3 below.

The time vector and simultaneity.
In summary, it is interesting to relate the time vector of an arbitrary event (*t, x*) on the RF, *K,* with the time vector of two other events; first, that associated with the BC presently at the location in question; secondly, a time vector of the origin of *K.* Thus, for *t*>*x/c* we relate the following three time vectors:

1. Time vector, $\vec{t} = \binom{t^{BC}}{x/c}$ for an arbitrary event (*t, x*) on *K.*
2. Time vector, $\vec{t} = \binom{t^{BC}}{0}$ for the identical event of a (possibly imagined) passing RF with a BC corresponding to the event (*t, x*) on *K*. (The BC on the passing RF is located at the origin of this RF, and thus, the *x* –coordinate here equals 0.)
3. Time vector for the event at the origin of *K,* having the same BC reading as the event (*x, t*). At the origin, we have *x* = 0, and so also this time vector equals $\vec{t} = \binom{t^{BC}}{0}$, (now referring to a vector on *K*).

Thus, we argue that $\vec{t} = \binom{t^{BC}}{0}$ is a logical choice when we shall specify the time vector of an event at the origin of *K,* considered to be simultaneous with the event (*x, t*). We refer to this as simultaneity of Type II. This differs from the standard simultaneity concept, Type I, "simultaneity in the perspective of", which claims simultaneity on *K* when we have same clock reading, *t,* (*i.e.* same absolute value of time vector.)

So our claim is that – having an arbitrary event – there are two BCs of interest: First, the one related to the event (on location), and secondly that at the origin of its own RF. Both these events have time vector $\binom{t^{BC}}{0}$, while the time vector of the event itself is equals $\binom{t^{BC}}{x/c}$. Equality of the first component indicates simultaneity.

Finally, it is of some interest to consider the limiting case, $t \to x/c$, (*i.e.* $w \to c$); which implies that the BC reading, t^{BC}, of the event approaches 0. In this limiting case, it has elapsed a time *x/c* on the RF, *K*; while for the 'moving' BC there has elapsed *no time.*

4.2 Time formulated as a complex variable

We can of course formulate the time vector for the event (*t, x*) as a complex variable. In polar form, we can write the vector $\vec{t}(t, w)$ in (10) as

$$\mathbf{t}(t, w) = te^{i\varphi}, \quad (w = x/t) \tag{15}$$

If we restrict to *w*<*c,* the argument, $\varphi \in (-\pi/2, \ \pi/2)$, is given by

$$\sin \varphi = w/c.$$

(more generally we could replace φ by $\varphi_{\_}$, where $\sin\varphi_{\_} = w_{\_}/c$.) Further , the magnitude, t equals the clock reading, and we interpret $w = x/t$ as the velocity relative to K of an (imagined) basic clock (BC), now having arrived at the position, x.

When $\varphi = 0$, we have $w = x = 0$. Then we are at the origin of K, and the relevant BC is the one located on K itself. In this case the time variable becomes a real number.

As specified in Section 4.1, the real part, $\mathrm{Re}(\mathbf{t}(\mathrm{t}, w)) = t\sqrt{1-(w/c)^2} = t\cos\varphi = t^{BC}$ gives the BC reading at the location, and this identifies simultaneity. The imaginary part, $\mathrm{Im}(\mathbf{t}(\mathrm{t}, w)) = t \cdot (w/c) = t\sin\varphi = x/c$ represents the distance (measured in terms of time required for a light flash) from the RFs own BC to the position in question.

Finally, we can generalize (12) to hold for a three-dimensional space, with coordinates (x, y, z). We then define w by $w = \sqrt{x^2+y^2+z^2}\ /t$. Thus, w still specifies a position (distance from origin) at time t on the RF K, and we still have $\sin\varphi = w/c$, using the new definition of w.

4.3 Relating time vectors of two RFs

We now consider how to relate the time vectors, $\vec{t}(t, w)$ of two different RFs, moving relative to each other. We return to the notation of Chapter 2, with two RFs K_0 and K_v. Relative to these RFs we have the 'corresponding' positions $x_0 = w_0 t_0$ and $x_v = w_v t_v$, and the LT is given as (3), (4). Further, we for all v; (also v=0) introduce

$$\sin\varphi_v = w_v/c, \text{ (for } w_v < c) \qquad (16)$$

As seen in Section 4.1 there are various ways to write the time vector, $\vec{t}(t_v, w_v)$. By use of (16), the *eqs.* (8) and (9) now further give

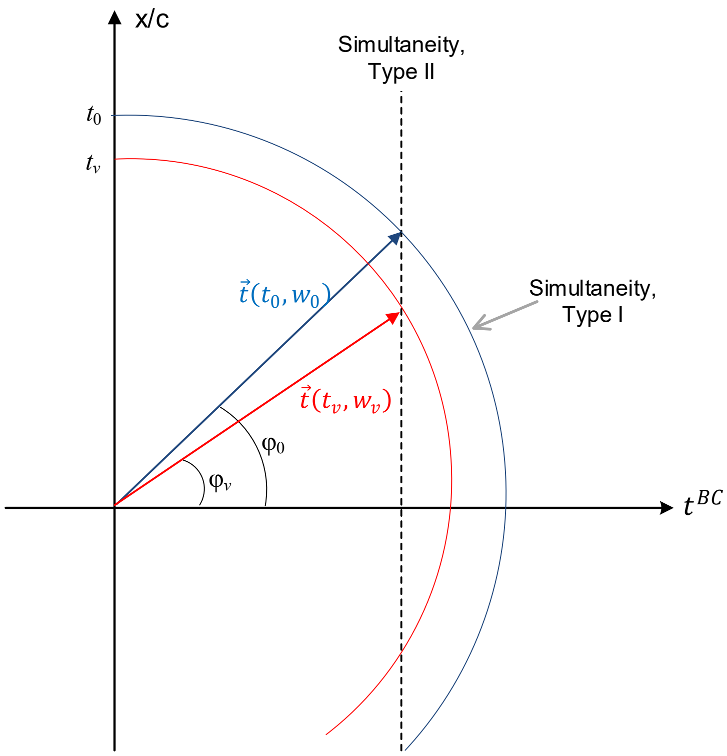

Figure 3 Time vector, $\vec{t}(t_0, w_0)$ on K_0 when its clocks read time t_0 (blue); and time vector, $\vec{t}(t_v, w_v)$ on K_v at the same position (red). We actually consider the same event, described by two different RFs (and being related by the LT). Thus, here we insert the time vectors of K_0 and K_v in the same co-ordinate system; both having a BC reading equal to t^{BC}.

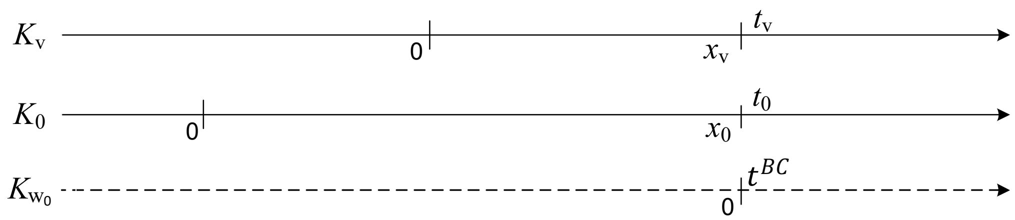

Figure 4 Two events (x_0, t_0) and (x_v, t_v) representing basic simultaneity, and the clock reading, t^{BC} of a BC at the same position. Origins of all RFs are marked with a zero, 0.

$$\vec{t}(t_v, w_v) = \binom{\cos\varphi_v}{\sin\varphi_v} t_v = \binom{1}{\tan\varphi_v} t^{BC} \quad (17)$$

We illustrate this result in Fig. 3, showing the time vector of one and the same event in the perspective of two RFs, K_0 and K_v. The most interesting feature is that the first component of the time vector is identical for all 'perspectives':

$$t_{v,1} = t_v \cos\varphi_v = t_v\sqrt{1-(w_v/c)^2} = t^{BC}, \text{(all } v) \quad (18)$$

This is actually an obvious result, since for any event there is just one BC present; we are actually referring to the same event, just described by two different RFs (and thus the LT provides the relation between the parameters).

We also have various expressions for the imaginary part of the complex time vector:

$$t_{v,2} = \frac{x_v}{c} = \left(\frac{w_v}{c}\right) t_v = t_v \sin\varphi_v = \tan\varphi_v\, t^{BC} \quad (19)$$

As a result we can easily write out a version of the LT relating the time vectors of K_0 and K_v, respectively; that is relating $\overrightarrow{t_v} = \binom{t_{v,1}}{t_{v,2}}$ and $\overrightarrow{t_0} = \binom{t_{0,1}}{t_{0,2}}$. As seen, various alternative expressions exist, (*cf.* Fig. 2 and also recall the essential result $t_{v,1} = t_{0,1} = t^{BC}$). We refer to Appendix B for further examples.

In this Section we have restricted to discuss identical events, described by two different RFs, see Fig. 4. We refer to this as *Simultaneity, Type II, Local*. However, we can apply the same approach to define simultaneity by utilizing different BCs (at different locations) but with the same clock reading, t^{BC}. We refer to this as *Simultaneity, Type II, At a distance, cf.* next chapter considering the travelling twin example.

4.4 Summary of time vector and simultaneity

To summarize: On any RF, *K,* we introduce a time vector related to an event (*t, x*), where *x* = *wt*, which splits the clock reading ('clock time'), *t* in two components, which for *t>x/c,* (*w<c*) equals:

$t_1 = t \cdot \sqrt{1-(w/c)^2} = t^{BC}$; the clock reading, of the (imagined) BC currently at this position

$t_2 = t \cdot w/c = x/c$; clock time required for a light flash to go from the RF's own BC (at the origin of this RF) to the current position.

So, both components ('dimensions') represent a 'distance in time' from the 'point of initiation'. Further, the absolute value of the time vector equals the clock reading, *t* of the event. We note that it makes sense to define the time vector as a complex variable, with the first component as the real part, and the second as the imaginary.

We point out that this time vector is suitable for specifying various forms of simultaneity:

1. Simultaneity Type I: Events with the same clock reading, *t.* This applies for the events of a single RF, and simply means that the synchronized clocks of this RF give the same clock readings. We

usually refer to this as simultaneity *in the perspective of this RF,* and thus, it occurs when time vectors (on this RF) have the same *absolute value*; *cf.* semicircles of Figs. 2 and 3.

2. Simultaneity Type II: Events with the same BC reading, t^{BC}. Thus, the (possibly imagined) BCs present at the location of the events have identical readings, (*i.e.* the complex time vectors have the *same real part*). We can apply this either to one single RF, or to events of several RFs, (which are moving relative to each other at constant speed).

   We could distinguish between the following cases:

   i. We have the standard situation that two different RFs describe the same event (*cf.* the LT). Then of course the clock reading, t^{BC} are identical. Actually we refer to the *same BC* on both RFs. In this situation, the LT will –as we know– describe the relation between the two time vectors. Here we may refer to Simultaneity *Type II, Local*; *cf.* Fig. 3.

   ii. We also here have events with the same t^{BC}, but these clock readings are carried out on different clocks (*i.e.* at different locations). In this case we refer to Simultaneity *Type II, At a distance*; see exampling next chapter. In principle, this could apply both for events on the same RF (Figure 2) and events related to two different RFs (Figure 3).

We note the limitation of the given approach: We have defined simultaneity relative to a specific 'point of initiation', only. However, if we first identify simultaneous events 'at a distance' relative to a given 'point of initiation', it should also be possible to define further simultaneities based on these events, now treated as new 'common' points of initiation. This could represent a considerable extension.

5 The travelling twin

The travelling twin paradox is frequently discussed, *e.g.* see Schuler and Robert (2014). As stated for instance in Mermin (2005) this illustrates that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together. We gave a lengthy discussion in Hokstad (2018), and now restrict to a short comment.

We start out with two synchronized clocks at the origins of two reference frames; the RF of the earth, and the RF of the rocket of the travelling twin. We note that both clocks are located at the origin of their RFs, and so both are basic clocks (BCs).

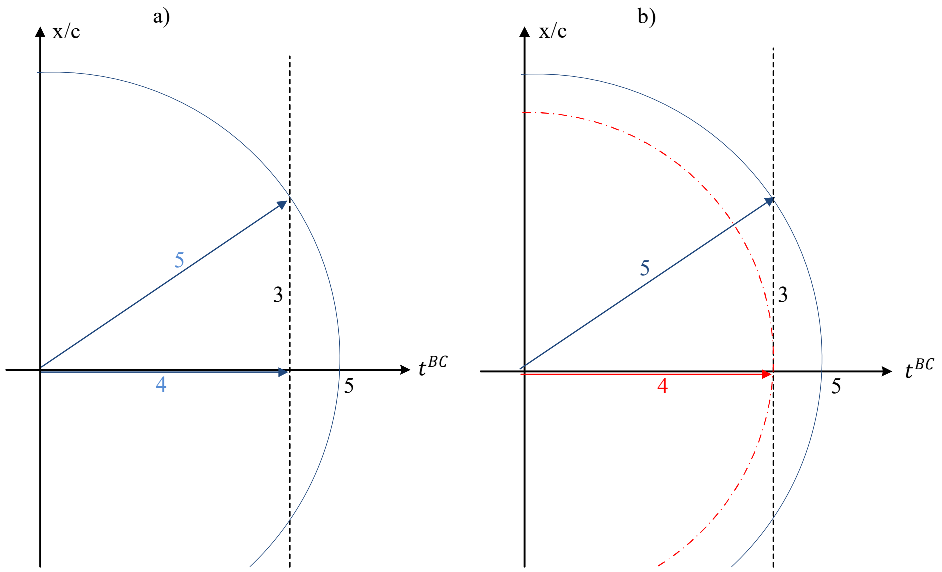

Figure 5 The time vectors $\vec{t} = \binom{3}{4} = 5\binom{0{,}6}{0{,}8}$ and $\vec{t} = \binom{4}{0} = 4\binom{1}{0}$ **by the arrival at the star of the travelling twin. a) Time vectors at the star and on the earth on the RF of the earthbound twin. b) Time vectors at the star: Red = travelling twin. Blue = RF of earthbound twin.**

Fig. 5 gives an illustration of relevant time vectors by the arrival at the star, using the numerical example of Mermin (2005). The distance from the earth to the star equals x_0 = 3 light years, *i.e.* x_0/c = 3 years, and the velocity of the rocket is $v = 0.6c$, giving $\sqrt{1-(v/c)^2} = 0.8$. Then the clock readings of the two RFs at the star by the twin's arrival are 4 and 5 years, respectively. First, Fig. 5.a) gives the time vectors on the earth and the star, respectively, 'in the perspective' of the earthbound twin. We know that on the star the clock of his RF reads 5 years, and so 'in his perspective' this is the clock readings all over his RF, (blue semicircle). However, the BC on the star at this instant, (which is the clock of his travelling twin) reads 4 years. Thus, following our claim regarding simultaneity Type II 'At a distance' (but on the same RF) of Section 4.4 this event on the star is simultaneous with his own clock (also being a BC) showing 4 years. Simultaneity in this sense occurs when the earthbound twin's clock at the star reads 5 years and his clock on the earth shows 4 years; at both those locations the BC reading equals 4 years.

Fig. 5.b) illustrates the two time vectors at the star: Red time vector for the travelling twin, and again the blue vector for the earthbound twin. These two time vectors of Fig. 5.b) represent a Simultaneity Type II 'Local' in Section 4.4.

Next, by combining the two results of Fig. 5. a) and b), we get that the clock of the travelling twin showing 4 years (on the star) is an event being simultaneous with the clock of the earthbound twin showing 4 years (on the earth). – Or in short, we claim these events to be simultaneous because the first component of their time vectors are equal (= t^{BC} = 4 years).

Note that our claim is that the arrival at the star is simultaneous with the the event that clock on the earth shows 4 years, (and not 5 years, that seems to be claim of some authors). However, the change of direction (and thus the change of RF) required for the travelling twin, will have a big impact here on the further development. So the above simultaneity result does not imply that the twins will have the same age also by their reunion, (as was unfortunately claimed in previous versions of the present paper); *cf.* last version of Hokstad (2018).

6 Conclusions

In the present work we consider implications of the Lorentz Transformation in the theory of special relativity. The clock reading of a reference frame (RF) is denoted, *t,* and we consider just one space coordinate, *x*. Related to any event (*t, x*) we define a time vector in two dimensions. We can formulate the vector as a complex variable, and there is a close link to the time-space of Minkowski.

The absolute value of the time vector equals the clock reading ('clock time'), *t* at the relevant position, and we can see this as a measure for the overall distance in time from the 'point of initiation', *i.e.* the time, 0 and the origins of the RFs having a common location.

This vector provides a means to define various forms of simultaneity. Time vectors on the same RF, having same absolute value will specify events that are simultaneous 'in the perspective of this RF. We refer to this as Simultaneity Type I.

However, we argue that we can also use this time vector to define a simultaneity Type II. There are two variants of this: 'Local' and 'At a distance'. Events having a time vector with the same real part will exhibit this form of simultaneity.

There is one fundamental claim here. We postulate an infinite set of (possibly imagined) RFs at constant relative speeds. At the origins of these RFs there are clocks which are synchronized at time 0, and for symmetry reasons we conclude that they remain synchronized. We refer to these clocks as basic clocks (BCs), and identical clock readings of these will represent simultaneity Type II.

The first component of the time vector equals the clock reading of this (imagined) BC. The second component equals the distance to the BC of the actual RF itself, measured as the time required for a light signal to go this distance.

The results apply for simultaneity relative to a common 'point of initiation'; however, extensions seem possible.

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Appendix A. Some notation

K_v = RF moving relative to a RF, K_0, at velocity v.

$K = K_0$; (the suffix 0 is in general dropped when just one RF is involved)

x_v = The position on K_v

t_v = Clock reading at position x_v on K_v, (often given as a function of $w_0 = x_0/t_0$)

$w_v = x_v/t_v$ (Used to specify a location x_v on K_v at a specific clock time t_v. Can also be interpreted as the velocity of an imagined RF K_{w_v}.)

$\varphi_v = \sin^{-1} w_v/c$

$w_- = \min(w, c)$, where $w = w_0$

$\varphi_- = \sin^{-1} w_-/c$

$t^{BC} = t^{BC}(t, x)$; The clock reading of the basic clock (BC) of a given event

v = Velocity of RF, K_v, relative to RF, K_0

When we have two RFs we apply the notation for K_0, x_0, … *etc.* for parameters related to the 'primary' RF. However, in cases of having just one RF, we rather use the notation, *K, x, ... etc.*

Appendix B

Fig. 3 presents an example where $0 < v < w_0$, and using (16), also $0 < w_v < w_0$; thus, $0 < \varphi_v < \varphi_0$. This is the situation illustrated in Fig. 4. The Fig. B1 illustrates some further cases (all with $v > 0$):

a) $v = w_0$. In this case $x_v = w_v = \varphi_v = 0$, and the origin of K_v is located at the relevant position; *i.e.* the BC of this event is actually located on K_v itself.

b) $v > w_0 > 0$. Using (16) we get $\varphi_v < 0$. So also $w_v < 0$, $x_v < 0$.

c) $w_0 = -w_v = w^*$, (and $x_v = -x_0$). We specify $v = v^*$. Here the blue and red semicircle coincide.

d) $v > v^*$. In this case, $w_v < -w_0$.

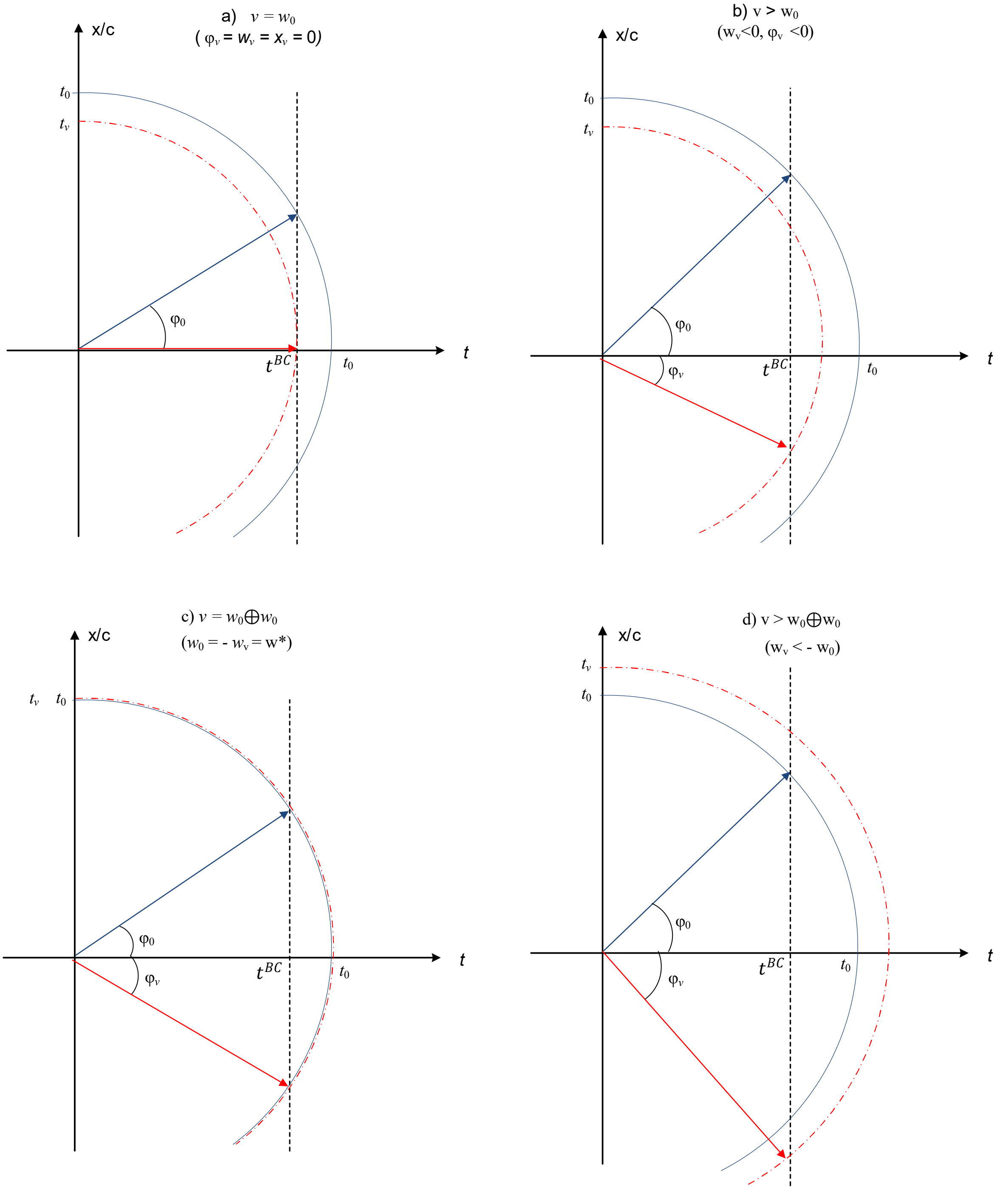

Figure B1 Further examples of a time vector, $\vec{t}(t_0, w_0)$ on K_0 (blue), and 'corresponding' time vector $\vec{t}(t_v, w_v)$ on K_v (red).