

# Infinitorun : part 1

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abstract

This note presents three integrals for  $\pi$ .

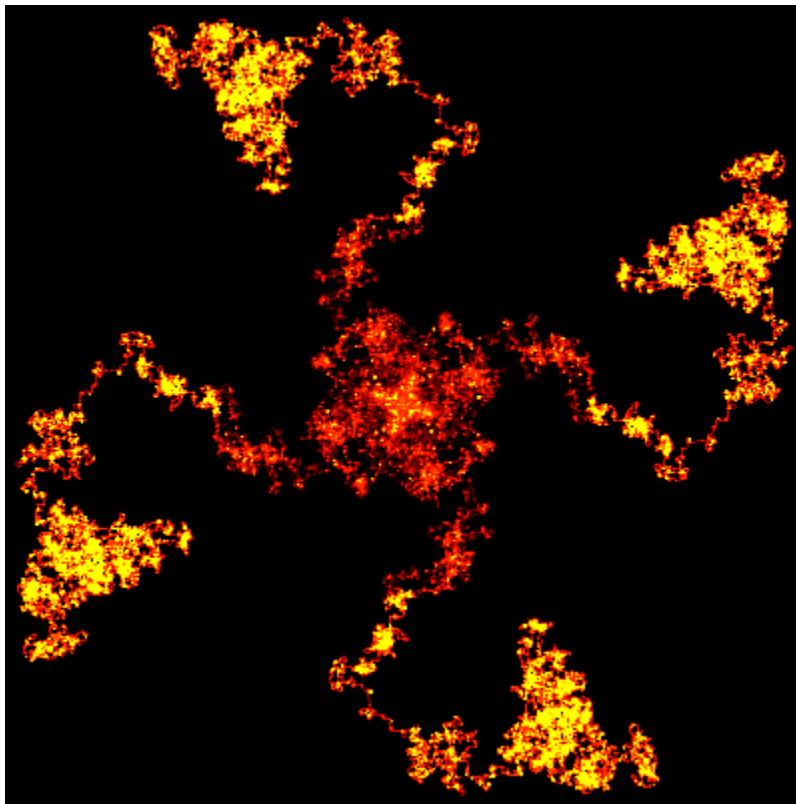


Fig.1 Attractor

### Three integrals for pi

$$\pi = \frac{9\sqrt{3}}{4} - 24 \int_0^{\sqrt{3}/4} \left\{ x^2 + \left( x^2 + \left( x^2 + \left( x^2 + \dots \right)^2 \right)^2 \right)^2 \right\} dx \quad (1)$$

$$\pi = 4\sqrt{2} - 2 - 32 \int_0^{\sqrt{2}/4} \left\{ x^2 + \left( x^2 + \left( x^2 + \left( x^2 + \dots \right)^2 \right)^2 \right)^2 \right\} dx \quad (2)$$

$$\pi = 6 - \frac{3\sqrt{3}}{2} - 48 \int_0^{1/4} \left\{ x^2 + \left( x^2 + \left( x^2 + \left( x^2 + \dots \right)^2 \right)^2 \right)^2 \right\} dx \quad (3)$$

### Some remarks

Notation:

$$f_1(x) = x^2 \quad (4)$$

$$f_2(x) = x^2 + (x^2)^2 \quad (5)$$

$$f_3(x) = x^2 + \left( x^2 + (x^2)^2 \right)^2 \quad (6)$$

$$f_4(x) = x^2 + \left( x^2 + \left( x^2 + (x^2)^2 \right)^2 \right)^2 \quad (7)$$

$$f_n(x) = x^2 + \underbrace{\left( x^2 + \left( x^2 + \left( x^2 + \dots \right)^2 \right)^2 \right)^2}_n \quad (8)$$

❖ Approximations:

Formula (1):

$$\pi 1_n = \frac{9\sqrt{3}}{4} - 24 \int_0^{\sqrt{3}/4} f_n(x) dx \quad , n = 1, 2, 3, \dots \quad (9)$$

$$\pi 1_n \rightarrow \pi \quad (10)$$

Formula (2):

$$\pi 2_n = 4\sqrt{2} - 2 - 32 \int_0^{\sqrt{2}/4} f_n(x) dx, n = 1, 2, 3, \dots \quad (11)$$

$$\pi 2_n \rightarrow \pi \quad (12)$$

Formula (3):

$$\pi 3_n = 6 - \frac{3\sqrt{3}}{2} - 48 \int_0^{1/4} f_n(x) dx, n = 1, 2, 3, \dots \quad (13)$$

$$\pi 3_n \rightarrow \pi \quad (14)$$

❖ Graphics:

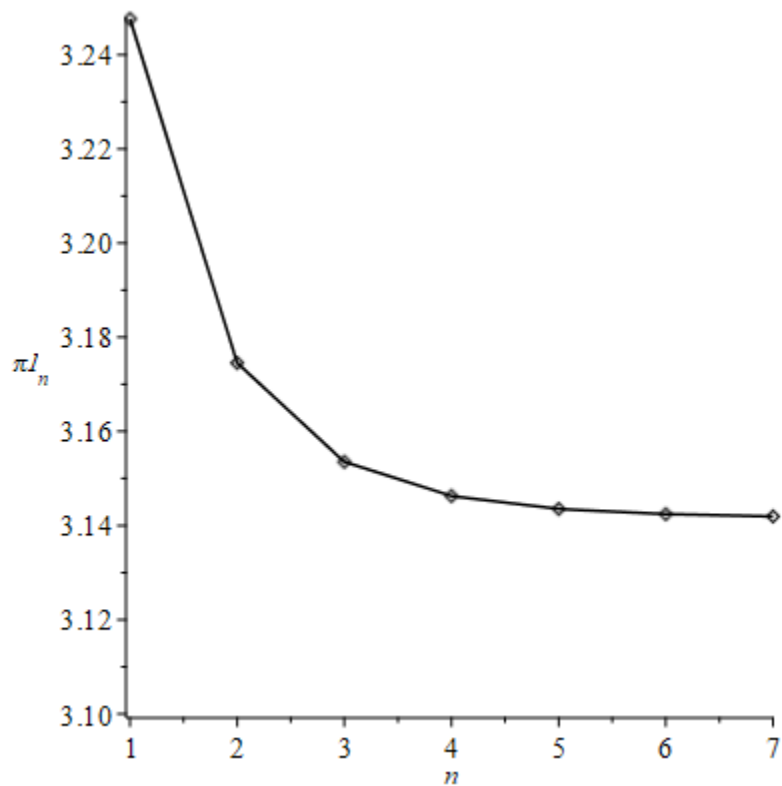


Fig. 2

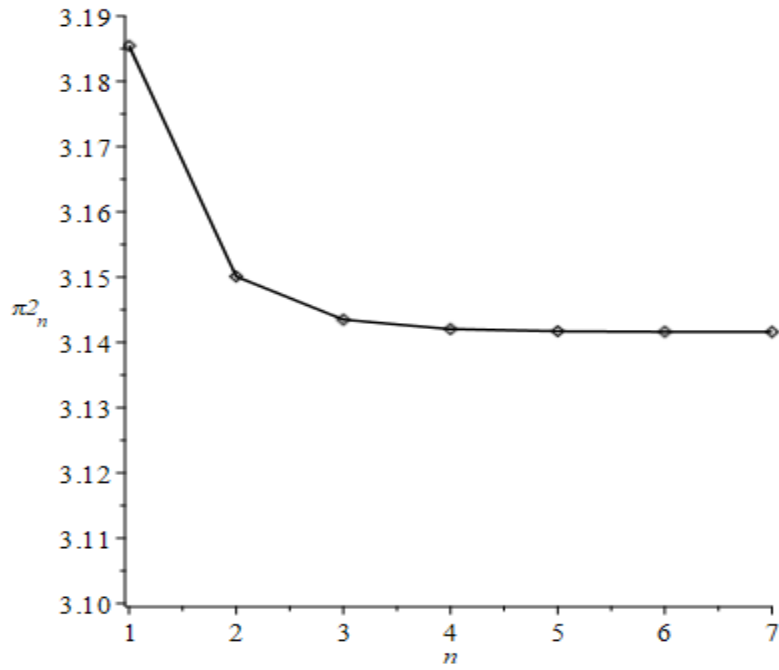


Fig. 3

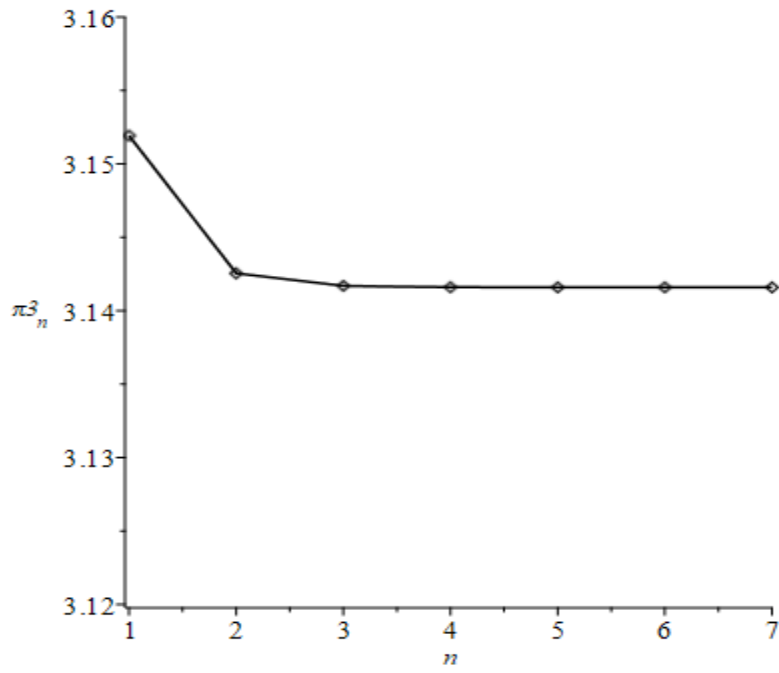


Fig. 4

❖ Evaluations:

$$\pi = 3.141592653589\dots \quad (15)$$

$$\pi_{10} = 3.141624488251\dots \quad (16)$$

$$\pi_{20} = 3.141592783204\dots \quad (17)$$

$$\pi_{30} = 3.141592653619\dots \quad (18)$$

Conclusion: the convergence is very slow.

## References

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