What we can know: Is Marcus du Sautoy right re Bell's theorem?

Gordon Watson*

Abstract

Here begins a precautionary tale from a creative life in STEM. Bringing an elementary knowledge of vectors to Bell (1964)—en route to refuting Bell's inequality and his theorem—we aim to help STEM students study one of the strangest double-errors in the history of science. To that end we question Marcus du Sautoy's claim that Bell's theorem is as mathematically robust as they come.

1 Preamble

1.1. (i) Bell's theorem is to some, 'the most profound discovery of science' (Stapp 1975:271), 'one of the few essential discoveries of 20th Century physics' (vdMST 1992: v). (ii) Bell's theorem (1964) claims—via Bell-(2) $\neq$ Bell-(3)—that two sensible equations cannot be equal under QM [Bell-(#) is short for Bell 1964:(#)]. (iii) du Sautoy (2016:170) claims that 'Bell's theorem is as mathematically robust as they come.' (iv) With a proof best read next to Bell (1964): we aim to refute both claims. (v) Bell (1964) is freely available (see References): please identify its unnumbered formulae as (11a),(14a)-(14c),(15a),(21a)-(21e),(23). (vi) Taking math to be the best logic, it may flow for several lines before we comment; often with ▲ (absurd) or ■ (QM-true). (vii) QM is our gold-standard for results.

1.2. Refuting other Bellian claims, Watson (2017d) introduces EPR-Bohm (EPRB)—the experiment in Bell's (1964)—at ¶¶21.-2.7. The key to our work there is this: rejecting inferences that are false in quantum settings, we posit the principle of true local realism: the union of true locality (after Einstein) and true (non-naive) realism (after Bohr). Given our QM-true results, we'll be pleased to address issues re our approach: to aid discussion, improvement, correction, our paragraphs, equations, figures, etc, are numbered; constructive and pointed critical comments are especially welcome.

1.3. In Bell (1964), P denotes an expectation (an average). We, reserving P for probabilities, often denote expectations via $\langle\cdot\rangle$; see LHS Bell-(3). Here, to make our work easier to follow wrt Bell (1964), we replace Bell's $P(\vec{a}, \vec{b})$ with $E(a, b)$—which is no bad thing—our a and b being unit-vectors.

1.4. Believing our analysis to be elementary—and QM-true—we'll happily provide additional notes in later versions; see ¶¶1.2 & 3.1. Here's how we set the scene for now, with $A_i = A(a, \boldsymbol{\lambda}_i)$; etc:

$$\text{Bell's inequality: } |E(a,b) - E(a,c)| - E(b,c) - 1 \leq 0. \blacktriangle \tag{1}$$

$$\text{Our inequality: } |E(a,b) - E(a,c)| + E(a,b)E(a,c) - 1 \leq 0. \blacksquare \tag{2}$$

$$\text{Bell's theorem: } E(a,b) = -\int d\lambda\, \rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda}) \neq -a \cdot b. \blacktriangle \tag{3}$$

$$\text{Our theorem: } E(a,b) = \frac{1}{n}\sum_{i=1}^{n} A_i B_i = -\int d\lambda\, \rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda}) = -a \cdot b. \blacksquare \tag{4}$$

· (1) is Bell-(15), Bell's (1964) inequality formatted per ¶1.3: (1) is doubly absurd and false.

· (2) corrects (1): QM-true, (2) is the result that Bell should have reached; see (7)-(12).

· (3) is Bell's theorem: inferred by Bell via (1) and QM, (3) is false because (1) is false.

· (4) is our refutation of Bell's theorem (3): (4) is QM-true, so (4) & (2) are QM-true together.

* eprb@me.com [Ex: BTR24Av3, 20171123.] Ref: BTR30Av1. Date: 20171125.

1.5. Sharing Bell's indifference (1964:195)—whether $\boldsymbol{\lambda}$ is continuous or discrete (we can work with both)—we include both possibilities in (4). The variant of Bell-(2) in (3) & (4) is Bell-(14): it is the form that Bell uses in his principal analysis; (14a)-(14c).

2 Introduction

2.1. A longer Introduction would discuss Bell's nomination for a Nobel Prize—providing context from his other achievements in QM—including the pioneering developments that followed his theorem. [On Einstein's side wrt locality: we question Bell's work on local-causality; which includes his theorem.]

2.2. From Bell (2004, cover): 'John Bell ... is particularly famous for his discovery of the crucial difference between the predictions of conventional quantum theory and the implications of local causality, a concept insisted on by Einstein.' [And insisted on by us: our point being that there is no difference.]

2.3. We believe a wider knowledge of Bell's efforts to resolve his 'action-at-a-distance' (AAD) dilemma (Bell 1990:7)—and of his 'don't be a sissy' have-a-go attitude (Mermin 2001:1)—will bring many students to life, and to a life, in STEM. [Accepting locality, contra Bell (1990:13), we reject AAD.]

3 Analysis

3.1. Encouraging students to study Bell (1964) deeply—and enjoy discovery—some small steps are not included here. [Depending on responses, such steps may be added as endnotes in later updates.]

3.2. En route to showing that (1) is doubly absurd—and thus (3) also—we first confirm an often-overlooked fact: (1) is absurd under QM. Thus, using our (4): (1) under QM becomes

$$|a \cdot c - a \cdot b| + b \cdot c - 1 \quad \leq \quad 0. \tag{5}$$

3.3. We now test (1), hence (5), using RHS (4) with an EPRB example. If $a \cdot b = b \cdot c = \frac{1}{2}$ & $a \cdot c = -\frac{1}{2}$,

$$\text{then LHS (1) and LHS (5)} = |-\tfrac{1}{2} - \tfrac{1}{2}| + \tfrac{1}{2} - 1 = \tfrac{1}{2} \nleq 0. \blacktriangle \tag{6}$$

3.4. Thus, via RHS (4)—with (6) QM-true—Bell's (1) and Bell's (3) are refuted; the latter via (4), which is (3)'s contradiction. Why is it so? As we'll see: Bell's derivation of (1) is itself absurd! ■

3.5. To see this, we start at the same place as Bell. (i) Given ¶1.5 above, we apply LHS (4) to LHS Bell-(14a). (ii) We randomly distribute $3n$ particle-pairs—using up to $3n$ detector-pairs—over randomized detector-settings $(a, b), (a, c), (b, c)$; (a, b) denoting the angle between a and b, etc. (iii) To avoid any chance of false inference—our STEM caution—we allow each particle-pair to be unique: and thus uniquely indexed $[i = 1, 2, ..., 3n]$ for ID purposes. (iv) Finally, we allow n to be such that—to a more-than-adequate accuracy; ie, satisfying serious critics—and for convenience in presentation:

$$\begin{aligned}
\text{LHS Bell(14a)} \quad &= \quad E(a,b) - E(a,c) && (7)\\
&= \quad \frac{1}{n}\sum_{i=1}^{n} A_i B_i - \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i} && (8)\\
&= \quad \frac{1}{n}\sum_{i=1}^{n} A_i B_i \,[\,1 - A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i}\,]. && (9)\\
\text{But } A_i B_i \quad &= \quad \pm 1: \text{ using Bell-(1) with our identifiers.} && (10)\\
\therefore \; |\, E(a,b) - E(a,c) \,| \quad &\leq \quad 1 - \frac{1}{n}\sum_{i=1}^{n} A_i B_i \cdot \frac{1}{n}\sum_{i=1}^{n} A_{n+i} C_{n+i} && (11)\\
&\leq \quad 1 - E(a,b)E(a,c): \text{ which is (2); and QM-true,} && (12)\\
\because \text{ for any } a, b, c \quad &: \quad |a \cdot c - a \cdot b| + (a \cdot b)(a \cdot c) - 1 \leq 0, \text{ using (4).} \blacksquare && (13)
\end{aligned}$$

3.6. With our results QM-true: Bell's inequality (1) and its consequents (5)-(6) are refuted via (7)-(12). Bell's theorem (3) is refuted via its contradiction—RHS (4)—in (12). nb: CHSH (1969) and others imply that (1) is Bell's theorem. Either way, it is refuted; CHSH is itself refuted at Watson 2017d:(41).

3.7. Given ¶3.6, with Bell's inequality and Bell's theorem refuted—and not his later ideas, which Watson (2017d) refutes—the key points of difference here (in our terms) are:

> 'The quantum mechanical expectation value $-a \cdot b$ [see RHS (3) & RHS (4)] cannot [sic] be represented, either accurately or arbitrarily, in the form of Bell-(2),' (p.199). (3) is true [sic], so (4) is not [sic] possible (p.196; line below Bell-(3). Any theory reproducing RHS (4)—the QM predictions—must be nonlocal [sic] (p.195). Since the QM predictions are well-founded, locality must be abandoned [sic]; thus, from Bell (1990:13): 'I step back from asserting that there is AAD and I say only that you cannot [sic] get away with locality. You cannot [sic] explain things by events in their neighbourhood.'

3.8. In sum—using the same technique that Bell used to derive his QM-false (1) & (3); but avoiding the pitfalls (our (2) & (4) being QM-true)—Bell's inequality (1) and Bell's theorem (3) are refuted.

3.9. For the record, the math-truth (and hence, here, the QM-truth) for (1)—and thus for (5)—is,

$$\text{for any } a, b, c \quad : \quad |a \cdot c - a \cdot b| - (b \cdot c) - 1 \leq \tfrac{1}{2}\,; \blacksquare \tag{14}$$

consistent with our result at (6). [Proving the limits of (13) & (14) is left as a STEM exercise.]

3.10. Also—from the note below Bell-(14b)—here's Bell's key error;

$$\text{Bell 1964:(14a)} \neq \text{Bell 1964:(14b); cf RHS (13) with RHS (6), etc.} \tag{15}$$

3.11. In other words: Bell's move (14a) - (14b)—using Bell-(1) incorrectly—is the source of the $\neq$ in (15). Here, from (1) & (2), is the physical significance of that $\neq$ and Bell's false move:

$$E(a,b)E(a,c) + E(b,c) \quad = \quad 0. \tag{16}$$

$$\text{ie, using (4)/QM, Bell has a constraint } \ (a \cdot b)(a \cdot c) - (b \cdot c) \quad = \quad 0 \text{ on detector-settings!} \blacktriangle \tag{17}$$

3.12. To be clear: (17), though absurd here under EPRB, tells us the constraint under which Bell's theorem and du Sautoy's claim hold true. Since (17) is an equality-to-zero, it's an interesting STEM exercise to find the limits of—and to understand the physical-significance of—such constraints.

3.13. Absurdities (1), (3), (6), (17) thus arise from Bell's misuse of Bell-(1) after Bell-(14a). QED. ■

3.14. So: (i) given the scenario in ¶3.5—given $3n$ particle-pairs, with from one to $3n$ detector-pairs available; at different sites—over varied settings equivalent to $(a, b), (a, c), (b, c)$; (ii) given our caution re false STEM inferences; (iii) given Bellian absurdities and our QM-truths; (v) and given du Sautoy (2016:170)—'Bell's theorem is as mathematically robust as they come'—we conclude with a question: Is Bell's theorem as mathematically robust as they come? Or have we missed something?

3.15. See Quora discussion: https://www.quora.com/Is-Marcus-du-Sautoy-right-re-Bells-theorem

4 Conclusions

> 'This was our dilemma: our analysis of EPRB led us to admit that, somehow, distant things are subtly connected, or at least not disconnected,' after Bell (1990:7). But there was hope: 'This action-at-a-distance business will pass. ... If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly. ... But I believe the questions will be resolved,' after Bell (1990:9).

4.1. Eliminating 22 math-expressions—between Bell-(3)-(12) and after Bell-(15)—Bell's ideas would be clearer still if our (3) is declared to be Bell's [invalid] theorem.

4.2. Students should then be encouraged to understand ¶3.5—nb: its particle-by-particle IDs and analysis is more elementary (and much less daunting) than it sounds—to thus find the many interesting errors in Bell-(14b)-(23). They might then join us and refute Bell's theorem on their own terms.

4.3. 'Britain's most famous mathematician'—du Sautoy (2016, cover)—may well say (p.170), 'Bell's theorem is as mathematically robust as they come.' But bound here, as we all are by QM—and by EPRB and true local realism—we find no basis for that conclusion here.

4.4. Indeed, in closing, we launch our own impossibility theorem. Given EPRB, QM, and true local realism [the union of true locality and true realism per Watson (2017d)]—ie, given the locality that we (with Einstein) accept; which Bell (1990:12-13) rejects—it is impossible that Bell-(14a) = Bell-(14b). Or have we missed something?

5 Acknowledgments

27 October 2017—for David Bohm (1917-1992)—the 25th anniversary of his passing.

I again thank those cited in Watson (2017d). I thank Phil Watson (Kariong) for suggestions to improve my writing, and Roger Mc Murtrie (again) for his collaboration. Also, John Bell (1928-1990)—my could-have-been collaborator—for his work on local-causality. [Could have been? In June 1989, seeking to contact John to gift him my simple refutation of his theorem—reading Mermin (1988)—I phoned two physics-professors. The first hung up on me a second time; the second convinced me that John was dead.] Reading Bell (1990:9) again, maybe some of us were being rather silly (at times): but so goes many a creative life in STEM.

6 References [DA = date accessed]

1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200.
http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf [DA20170328]

2. Bell, J. S. (1990). "Indeterminism and nonlocality." Transcript of 22 January 1990, CERN Geneva. Driessen, A. & A. Suarez (1997). Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God. A. 83-100.
http://www.quantumphil.org./Bell-indeterminism-and-nonlocality.pdf [DA20170328]

3. Bell, J. S. (2004). Speakable and Unspeakable in Quantum Mechanics. Cambridge, Cambridge University.

4. CHSH (1969). "Proposed experiment to test local hidden-variable theories." Physical Review Letters 23 (15): 880-884.

5. du Sautoy, M. (2016). What We Cannot Know. London, 4th Estate.

6. Mermin, N. D. (1988). Spooky actions at a distance: Mysteries of the quantum theory. The Great Ideas Today 1988. M. J. Adler. Chicago, Encyclopædia Britannica Inc: 2-53.

7. Mermin, N. D. (2001). "Whose knowledge?"
https://arxiv.org/pdf/quant-ph/0107151.pdf [DA20171116]

8. Stapp, H. P. (1975). "Bell's theorem and world process." Il Nuovo Cimento, 29B (2):270-276.

9. vdMST (1992). Bell's Theorem and the Foundations of Modern Physics A. van der Merwe, F. Selleri, G. Tarozzi, Eds. Singapore, World Scientific: v-vi.

10. Watson, G. (2017d). "Bell's dilemma resolved, nonlocality negated, QM demystified, etc."
http://vixra.org/abs/1707.0322 [DA20171107]