A concept for simultaneity in the STR

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Abstract. Simultaneity is a key concept in the special theory of relativity (STR), considering two reference frames moving relative to each other at constant speed, v. We introduce a new approach to simultaneity and its relation to clock readings and position. The two clocks at the origins play a crucial role. We synchronize these at time 0, and refer to this event as the 'point of initiation'. From symmetry reasons, we conclude that these two clock readings –at least in some sense – represent simultaneity. The second element of our approach is a new version of the Lorentz transformation; where we in a new way combine the four (time, space) variables at a specific position, giving a state variable of two orthogonal components. The resulting description of simultaneity is well suited to analyse and conclude on the travelling twin case. Further, we provide a generalization; specifying a two dimensional time vector. The absolute value of the time vector specifies a measure of distance (in time) from the point of initiation. We utilise this distance to specify simultaneity of events related to the two reference frames; simultaneity corresponding to identical distances; *i.e.* time vectors having same absolute value (='total distance'). We can also give the time vector as a complex number, where the real part equals the clock reading at the relevant position.

Key words: Time dilation; simultaneity, symmetry, Lorentz transformation; time vector, travelling twin.

1 Introduction

When reference frames (RFs) are moving relative to each other, the definition of *simultaneity* becomes crucial. Of course, we have the 'basic simultaneity'; *i.e.* simultaneity of events occurring at the same instant *and* same location, but these are actually the same event, just seen in the perspective of two different RFs. For events at a distance we can essentially observe simultaneity from the 'perspective' of a certain RF. The events where the synchronized clocks of a RF show the same time are simultaneous *in the perspective of this reference frame.*

In general, one may define simultaneity by use of light rays; *e.g.* see standard textbooks, [1], [2], but it seems not to be a unique definition of distant simultaneity. Here we present an approach, having two main elements:

- At the origins of each RF there is a clock. We synchronized them at time zero, and due to symmetry, they remain synchronized. So we can utilise them to define distant simultaneity.
- The two RFs will together have four (time, space) variables at a specific location. These are related through the Lorentz transformation. We here combine these four variables in a new way and obtain two state vectors, related by an orthogonal transformation; (*i.e.* a version of the Lorentz transformation).

This presentation directly provides a tool to handle the travelling twin case, presenting a solution to this paradox. Further, we generalize the approach; specifying time as a two dimensional vector. Given as a complex number in polar form its magnitude represents the 'distance' from the initial synchronization at time 0; *i.e.* from the 'point of initiation'. Events with the same magnitude for this time variable implies simultaneity (at least in a certain sense). The real part of the time variable equals the clock reading of the event. The imaginary part represents the distance in space from the 'point of initiation'- measured by the time required for a light signal.

The new approach given here is based on results presented in [3] and [4].

2 Foundation

We present some basic assumptions, and give both the standard and a new version of the Lorentz transformation. Throughout we restrict to consider one space coordinate.

2.1 Basic assumptions and notation

We start out with a reference frame (RF), K_0, where the position along the x-axis is denoted x_0 and the clock reading of any of its synchronized clocks is denoted, t_0.

Further, there is a reference frame, K_v (with the same orientation), moving along the x-axis of K_0 at velocity v. So K_0 is moving along the x-axis of K_v at velocity $-v$. On K_v we have

x_v = The position (along the x-axis), that corresponds to (has same location as) x_0 when the clock reading on K_0 equals t_0.

t_v = Clock reading at position x_v on K_v at the instant when x_v corresponds to x_0.

Observers (observational equipment) on both of these two RFs agree on these four observations. Further

- There is a complete *symmetry* between the two reference frames K_0 and K_v; the frames being identical in all respects.
- The clock at $x_v = 0$ and the clock at $x_0 = 0$ will at time $t_v = t_0 = 0$ be at the same location, and they are then synchronized. We refer to this as the 'point of initiation'.
- We will choose the *perspective* of one RF, usually K_0, and refer to this as the *primary* system. The time on this primary RF is at any position, x_0 given as a constant, t_0. In contrast, at a certain time, t_0 on the primary system, the observed time, t_v on the other ('secondary') RF will depend on the location where the time reading is carried out.

2.2 The Lorentz transformation (LT) and time dilation

The Lorentz transformation represents the fundament for our discussion of time dilation. From the above notation the Lorentz transformation takes the form

$$t_v = \frac{t_0 - \frac{v}{c^2}x_0}{\sqrt{1-(\frac{v}{c})^2}} \tag{1}$$

$$x_v = \frac{x_0 - vt_0}{\sqrt{1-(\frac{v}{c})^2}} \tag{2}$$

Two special cases are of particular interest. Recall that the first clock comparison is carried out at the at origins $x_v = x_0 = 0$ when $t_v = t_0 = 0$. Now we consider two specific choices for time and location for a second time comparison.

First we compare the clock located at $x_v = 0$ on K_v with the passing clocks on K_0, showing time t_0. This clock on K_0 must according to (2) have position $x_0 = vt_0$, and (1) gives the following relation between these two clock readings

$$t_v = t_0 \sqrt{1-(v/c)^2} \tag{3}$$

This equals the standard time dilation formula.

Secondly, we can compare the clock located at $x_0 = 0$, on K_0 with a passing clock on K_v (at position $x_v = -vt_v$), and now (1) gives

$$t_v = t_0 / \sqrt{1-(v/c)^2} \tag{4}$$

as the relation between t_0 and t_v. These relations, (3), (4) are apparently contradictory, and we prefer to formulate them in compact form as

$$t^{SC} = t^{MC} \sqrt{1-(v/c)^2} \tag{5}$$

where

t^{SC} = Second clock reading for the single clock (SC) used both at the first and the second time comparison; (*i.e.* reading of a clock located at the origin of a RF).

t^{MC} = Second clock reading on the RF using multiple clock (MC); so here we are not using the same clock as applied at the point of initiation.

We will use the relation (5) in the next chapter, discussing simultaneity. Now we restrict to point out that SC and MC are not features of the RF but of the clocks used for a specific time comparison. Both of the RFs can apply a SC for a certain time comparison, and thus might conclude that it is a system where 'time goes slower'. However, the same RF can also apply MC for another time comparison and then the conclusion would that time 'goes faster' on this RF. We therefore prefer to avoid phrases like 'moving clock goes slower'. It is the observational principle, *i.e.* choice of clocks for the time comparisons, that matters.

2.3 An alternative formulation of the Lorentz transformation

We now proceed to replace v in the Lorentz transformation (LT), (1), (2) with an angle, θ, given by

$$\sin\theta = v/c,$$

implying that

$$\cos\theta = \sqrt{1-(\frac{v}{c})^2}.$$

Now the LT (1), (2) can be formulated as:

$$t_v \cdot \cos\theta = t_0 - (x_0/c)\cdot\sin\theta \qquad (6)$$

$$x_v/c \cdot \cos\theta = x_0/c - t_0\cdot\sin\theta \qquad (7)$$

Thus, some rotation of the (time, space) coordinates is involved here. As we restrict to consider one space coordinate the LT involves four state variables, t_0, x_0, t_v and x_v. It is now important to observe that if we specify any two of these four variables, the other two will be given by the LT.

In particular, the standard version of the LT gives (t_v, x_v) expressed by (t_0, x_0), or *vice versa*. But similarly, we could reformulate the LT to give a relation between (t_0, t_v) and (x_0, x_v). And – as a third possibility – we can formulate the LT as a relation between (t_0, x_v) and (t_v, x_0). Here we follow up on this third possibility. First, by combining (6) and (7), we can replace (6) by

$$t_v = t_0\cos\theta - (x_v/c)\cdot\sin\theta \qquad (8)$$

Now (7) and (8) give a new version of the Lorentz transformation, which in matrix form becomes

$$\begin{pmatrix} t_v \\ x_0/c \end{pmatrix} = A\begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} \qquad (9)$$

Here the transformation matrix,

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad (10)$$

is orthogonal as

$$A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \qquad (11)$$

Next we introduce the 'time vectors'

$$\vec{t_1} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} \qquad (12)$$

$$\vec{t_2} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} \qquad (13)$$

and the relation (9) is then written

$$\overrightarrow{t_2} = \overrightarrow{At_1} \tag{14}$$

which we denote the orthogonal version of the Lorentz transformation. A nice feature of this formulation is that it represents a rotation, θ, of the $(t_0,\ x_v/c)$ plane, (with the components t_0 and x_v/c being orthogonal). Similarly, the vector $\overrightarrow{t_1} = \binom{t_0}{x_v/c}$ will be given by $\overrightarrow{t_2} = \binom{t_v}{x_0/c}$, using the same rotation in opposite direction, *i.e.* replacing *v* by *–v,* (and *θ* by -*θ*) in *A*.

We stress that the vectors, $\overrightarrow{t_1}$ and $\overrightarrow{t_2}$ provide identical information. They define the same event related to K_0 and K_v. Which of them we choose to describe the event could depend on which RF we specify as the primary one. The first component for both of these vectors equals the clock reading at one of the RFs. The second component equals the position of the other RF divided by *c* for the event in question. So this component equals the time for a light flash to go this distance. (For instance, a light flash occurring at the 'point of initiation' will arrive at position x_0 on K_0 exactly at this time x_0/c.) Therefore, both components of the vectors $\overrightarrow{t_1}$ and $\overrightarrow{t_2}$ represent time, and so we refer to them as time vectors. We could say that the two vectors of the specified event represent two aspects of the 'distance' in time, as measured from the 'point of initiation'.

3 Simultaneity in STR

Within the framework of STR it seems most common to verify simultaneity across reference frames by use of light rays. This differs from the approach presented here.

3.1 Concepts of simultaneity

It hardly exists a concept of 'global' simultaneity within the STR. However, there are some types of simultaneity with limited applicability. First, we refer to so-called *basic simultaneity* of events, meaning that the events occur at the same instant *and* the same location. Actually, we consider this to be the same event, just expressed by variables of different reference frames. So we note that basic simultaneity occurs in spite of the fact that the relevant clocks give different time readings for the event.

When the events occur at different locations one could refer to the weaker concept of *simultaneity by perspective.* We will say that events that show the same time (*t*) - as measured on a specific reference frame - are simultaneous in the perspective of this frame. However, the LT implies that this simultaneity will differ in the perspective of the different reference frames. Therefore, such a concept could seem rather useless.

Now in [3] we discussed the use of an auxiliary reference frame as a tool to define a simultaneity by perspective, and that proved useful. We simply postulate an auxiliary RF with origin always located at the midpoint between our two 'basic' RFs. In cases of strict symmetry, we argue that simultaneity in the perspective of this auxiliary reference frame also provide a valid symmetry for the two basic reference frame; and this symmetry implies a simultaneity at a distance. In particular, we strongly argued that this approach provides a solution to the travelling twin example.

Here we will pursue a slightly different approach based on the same symmetry, and utilize the discussions of the previous chapter. Now we point out that an essential requirement for the use of the LT is that we start out with three sets of synchronizations.

- First, all clocks on the first RF, K_0;
- Next, all clocks on the second RF, K_v;
- Third, the two clocks at the origins at time 0, (actually representing *basic simultaneity*).

Obviously, we assume that all clocks on K_0 *remain* synchronized; as do also the clocks on K_v. Further, we will now argue that the two clocks at the origins of K_0 and K_v - being synchronized at time 0 - will remain synchronized. They are moving away from each other at speed, *v,* but in our model of complete symmetry, there is no way to claim that one of the two clocks goes faster than the other. We have the standard phrase 'moving clock goes slower', but that is when the 'moving clock' is compared with passing clocks, and not with the other clock at the origin; *cf.* detailed discussions in [3].

So our claim is that when the two clocks at the origin show the same time, this correspond to simultaneous events 'at a distance'. We follow up on this in the next section.

3.2 Simultaneity of moving clocks

Now we first apply the general result of Section 2.3 at two specific positions. First Position I equals the position of the origin of K_0, *i.e.* the clock at location $x_0 = 0$. The Position II is at the other origin, $x_v = 0$ of K_v. At both positions, there is an event both at K_0 and K_v; giving four variables, which we now specify in the general notation. First, the variables of Position I:

Position I: At the origin, $x_0 = 0$ of K_0.

i. (t_0, x_0): Clock reading, t_0 of the clock at origin, $x_0 = 0$ on K_0

ii. (t_v, x_v): Clock reading, $t_v = t_0/\sqrt{1-(v/c)^2}$ at the corresponding position $x_v = -vt_v$ on K_v

As discussed in the Section 2.3 we can summarize these four values in the two vectors (see (12), (13)):

$$\vec{t_1} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ -v/c \end{pmatrix} t_v = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} t_v \tag{15}$$

$$\vec{t_2} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} = A\vec{t_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_v \tag{16}$$

Similarly, for the variables of Position II we have:

Position II: At the origin, $x_v = 0$ of K_v.

iii. (t_v, x_v): Clock reading, t_v of the clock at origin, $x_v = 0$ on K_v

iv. (t_0, x_0): Clock reading, $t_0 = t_v/\sqrt{1-(v/c)^2}$ at the corresponding position $x_0 = vt_0$ on K_0

Further, we summarize also these four variables in two vectors, then getting

$$\vec{t_1} = \begin{pmatrix} t_0 \\ x_v/c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t_0 \tag{17}$$

$$\vec{t_2} = \begin{pmatrix} t_v \\ x_0/c \end{pmatrix} = \begin{pmatrix} \sqrt{1-(v/c)^2} \\ v/c \end{pmatrix} t_0 = A\vec{t_1} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} t_0 \tag{18}$$

Now we consider the relation between the four events *i.-iv.* The two events *i.* and *ii.* of Position I represent 'basic simultaneity', *i.e.* 'same location, same time'. This is also the case for the two events *iii.* and *iv.* of Position II. In general, however, the two events at Position I and the two at Position II are not related.

But now we specify the events of Positions I and II in a symmetric way, which implies simultaneity for all four events. Thus, we restrict to the case that the two clocks at the origins, (which we synchronized at time 0), shall show the same time. Thus, clock reading, t_0 of Position I is equal to the clock reading, t_v of Position II. Then we have the symmetric situation introduced in Section 3.1, and so also events *i.* and *iii.* are simultaneous due to symmetry. Therefore, we can claim that all four events, *i.* – *iv.* are simultaneous, at least in some sense. Further, the time vectors of (15), (16) are essentially identical to the two time vectors (17), (18); (just interchanging their order).

Now there is a minor asymmetry here, as the position of event *ii*. equals $x_v = -vt_v$, (*eq.* (15), whilest the position of event *iv*. equals $x_0 = vt_0$ (without the minus sign); *cf.* (18). However, this difference appears since we here let K_0 and K_v have the *same* direction. If we define these RFs to have opposite directions, the minus sign of (15) would disappear, and there would be a complete symmetry of time vectors; just interchanging vector 1 and 2 when we go from Position I to Position II.

Therefore the vectors (15), (16) defined at the origin of K_0 are essentially the same as the vectors (17), (18) defined at the origin of K_v. So in exactly in the same way as we in Section 2.2 combined (3) and

(4) into the more informative *eq.* (5), we now replace the two sets (15), (16) and (17), (18) by the equations

$$\overrightarrow{t_a} = \binom{t^{MC}}{x^{SC}/c} = \binom{1}{0} t^{MC} \tag{19}$$

$$\overrightarrow{t_b} = \binom{t^{SC}}{x^{MC}/c} = \binom{\sqrt{1-(v/c)^2}}{v/c} t^{MC} \tag{20}$$

So rather than $\overrightarrow{t_1}$ and $\overrightarrow{t_2}$, we apply $\overrightarrow{t_a}$ and $\overrightarrow{t_b}$. Of course, we also now have $\overrightarrow{t_b} = A\overrightarrow{t_a}$. The two new vectors, $\overrightarrow{t_a}$ and $\overrightarrow{t_b}$ apply for both Position I and II, (*i.e.* the origin of one of the RFs). Note that we choose v/c in *eq.* (20) as positive or negative, depending on which of the two origins we refer to.

As in Section 2.2 we apply notation t^{SC} = clock reading of the clock actually positioned at an origin, and t^{MC} = clock reading of the corresponding clock on the other RF. Similarly, x^{SC} and x^{MC} represent the positions corresponding to these clock readings, (and so $x^{SC} = 0$).

We observe that the first component of each time vector equals the clock reading of one of the two RFs. The second component represents the distance from the origin of a reference frame; measured by velocity of speed of light.

So we can interpret all four components as some 'distance in time' from the point of initiation. We also note that both vectors have the same absolute value, t^{MC}, which we might interpret as the 'total distance'. So, in summary, the two time vectors (19), (20), will by the LT essentially give identical information, and they apply for both of the origins. We now suggest that one demonstrates simultaneity of these four events by considering absolute value of these time vectors. Thus, we postulate that one can apply a time vector similar to (19), (20) to define simultaneity. This will be the topic of Chapter 4. First we discuss the travelling twin example.

3.3 The travelling twin

As stated for instance in Mermin [2] the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together. We will – as we also understand [2] - consider the idealised situation. That is, we restrict to that part of the travel when TSR applies – and thus assume complete symmetry between the two twins. The example was discussed at length in [3], and we here just give a short comment.

We start out with two synchronized clocks at the origins of two reference frames; the RF of the earth, and the RF of the rocket of the travelling twin. This is exactly the situation described above. In our symmetry argument we conclude that the two clocks remain synchronized in the sense that identical readings of these particular clocks imply simultaneity.

By the arrival of the twin to the 'star', both twins observe two clock readings, their own clock, and the clock on the other passing frame. The four (time, space) readings at these two positions are identical, and at both places give rise to two 'time vectors'; see discussion of Section 3.2.

Ch. 10 of Ref. [2], gives the following numerical example. The distance between earth and the 'star' equals $x_0 = 3$ light years, and the rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the reference frame of the earth/star, the rocket reaches the star at time, $t_0 = x_0/v = 5$ years. The clock on the rocket is located at $x_v = 0$, and at the arrival at the star this clock reads $t_v = t_0 \cdot \sqrt{1-(v/c)^2} =$ 4 years.

So ref. [2] – as seems common in the literature - claims that the travelling twin ages 4 years during 'the same time' as the earthbound twin ages 5 years. We will, however, conclude differently. We claim that simultaneity is not handled properly in these arguments. Of course it is correct that 'in the perspective' of one twin the other twin ages more slowly; but this argument is valid for both twins; *cf.* the so-called Dingle's question [5], [6]. As seen above, we will from symmetry reasons argue that the clock of the travelling twin, which shows 4 years (at his arrival to the star), occurs simultaneously with the event that the clock of the earthbound twin shows 4 years. So by the return they have both aged 8 years. Also see further, detailed arguments in [3].

4 Time vector and a generalized concept of simultaneity

4.1 A generalized time vector

Chapter 3 provided a result just for two very specific positions, *i.e.* for the positions of the two clocks initially synchronized at the origins of the RFs. There is one rather obvious generalization of this result. We introduce a vector

$$\overrightarrow{t(w)} = \begin{pmatrix} \sqrt{1-(w/c)^2} \\ w/c \end{pmatrix} t^{MC} \tag{21}$$

where t^{MC}, as before, equals the clock reading at the same position as one of the origins; but not on the RF of the origin itself. Due to the symmetry we are free to choose any of the two origins for this. We observe that (19) and (20) come out as special cases of this vector:

$$\overrightarrow{t(0)} = \overrightarrow{t_a}$$

$$\overrightarrow{t(v)} = \overrightarrow{t_b}$$

Further, we can now interpret $\overrightarrow{t(w)}$ as the time vector of an arbitrary position in between Position I and II. First we choose one RF and a specific t^{MC} on this. Next we specify a x^{MC} on the same RF, and let $w = x^{MC} / t^{MC}$. Then (21) gives the time vector corresponding to the event (t^{MC}, x^{MC}), and we may also obtain the remaining two (time, space) variables at this position from the (orthogonal) Lorentz transformation.

Now consider the interpretation of the parameter, w, which obviously refers to a velocity. We simply imagine that there is a third RF, (K_w) - with relative speed, w – and also having a clock at its origin at the 'point of initiation'. This clock will read time, $t_w = t^{MC}\sqrt{1-(w/c)^2}$ at the instant when our chosen RF reads time, t^{MC} at the position, x^{MC}. Thus, this t_w corresponds to t^{SC} of *eq.* (5), and $\sqrt{1-(w/c)^2}$ just equals the time dilation factor for a RF at relative speed, w. Further, we argue that also this third clock remains synchronized with the two other clocks at the origins (of K_0 and K_v, respectively). Thus, the new event with time vector given in (21) is simultaneous with the events of Position I and II, for any w, satisfying $0 < w < v$.

We see that the absolute value of the time vector generally equals

$$\left|\overrightarrow{t(w)}\right| = t^{MC}$$

We now introduce this absolute value as a general measure of 'distance' from the 'point of initiation', and so this provides a means to define simultaneity. We postulate that events - related to any of the two specified RFs - are simultaneous if their time vector (21) have the same absolute value.

4.2 Time formulated as a complex number

We can of course also write the time vector (21) as a complex number. The magnitude equals t^{MC}, and we define the argument, θ_w by

$$\sin\theta_w = w/c$$

Then, we write time as a complex number, in polar form:

$$t(w) = t^{MC} e^{i\theta_w}$$

As already stated, we interpret the magnitude, t^{MC} as the distance from the point of initiation, and so this parameter defines simultaneity. The argument, θ_w, represents the direction of the time variable, and is given by the speed, w of an (imagined) clock located at the position in question. This provides synchronization with the clocks at the origins of K_0 and K_v. That is, the velocity, w of this third clock is adapted to provide that the event specified by (21) is simultaneous with the events at Position I and II.

Further, the real and imaginary part of the time variable equals

$$\text{Re}\,(t(w)) = \sqrt{1-(w/c)^2}\,t^{MC} = \cos\theta_w\, t^{MC}$$

$$\text{Im}\,(t(w)) = (w/c)t^{MC} = \sin\theta_w\, t^{MC} = x^{MC}/c$$

Again: he real part represents the clock reading of the chosen RF, (at the position of the origin of the other RF). The imaginary part represents the distance in space from the 'point of initiation', but measured by the time required for light to go this distance. So we could say that both the real and imaginary part represent aspects of distance from the 'point of initiation'.

5 Conclusions

The Lorentz transformation relates the (time, space) coordinate of one reference frame (RF) with the (time, space) coordinate at the same position on the other RF. In the approach of the present paper we define a state vector, which combines the time variable of one RF with the space variable of the other. Thus, we obtain two vectors, related by an orthogonal version of the Lorentz transformation, (and thus provide the same information). We can interpret both components of these state vectors as time, and the time vector provides a measure of distance from the 'point of initiation', (*i.e.* time 0 when the origins of the two RFs had a common location). Therefore, we refer to the vector as a 'time vector'.

The vectors have nice features. They have orthogonal components and are related by an orthogonal transformation (rotation) in the two-dimensional (time) space. The first component of each vector equals the clock time. We can further interpret the absolute value as a measure of the overall distance in time from the 'point of initiation'; thus providing a means to define simultaneity.

A basic claim here is that the two clocks at the origin of the two reference frames at time 0 will remain synchronized also after time 0. Due to the inherent symmetry of the movements of the TSR, we conclude that the same clock reading of these two clocks must correspond (at least in some sense) to simultaneity.

This symmetry of the two clocks at the origin also makes it easy to conclude on the travelling twin example. In my opinion, previous discussions of this case has not handled simultaneity properly.

We generalise the time vector to represent time at any position in between the two origins (at Position I and II). We further present the time vector as a complex number. The real part of the time vector equals the clock time. The imaginary part is a measure of distance in terms of speed of light. The magnitude still represents the 'distance' from the initial point of synchronization at time 0. The time variable reduce to a real variable when the RFs are at rest relative to each other.

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