Conjecture that any square of a prime can be obtained through an unusual operation on the numbers 360k+72

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Abstract. In this paper I make the following conjecture: The square of any odd prime can be obtained from the numbers of the form 360*k + 72 in the following way: let d1, d2, ..., dn be the (not distinct) prime factors of the number 360*k + 72; than for any square of a prime p^2 there exist k such that (d1 - 1)*(d2 - 1)*...*(dn - 1) + 1 = p^2. Example: for p^2 = 13^2 = 169 there exist k = 17 such that from 360*17 + 72 = 6192 = 2^4*3^2*43 is obtained 1^4*2^2*42 + 1 = 169. I also conjecture that any absolute Fermat pseudoprime (Carmichael number) can be obtained through the presented formula, which attests again the special relation that I have often highlighted between the nature of Carmichael numbers and the nature of squares of primes.

Conjecture:

The square of any odd prime can be obtained from the numbers of the form 360*k + 72 in the following way: let d1, d2, ..., dn be the (not distinct) prime factors of the number 360*k + 72; than for any square of a prime p^2 there exist k such that (d1 - 1)*(d2 - 1)*...*(dn - 1) + 1 = p^2.

The less k for ten squares of odd primes:
(obtained for k up to 100)

: p^2 = 3^2 = 9 is obtained for k = 1 because from 360*1 + 72 = 432 = 2^4*3^3 is obtained 1^4*2^3 + 1 = 9;

: p^2 = 5^2 = 25 is obtained for k = 11 because from 360*11 + 72 = 4032 = 2^6*3^2*7 is obtained 1^6*2^2*6 + 1 = 25;

: p^2 = 7^2 = 49 is obtained for k = 4 because from 360*4 + 72 = 1512 = 2^3*3^3*7 is obtained 1^3*2^3*6 + 1 = 49;

: p^2 = 11^2 = 121 is obtained for k = 6 because from 360*6 + 72 = 2232 = 2^3*3^2*31 is obtained 1^3*2^2*30 + 1 = 121;
\[ p^2 = 13^2 = 169 \] is obtained for \( k = 17 \) because from 
\[ 360 \times 17 + 72 = 6192 = 2^4 \times 3^2 \times 43 \] 
is obtained 
\[ 1^4 \times 2^2 \times 42 + 1 = 169; \]

\[ p^2 = 17^2 = 289 \] is obtained for \( k = 18 \) because from 
\[ 360 \times 18 + 72 = 6552 = 2^3 \times 3^2 \times 7 \times 13 \] 
is obtained 
\[ 1^3 \times 2^2 \times 6 \times 12 + 1 = 289; \]

\[ p^2 = 23^2 = 529 \] is obtained for \( k = 40 \) because from 
\[ 360 \times 40 + 72 = 14472 = 2^3 \times 3^3 \times 67 \] 
is obtained 
\[ 1^3 \times 2^2 \times 6 \times 66 + 1 = 529; \]

\[ p^2 = 29^2 = 841 \] is obtained for \( k = 42 \) because from 
\[ 360 \times 42 + 72 = 15192 = 2^3 \times 3^2 \times 211 \] 
is obtained 
\[ 1^3 \times 2^2 \times 210 + 1 = 841; \]

\[ p^2 = 31^2 = 961 \] is obtained for \( k = 48 \) because from 
\[ 360 \times 48 + 72 = 17352 = 2^3 \times 3^2 \times 241 \] 
is obtained 
\[ 1^3 \times 2^2 \times 240 + 1 = 961; \]

\[ p^2 = 41^2 = 1681 \] is obtained for \( k = 84 \) because 
from 
\[ 360 \times 84 + 72 = 30312 = 2^3 \times 3^2 \times 421 \] 
is obtained 
\[ 1^3 \times 2^2 \times 420 + 1 = 1681. \]

**Conjecture:**

Any Carmichael number can be obtained from the numbers of the form \( 360 \times k + 72 \) in the following way: let \( d_1, d_2, \ldots, d_n \) be the (not distinct) prime factors of the number \( 360 \times k + 72 \); then for any Carmichael number \( C \) there exist \( k \) such that 
\[ (d_1 - 1)(d_2 - 1)\ldots(d_n - 1) + 1 = C. \]

**The less \( k \) for two Carmichael numbers:**

( obtained for \( k \) up to 100)

\[ C = 561 \] is obtained for \( k = 85 \) because from 
\[ 360 \times 85 + 72 = 30672 = 2^4 \times 3^3 \times 71 \] 
is obtained 
\[ 1^4 \times 2^3 \times 70 + 1 = 561; \]

\[ C = 1729 \] is obtained for \( k = 96 \) because from 
\[ 360 \times 96 + 72 = 34632 = 2^3 \times 3^2 \times 13 \times 37 \] 
is obtained 
\[ 1^3 \times 2^2 \times 12 \times 36 + 1 = 1729. \]