Refutation of realizability Semantics for QML

From:

Rin, B.G.; Walsh, S. (2016). Realizability semantics for quantified modal logic: generalizing Flagg's 1985 construction. arxiv.org/pdf/1510.01977.pdf

We assume the apparatus of the Meth8 modal logic model checker implementing variant system VŁ4. Meth8 allows to mix four logical values with four analytical values. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	p=p	T	Tautology	proof	11	3
2	p@p	F	Contradiction	absurdum	00	0
3	%p>#p	N	Non-contingency	truth	01	1
4	%p<#p	C	Contingency	falsity	10	2

LET: ~ Not; + Or; & And; \ Not and; > Imply; < Not imply; = Equivalent to;
@ Not equivalent to; # all ∀; % some ∃; (p@p) 00 zero; (p=p) 11 one

Results are the repeating proof table(s) of 16-values in row major horizontally.

"The resulting semantics generalize the important but little-understood construction of Flagg (1985), whose goal was to provide a consistency proof of Epistemic Church's Thesis together with epistemic arithmetic, a modal rendition of first-order arithmetic. *Epistemic Church's Thesis* (ECT) is the following statement:

(1.1) $[\Box\ (\forall n \exists m \Box] \varphi(n,m))] \Rightarrow [\exists e \Box] \forall n \exists m \exists q (T(e,n,q) \wedge U(q,m) \wedge \Box \varphi(n,m))]$" (1.1)

LET: pqtuwxy pqtuemn

#((#y&%x)&(#p&(y&x))) >
((%w&#(#y&(%x&(%x&%q))))&(((t&(w&(y&q)))&(u&(q&x)))&(#p&(y&x)))) ;
TTTT TTTT TTTT TTTT, TCTC TCTC TCTC TCTC, TCTT TCTT TCTT TCTT (1.2)

"EZF ... is built from Q_{eq}.S4 by the addition of the following axioms: ...

II. Induction Schema: $[\forall x((\forall y \in x\ \varphi(y)) \Rightarrow \varphi(x))] \Rightarrow [\forall x \varphi(x)]$" (2.1)

LET: pxy φxy

We distribute the quantification in the antecedent to ensure clarity.

((#x&((#y<x)&(p&y)))>(#x&(p&x))) > (#x&(p&x)) ;
FFFF FFFF FFFF FFFF, FNFN FNFN FNFN FNFN (2.2)

"III. Scedrov's Modal Foundation: $[\Box\forall x(\Box(\forall y\in x\varphi(y))\Rightarrow\varphi(x))]\Rightarrow[\Box\forall x\varphi(x)]$" (3.1)

We distribute the quantification in the antecedent to ensure clarity.

```
(#(#x&#((#y<x)&(p&y)))>(#x&(p&x))) > #(#x&(p&x)) ;
FFFF FFFF FFFF FFFF, FNFN FNFN FNFN FNFN                    (3.2)
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Eqs. 1.2, 2.2, and 3.2 as rendered are *not* tautologous. Eqs. 2.2 and 3.2 result in the same truth table because Eq. 3.2 reduces to Eq. 2.2.

We did not test subsequent axioms.

This means respectively that the following are not theorems: Epistemic Church's Thesis; EZF induction schema; and Scedrov's modal foundation.

What follows is that Flagg's construction, Goodman's intensional set theory, and epistemic logic are suspicious.