Neutrino mass phases and the CKM matrix

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Abstract

The Brannen neutrino mass triplet extends Koide's rule for the charged leptons, which was used to correctly predict the τ mass. Assuming that Koide's rule is exact, we consider the small deviation from the $2/9$ lepton phase, noting a striking connection to the CKM matrix. An estimate for the fine structure constant α is included.

Neutrino oscillations [1][2] indicate a triplet of non zero masses for active neutrinos. Any low energy relation between the masses in a triplet is a valuable clue to a theory beyond the Standard Model. Decades ago, Koide [3][4] found a relation for the charged lepton masses, that was later extended by Brannen [5][6] to the neutrino masses using density matrices. Both charged lepton and neutrino triplets are given as the eigenvalues of a circulant Hermitian mass matrix M. Let

$$\sqrt{M} = \frac{\sqrt{\mu}}{r} \begin{pmatrix} r & e^{i\theta} & e^{-i\theta} \\ e^{-i\theta} & r & e^{i\theta} \\ e^{i\theta} & e^{-i\theta} & r \end{pmatrix}, \tag{1}$$

where μ is a scale parameter. For all leptons, the Koide rule corresponds to $r = \sqrt{2}$. For the charged leptons, 3μ happens to equal the proton mass m_p. For neutrinos, a global fit to data gives $\mu = 0.01$eV. Mass eigenvalues take the form

$$m_k = \mu(1 + \sqrt{2}\cos(\theta + \frac{2\pi k}{3}))^2 \tag{2}$$

for $k = 1, 2, 3$. Phenomenologically, the charged lepton phase θ_l is close to $2/9$ [5] while the neutrino phase is $2/9 + \pi/12$, as if the neutrino masses are governed by a phase triplet

$$\theta_\nu = \frac{2}{9} + \epsilon + \frac{\pi}{12} \tag{3}$$

for $\epsilon \sim 10^{-7}$. The $\pi/12$ is a fundamental arithmetic geometric phase for spinors. The $2/9$ phase is associated with electric charge, since the phases $2/27$ and $4/27$ give the Koide triplets for the quarks [7]. The diagonalisation of M uses the 3×3 quantum Fourier transform [8], that is conjugation by

$$F = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix}, \tag{4}$$

where ω is the primitive cubed root of unity. The connection between F and the tribimaximal mixing matrix [9] is discussed in [10]. One expects the square root $\sqrt{M}$ to be the fundamental degree of freedom by color gravity duality in a motivic formulation for nonperturbative amplitudes. Neutrino masses are now well constrained by observation [11].

For any prime power dimension p, the Fourier transform matrix is viewed as a column set of eigenvectors for one Pauli operator, as is the identity matrix I. In dimension 2, the three Pauli matrices give

$$F_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad R_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{5}$$

There always exists a circulant R that generates p out of the $p+1$ mutually unbiased bases, all but F. For $p = 3$,

$$R = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & 1 \\ 1 & 1 & \omega \\ \omega & 1 & 1 \end{pmatrix}. \tag{6}$$

Since $R^3 = iI$, R is a matrix representation of the phase $\delta = \pi/6$. In a six dimensional system that combines one qubit ($p = 2$) and one qutrit ($p = 3$), the tensor product $R \otimes R_2$ stands for the arithmetic phase $\pi/12$ [5].

In the eigenvalues of (2), $\theta = 0$ gives a degenerate pair of masses while the $\theta = \delta$ triplet includes the mass scale μ. We therefore define a basic dimensionless mass splitting parameter

$$\Delta \equiv m_{\theta=0} - m_{\theta=\delta} = \frac{1}{2} + 2\sqrt{2} - \sqrt{6}. \tag{7}$$

If this mass comes from a Koide triplet, the phase satisfies a relation

$$\Delta = (1 + \sqrt{2}\cos(\varpi + \theta + \frac{\pi}{12}))^2, \tag{8}$$

which has a solution

$$\theta = 0.21760 \text{ rad} \equiv \frac{2}{9} - \kappa. \tag{9}$$

Now compare θ to the three neutrino phases in (3). The phase differences are shown in Table 1. Observe the striking similarity to the three Euler angles of the CKM matrix. An analogous possibility is also given for the MNS matrix.

In the ribbon scheme for leptons and quarks, one expects some form of quark lepton complementarity, since leptons are easily transformed into quarks with ribbon twist operators.

Now consider the normalised determinant of (1),

$$D(\theta) \equiv -1 + \sqrt{2}\cos(3\theta), \tag{10}$$

arising as a cubic in r. Since D is invariant under the Fourier transform, it is a product $\pm\sqrt{m_1 m_2 m_3}$ of eigenvalues. The range of $D+1$ is centred around zero

Table 1: phase differences (°)

	ϵ	2/9	$\pi/12$
$2/9-\kappa$	12.5	0.26	2.5
$2/9+\kappa$	13.0	0.24	2.0
	ϵ	12/27	$\pi/3$
$4/27-\kappa$	8.2	35.7	51.2
$4/27+\kappa$	8.7	36.2	51.7

precisely when $r=\sqrt{2}$, and this $\sqrt{2}$ parameter defines the tribimaximal mixing matrix [9] as a derivative of F [10]. The diagonal element r for the circulant M is

$$r_M = \frac{4}{\sqrt{5-4D}}. \tag{11}$$

The condition $D=0$ is obtained for $\theta=\pm\pi/12$. Allowing for a $-\pi/12$ in (3), we obtain a set N of mirror neutrino masses, which were used in [12][13] to study the mirror neutrino CMB cosmology [13].

Neutrinos are neutral because their ribbon diagrams have no charge twists [10]. In the categorical ribbon scheme, one full twist stands for a unit of charge. To estimate the fine structure constant using this charge, consider quantum data for Jones invariants from Chern-Simons field theory [14]. Empirically, we take the Hopf link invariant at $5\pi/6$, which traces over a twist at a phase in the δ triplet, to obtain [15]

$$\sqrt{\alpha} = 4\cosh(\frac{2\pi}{5\pi/6+1}), \tag{12}$$

roughly giving $\alpha = 137.09567$.

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