Bell's theorem refuted for STEM students

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Abstract

Bringing an elementary knowledge of vectors to Bell (1964), we amend Bell's inequality, reveal his mistake, refute his theorem: all in the hope of helping STEM students study one of the most famous—and strangest—works in the history of physics. For who else but Bell uses flawed approximations of unnecessary experiments to invalidate the flawed use of a fact: and then rejects that fact? Here begins a precautionary tale from a creative life in STEM.

1 Preamble

1.1. Key texts are freely available online; see References. For an extended preamble, see Watson (2017d). There, at ¶¶2.1-2.7 and by way of background here, we introduce EPR-Bohm (EPRB): the famous experiment discussed in Bell (1964). The key to our work is this: we simply reject inferences that are false in quantum settings. This leads us to the principle of true local realism: the union of true locality (after Einstein) and true (non-naive) realism (after Bohr). All our results then accord with QM's demystification and experiment, so we'll be pleased to address any concerns re our work.

1.2. This essay is best read alongside Bell (1964). To facilitate discussion: (i) a short-form reference to Bell's (1) is a reference to Bell 1964:(1), etc; (ii) after Bell's (14), identify the three unnumbered expressions as (14a)-(14c). Our focus will now be on Bell's famous inequality (15). This fame derives from Bell's widely-accepted claim that, given his (15) and QM's validity: his sensible (2) cannot equal his valid QM-based RHS (3). We refute Bell's claim—famously known as Bell's theorem—below.

1.3. Bell (1964) uses P to denote an expectation whereas we, reserving P for probabilities, denote expectations via $\langle\cdot\rangle$. Here, however, to make our analysis easier to follow wrt Bell (1964), we replace Bell's $P(\vec{a},\vec{b})$ with $E(a,b)$ [a and b being unit-vectors]. [Comments in [·] are there in case of need.]

2 Analysis

2.1. We first reformat Bell's famous inequality, 1964:(15); en route to showing it to be doubly absurd.

$$|E(a,b) - E(a,c)| - E(b,c) - 1 \quad \leq \quad 0. \tag{1}$$

2.2. We next confirm that (1) is absurd under QM. Using RHS Bell's (3), (1) under QM becomes:

$$|a\cdot c - a\cdot b| + b\cdot c - 1 \quad \leq \quad 0. \tag{2}$$

2.3. We now check (2), hence (1), using *any* valid EPRB example; say: $a\cdot b = b\cdot c = \frac{1}{2}\,;\ a\cdot c = -\frac{1}{2}$.

$$\text{So LHS (2)} = |a\cdot c - a\cdot b| + b\cdot c - 1 = \tfrac{1}{2} \nleq 0.\ \blacktriangle \tag{3}$$

2.4. So Bell's famous (1) is absurd under QM: for what was claimed in (1)-(2) is absurd in (3)! How can that be? Well, as we'll see next: Bell's derivation of famous (1) is also absurd a second time! ■

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2.5. To see this, we take what we believe to be the easier leg of Bell's indifference (1964:195)—though we can handle both—and we allow each $\boldsymbol{\lambda}$ to be discrete; ie, quantum-like and thus more physically significant for us. We then randomly distribute $3n$ particle-pairs over the detector settings $(a, b), (a, c), (b, c)$—using $3n$ detector-pairs if we wish—n being sufficient to deliver an adequate accuracy; ie, one that satisfies any critic. [See Watson (2017d: ¶¶2.22-2.28) for help; if you must.]

2.6. Thus, using a not-too-difficult particle-by-particle analysis of EPRB—and the discrete version of (2)—we derive an inequality that is true under both QM and mathematics [Watson 2017d:(39)]:

$$|\,E(a,b) - E(a,c)\,| + E(a,b)E(a,c) - 1 \quad \leq \quad 0.\blacksquare \tag{4}$$

2.7. NB: (4) is valid under EPRB, QM and mathematics because,

$$\text{for any } a,b,c\colon |\,a\cdot c - a\cdot b\,| + (a\cdot b)(a\cdot c) - 1 \quad \leq \quad 0.\blacksquare \tag{5}$$

2.8. On the other hand: Bell's famous (15)—as in (1)—exceeds its claimed limit there (≤ 0) because the related mathematical fact—to be clear, also relevant under EPRB and QM—is this:

$$\text{for any } a,b,c:|\,a\cdot c - a\cdot b\,| + (b\cdot c) - 1 \quad \leq \quad \tfrac{1}{2};\blacksquare \tag{6}$$

which is the result we obtained in (3). [Proofs of (5) and (6) are left as STEM exercises.]

2.9. So now, to be clear, here's Bell's (1964:199) claim—the basis for his theorem—in our terms. *The quantum mechanical expectation value $E(a,b) = -a\cdot b$ cannot be represented, either accurately or arbitrarily, in the form of Bell's (2).*

$$\text{ie: } E(a,b) = \smallint d\lambda\, \rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})B(b,\boldsymbol{\lambda}) = -\smallint d\lambda\, \rho(\boldsymbol{\lambda})A(a,\boldsymbol{\lambda})A(b,\boldsymbol{\lambda}) \neq -a\cdot b; \tag{7}$$

which—by combining the line below his (3) with "This is the theorem" from Bell (2004: 65)—is (in our view) Bell's [officially endorsed] theorem: extended, for future convenience, by our use of Bell's (13).

2.10. A question with but one answer thus arises [since (for us) Bell's theorem (7) is still unproven]: is (7) correct, and the source of (3) as Bell claims; or is (7) wrong, despite (3)? The latter is the case!

2.11. In essence, to explain (3), (6), and the absurdity of his famous (1) under QM, Bell recruits (7) with its $\neq$. However, using the physically-significant discrete form of Bell's (2) as above, we counter via valid (3), (4), (5), (6). *And—using RHS of Bell's* (3)—we obtain the correct generalized QM results for each of them: ie, generalized by being valid over any a, b, c.

2.12. Thus: against Bell's inequality (15) and his absurd results under *some* a, b, c, we derive several inequalities, each devoid of absurdity under *any* a, b, c. [See also LHS Watson 2017d:(41).]

2.13. So: *using the same technique that Bell used to derive his QM-invalid theorem* (7) with its $\neq$, we derive several relevant inequalities: all of which are QM-true. So—on Bell's own terms—Bell's theorem (7) is refuted! [To be clear: the $\neq$ in (7) should be $=$. See also Watson 2017d:(24), (66), (80); with shorter forms (using delta-functions) available.]

2.14. Further, reinforcing our case, we factually pinpoint Bell's error. That fact is this:

$$\text{Bell 1964:(14a)} \neq \text{Bell 1964:(14b)}. \tag{8}$$

2.15. For that note below Bell's (14b) tells us what he used: and the scenario sketched in ¶2.5 exposes the error in (and the impossibility of) such use. (Hint: we have from one to $3n$ detector-pairs available; at different sites and with variable settings, if we wish.) [See Watson 2017d:(36).]

3 Conclusions

> 'This was our dilemma: our analysis of EPRB led us to admit that, somehow, distant things are subtly connected, or at least not disconnected,' after Bell (1990:7). But there was hope: 'This action-at-a-distance business will pass. ... If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly. ... But I believe the questions will be resolved,' after Bell (1990:9).

3.1. Eliminating 22 math-expressions—between (3)-(12) and after (15)—we see Bell's ideas more clearly; clearer still if Bell's (2)-(3)+(claim) is declared to be his theorem and written as in (7).

3.2. Students should then be encouraged to understand ¶2.5—nb: its subtle particle-by-particle analysis is more elementary (and much less daunting) than it sounds—and find the many interesting errors in Bell's (14b)-(22). They should then see that the key to our analysis is this: we simply reject inferences (which includes math) that are false in quantum settings; many of which are also false in classical settings. [Think of Malus' experiments (c.1808); see Watson 2017d: ¶2.18, etc.] They might then join us in refuting Bell's theorem; as here and elsewhere.

3.3. Widely regarded as a leading mathematician in UK, du Sautoy (2016:170) may well say: 'Bell's theorem is as mathematically robust as they come.' But Bell's math is conducted in the context of EPRB and STEM; and in that context Bell's math fails due to flawed analysis based on an unphysical assumption: not due to LHS (7) above. [See the approach at Watson (2017d: ¶2.22-2.28).]

3.4. Well may we say: Bell (1964) uses flawed approximations of unnecessary experiments to invalidate his flawed use of a mathematical fact; and then rejects the fact. For we include the examples under '3 Illustration' and the fact is that his (2) is valid. But wider knowledge of Bell's battle to resolve his dilemma re 'action-at-a-distance' and locality (Bell 1990:7)—and of his little-known 'don't be a sissy' have-a-go attitude (Mermin 2001:1)—will surely bring more students to life, and to a life, in STEM.

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