Modeling Virus as Elastic Sphere in Newtonian Fluid based on 3D Non-Stationary Navier-Stokes Equations

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ABSTRACT

Although virus is widely known to significantly affect many biological form of life, its physical model is quite rare. In this paper, we present a mathematical model of virus as an elastic sphere in a Newtonian fluid, i.e. via 3D Non-Stationary Navier-Stokes equations. We also obtain a numerical solution by the help of Mathematica 11.

Keywords: Newtonian fluid, virus model, 3D Navier-Stokes, nonlinear physics, computational physics

1. Introduction

Although virus is widely known to significantly affect many biological form of life, its physical model is quite rare. In a paper, L.H. Ford wrote: “Two simple models for the particle are treated, a liquid drop model and an elastic sphere model. Some estimates for the lowest vibrational frequency are given for each model. It is concluded that this frequency is likely to be of the order of a few GHz for particles with a radius of the order of 50nm.”

In this regard, experiments on the acoustic vibrations of elastic nanostructures in fluid media have been used to study the mechanical properties of materials, as well as for mechanical and biological sensing. The medium surrounding the nanostructure is typically modeled as a Newtonian fluid.

In 2015, Vahe Galstyan, On Shun Pak and Howard A. Stone published a paper where they discuss breathing mode of an elastic sphere in Newtonian and complex fluids.[2] They use a linearized version of Navier-Stokes equations.

In this paper we will also discuss a Newtonian fluid, i.e. 3D Navier-Stokes equations. It is our hope that the new proposed method can be verified with experiments.

2. Mathematical model and computational solution of 3D Navier-Stokes

In 2015, Vahe Galstyan, On Shun Pak and Howard A. Stone published a paper where they discuss breathing mode of an elastic sphere in Newtonian and complex fluids.[2] They consider the purely radial vibration of an elastic sphere of radius R in a compressible viscous fluid. This spherically symmetric motion is also called the breathing mode. The displacement field of the elastic fluid medium is governed by the Navier equation in elasticity.

They use a linearized version of Navier-Stokes equations, as follows:[2]

$$\rho_v \frac{\partial v}{\partial t} = -\nabla p + \eta \nabla^2 v + \left(\kappa + \frac{\eta}{3} \right) \nabla, \quad (1)$$

where ρ_v is the density of the fluid, η is the shear viscosity, κ is the bulk viscosity, and p is the thermodynamic pressure.

Now, instead of using linearized NS equations as above, we will discuss a numerical solution of 3D Navier-Stokes equations based on Sergey Erhskov's papers [4][5].

In fluid mechanics, there is an essential deficiency of the analytical solutions of Navier–Stokes equations for 3D case of non-stationary flow. The Navier-Stokes system of equations for incompressible flow of Newtonian fluids should be presented in the Cartesian coordinates as below (under the proper initial conditions):[4]

$$\nabla . \vec{u} = 0, \quad (2)$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \nu \cdot \nabla^2 \vec{u} + \vec{F}, \quad (3)$$

Where u is the flow velocity, a vector field; ρ is the fluid density, p is the pressure, v is the kinematic viscosity, and F represents external force (per unit mass of volume) acting on the fluid.[4]

In ref. [4], Ershkov explores the ansatz of derivation of non-stationary solution for the Navier–Stokes equations in the case of incompressible flow, which was suggested earlier. In general case, such a solution should be obtained from the mixed system of 2 coupled Riccati ordinary differential equations (in regard to the time-parameter t). But instead of solving the problem analytically, we will try to find a numerical solution.

The coupled Riccati ODEs read as follows:[4]

$$a' = \frac{w_y}{2} \cdot a^2 - (w_x \cdot b) \cdot a - \frac{w_y}{2}(b^2 - 1) + w_z \cdot b, \quad (4)$$

$$b' = -\frac{w_x}{2} \cdot b^2 - (w_y \cdot a) \cdot b - \frac{w_x}{2}(a^2 - 1) + w_z \cdot a \quad (5)$$

First, equations (4) and (5) can be rewritten in the form as follows:

$$x(t)' = \frac{v}{2} \cdot x(t)^2 - (u \cdot y(t)) \cdot x(t) - \frac{v}{2}(y(t)^2 - 1) + w \cdot y(t), \quad (6)$$

$$y(t)' = -\frac{u}{2} \cdot y(t)^2 - (v \cdot x(t)) \cdot y(t) - \frac{u}{2}(x(t)^2 - 1) + w \cdot x(t) \quad (7)$$

Then we can put the above equations into Mathematica expression:[3]

```
v=1;
u=1;
```

```
w=1;
{xans6[t_], vans6[t_]}=
{x[t],y[t]}/.Flatten[NDSolve[{x'[t]==(v/2)*x[t]^2-(u*y[t])*x[t]-(v/2)*(y[t]^2-1)+w*y[t], y'[t]==-(u/2)*y[t]^2-(v*x[t])*y[t]-(u/2)*(x[t]^2-1)+w*x[t], x[0]==1,y[0]==0}, {x[t],y[t]},{t,0,10}]]

graphx6 = Plot[xans6[t],{t,0,10}, AxesLabel->{"t","x"},PlotStyle->Dashing[{0.02,0.02}]];
Show[graphx6,graphx6]
```

The result is as shown below:[3]

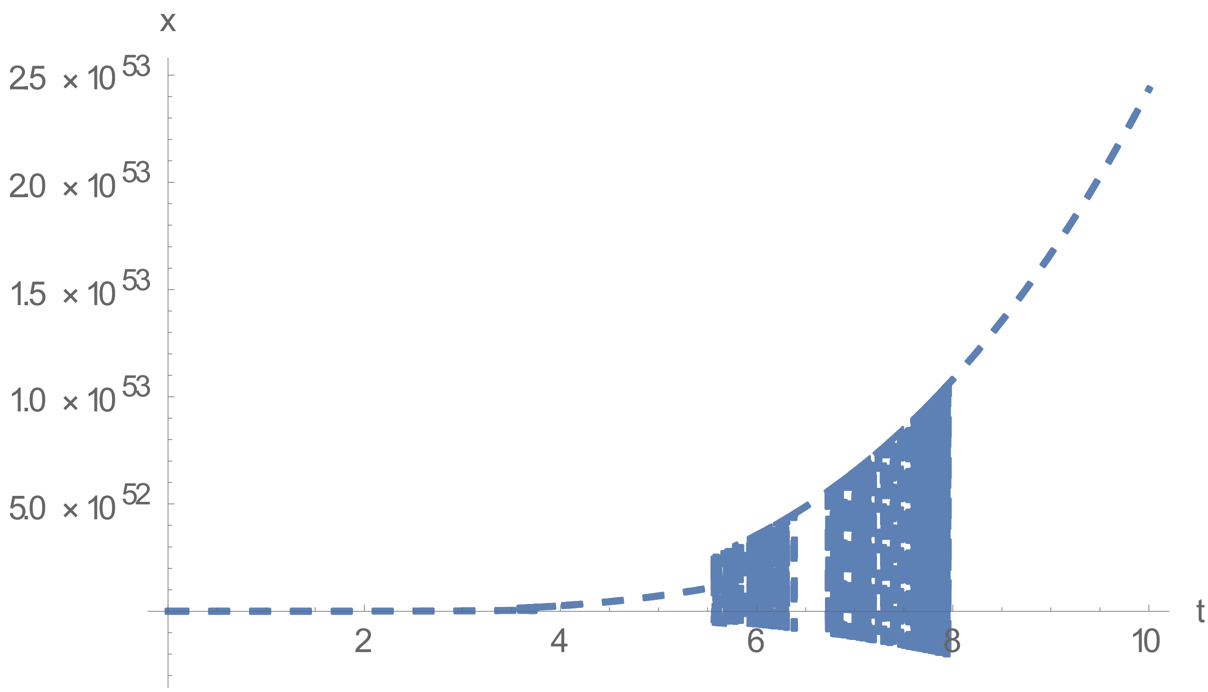

DIAGRAM 1. Graphical plot of solution for case v=u=w=1. See [3]

3. Concluding Remarks

In this paper we review 3D non-stationary Navier-stokes equations obtained by Ershkov, as a model of virus as an elastic sphere in Newtonian fluid, and we solve the equations numerically with Mathematica 11.

It is our hope that the above numerical solution of 3D Navier-Stokes equations can be found useful in biological modeling of virus.

Document history:

- version 1.0: 26 nov. 2017, pk.4:28

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