Neutrino mass phases and the CKM matrix

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Abstract

The Brannen neutrino mass triplet extends Koide's rule for the charged leptons, which was used to correctly predict the τ mass. Assuming that Koide's rule is exact, we consider the small deviation from the 2/9 lepton phase, noting connections to the CKM matrix and arithmetic information. An estimate for the fine structure constant α is included.

Neutrino oscillations [1][2] indicate a triplet of non zero masses for active neutrinos. Any low energy relation between the masses in a triplet is a valuable clue to a theory beyond the Standard Model. Decades ago, Koide [3][4] found a relation for the charged lepton masses, that was later extended by Brannen [5][6] to the neutrino masses using density matrices. Both charged lepton and neutrino triplets are given as the eigenvalues of a circulant Hermitian mass matrix M. Let

$$\sqrt{M} = \frac{\sqrt{\mu}}{r} \begin{pmatrix} r & e^{i\theta} & e^{-i\theta} \\ e^{-i\theta} & r & e^{i\theta} \\ e^{i\theta} & e^{-i\theta} & r \end{pmatrix}, \tag{1}$$

where μ is a scale parameter. For all leptons, the Koide rule corresponds to $r = \sqrt{2}$. For the charged leptons, 3μ happens to equal the proton mass m_p. For neutrinos, a global fit to data gives $\mu = 0.01\text{eV}$. Mass eigenvalues take the form

$$m_k = \mu(1 + \sqrt{2}\cos(\theta + \frac{2\pi k}{3}))^2 \tag{2}$$

for $k = 1, 2, 3$. Phenomenologically, the charged lepton phase θ_l is close to $2/9$ [5] while the neutrino phase is $2/9 + \pi/12$, as if the neutrino masses are governed by a phase triplet

$$\theta_\nu = \frac{2}{9} + \epsilon + \frac{\pi}{12} \tag{3}$$

for $\epsilon \sim 10^{-7}$. The $\pi/12$ is a fundamental arithmetic geometric phase for spinors. The $2/9$ phase is associated with electric charge, since the phases $2/27$ and $4/27$ give the Koide triplets for the quarks [7]. However, since the neutrinos also carry the $2/9$, the rational charge should be viewed as a derivation of charge from quantum gravity. One possibility is

$$\frac{2}{9} = \frac{2\pi}{27} \cdot \frac{3}{\pi} \tag{4}$$

where 27 is the dimension of the exceptional Jordan circulant algebra, and $3/\pi$ is the *apparent* dark energy/matter density fraction by the black hole pair production argument of Riofrio [8]. Then $4/27$ is $2/3$ $(= 18/27)$ of $2/9$. Neutrinos presumably lose the charge because the left and right (mirror) factors of $2/9$ cancel. The diagonalisation of M uses the 3×3 quantum Fourier transform [9], that is conjugation by

$$F = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix}, \tag{5}$$

where ω is the primitive cubed root of unity. The connection between F and the tribimaximal mixing matrix [10] is discussed in [11]. One expects the square root $\sqrt{M}$ to be the fundamental degree of freedom by color gravity duality in a motivic formulation for nonperturbative amplitudes. Neutrino masses are now well constrained by observation [12].

For any prime power dimension p, the Fourier transform matrix is viewed as a column set of eigenvectors for one Pauli operator, as is the identity. In dimension 2, the three Pauli matrices give

$$F_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad R_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{6}$$

There always exists [13][14] a circulant R that generates p out of the $p+1$ mutually unbiased bases, all but F. For $p = 3$,

$$R = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & 1 \\ 1 & 1 & \omega \\ \omega & 1 & 1 \end{pmatrix}. \tag{7}$$

Since $R^3 = iI$, R is a matrix representation of the phase $\delta = \pi/6$. In a six dimensional system that combines one qubit ($p = 2$) and one qutrit ($p = 3$), the tensor product $R \otimes R_2$ stands for the arithmetic phase $\pi/12$ [5].

In the eigenvalues of (2), $\theta = 0$ gives a degenerate pair of masses while the $\theta = \delta$ triplet includes the mass scale μ. We therefore define a basic dimensionless mass splitting parameter

$$\Delta \equiv m_{\theta=0} - m_{\theta=\delta} = \frac{1}{2} + 2\sqrt{2} - \sqrt{6}. \tag{8}$$

If this mass comes from a Koide triplet, the phase satisfies a relation

$$\Delta = (1 + \sqrt{2} \cos(\varpi + \theta + \frac{\pi}{12}))^2, \tag{9}$$

which has a solution

$$\theta = 0.21760 \,\mathrm{rad} \equiv \frac{2}{9} - \kappa. \tag{10}$$

Now compare θ to the three neutrino phases in (3). The phase differences are shown in Table 1. Observe the striking similarity to the three Euler angles of the CKM matrix [15][16]. Letting [11]

$$\rho = 0.611 = \frac{\pi}{4} - \frac{2}{3} \cdot \frac{\pi}{12} = \frac{7\pi}{36} \tag{11}$$

be a quark deformation of a tribimaximal angle, we get an analogous possibility for the PMNS matrix, providing an empirical argument for the importance of the $\pi/12$, as shown in

TABLE I. phase differences (°)

	ϵ	$2/9$	$\pi/12$
$2/9$	12.7	0	2.3
$2/9-\kappa$	12.5	0.3	2.0
$2/9+\kappa$	13.0	0.3	2.5
	ρ	$4/27$	$\rho+\pi/12$
0	35.0	8.5	50.0

Table 1. If these quark and lepton mass phases are exact, this leaves at most five parameters for *all* masses and mixing phases: an r value for the quarks, κ (which is close to $\pi/729$), two mass scale ratios, and one unknown correction. It is likely that this set can be reduced further, leaving *only* the two mass scale ratios μ_l/μ_ν and μ_q/μ_ν.

Koide et al [17] develop a Yukawaon model that accurately recovers mixing matrices and quark and lepton masses with only six parameters. In the ribbon scheme [11] for leptons and quarks, one expects some form of quark lepton complementarity, since leptons are easily transformed into quarks with ribbon twist operators.

Now consider the normalised determinant of (1),

$$D(\theta) \equiv -1 + \sqrt{2}\cos(3\theta), \tag{12}$$

arising as a cubic in r. Since D is invariant under the Fourier transform, it is a product $\pm\sqrt{m_1 m_2 m_3}$ of eigenvalues. The range of $D+1$ is centred around zero precisely when $r=\sqrt{2}$, and this $\sqrt{2}$ defines the tribimaximal mixing matrix [10] as a derivative of F [11]. The diagonal element r for the circulant M is

$$r_M = \frac{4}{\sqrt{5-4D}}. \tag{13}$$

The condition $D=0$ is obtained for $\theta=\pm\pi/12$. Allowing for a $-\pi/12$ in (3), we obtain a set of mirror masses, used in [18][19] to study the mirror neutrino CMB cosmology [19]. At $D+1=0$ (ie. the phase δ) and $r=\sqrt{2}$, the Brannen rule for neutrinos gives

$$m_1 m_2 m_3 = \frac{1}{729}(\sqrt{m_1}+\sqrt{m_2}-\sqrt{m_3})^6. \tag{14}$$

Allowing the mass differences between θ and $(\theta+\delta)$ to define

$$\Delta(\theta) = \frac{1}{2}\cos^2\theta + \sqrt{2}\sin\theta + (2\sqrt{2}-\sqrt{6})\cos\theta + \frac{\sqrt{3}}{4}\sin 2\theta, \tag{15}$$

we obtain the Weierstrass cubic for $\mathbf{D}^2 = 4m_1m_2m_3$,

$$\mathbf{D}^2 = 4\Delta^3 + 24\Delta - 14. \tag{16}$$

Neutrinos are neutral because their ribbon diagrams have no charge twists [11]. In the categorical ribbon scheme, one full twist stands for a unit of charge. To estimate the fine structure constant using this charge, consider quantum data for Jones invariants from Chern-Simons field theory [20]. Empirically, we take the Hopf link invariant at $5\pi/6$, which traces over a twist at the minimal mass in the δ triplet, to obtain [21]

$$\sqrt{\alpha} = 4\cosh(\frac{2\pi}{5\pi/6+1}), \tag{17}$$

roughly giving $\alpha = 137.096$.

These results motivate a further study of ribbon graphs in color gravity theories, where the obviously arithmetic nature of neutrino triplets plays an important role.

[1] B. Pontecorvo, Sov. Phys. JETP, 7, 172, 1958

[2] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870, 1962

[3] Y. Koide, Phys. Rev. D28, 1, 252-254, 1983

[4] Y. Koide, Phys. Lett. B120, 161-165, 1983

[5] C. A. Brannen, papers at viXra; http://www.brannenworks.com/

[6] C. A. Brannen, https://carlbrannen.wordpress.com/

[7] M. D. Sheppeard, old posts at the blog *Arcadian Functor*

[8] L. Riofrio, http://riofriospacetime.blogspot.com

[9] M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information*, Cambridge U. P., 2000

[10] P. F. Harrison, D. H. Perkins and W. G. Scott, arXiv:hep-ph/0202074

[11] M. D. Sheppeard, papers at viXra

[12] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola and J. W. F. Valle, arXiv:1708.01186

[13] J. Schwinger, Proc. Nat. Acad. Sci. USA 46, 570, 1960

[14] M. Combescure, J. Math. Phys. 50, 032104, 2009

[15] N. Cabibbo, Phys. Rev. Lett. 10, 531, 1963

[16] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652, 1973

[17] Y. Koide and H. Nishiura, Phys. Rev. D92, 111301(R), 2015

[18] G. Dungworth and M. D. Sheppeard, viXra:1102.0010

[19] G. Dungworth and M. D. Sheppeard, viXra:1711.0119

[20] E. Witten, Nucl. Phys. B311, 46-78, 1988

[21] M. D. Sheppeard, old posts as Kea on PhysicsForums