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Another Note on Paraconsistent Neutrosophic Sets

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Abstract: In an earlier paper, we proved that Smarandache's definition of neutrosophic paraconsistent topology is neither a generalization of Çoker's intuitionistic fuzzy topology nor a generalization of Smarandache's neutrosophic topology. Recently, Salama and Alblowi proposed a new definition of neutrosophic topology, that generalizes Çoker's intuitionistic fuzzy topology. Here, we study this new definition and its relation to Smarandache's paraconsistent neutrosophic sets.

Keywords: logic; set-theory; topology; Atanassov's intuitionistic fuzzy sets

1. Introduction

In various papers, Smarandache [1,2] has generalized Atanassov's intuitionistic fuzzy sets [3] to neutrosophic sets.

Çoker [4] defined and studied intuitionistic fuzzy topological spaces.

On the other hand, various authors including Priest et al. [5] worked on paraconsistent logic, that is, logic where some contradiction is admissible. We refer the reader to studies of References [6–8] as well as the work on paraconsistent fuzzy logic conducted in Reference [9].

Smarandache [2] also defined neutrosophic paraconsistent sets, and proposed a natural definition of neutrosophic paraconsistent topology.

In an earlier paper [10], we proved that this Smarandache's definition of neutrosophic paraconsistent topology is neither a generalization of Çoker's intuitionistic fuzzy topology nor of Smarandache's general neutrosophic topology.

Recently, Salama and Alblowi [11] proposed a new definition of neutrosophic topology that generalizes Çoker's intuitionistic fuzzy topology.

In this paper, we study this new definition and its relation to Smarandache's paraconsistent neutrosophic sets.

The interest of neutrosophic paraconsistent topology was previously shown by us [12] (Section 4).

2. Materials and Methods

First, we present some basic definitions:

Robinson [13] developed the non-standard analysis, a formalization of analysis and a branch of mathematical logic, which rigorously defines infinitesimals. Formally, a number x is said to be infinitesimal if for all positive integers n, one has $|\, x \,| < 1/n$. Let $\varepsilon \geq 0$ be such an infinitesimal number. The hyper-real number set is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let us consider the non-standard finite numbers (1+) = 1 + ε, where "1" is its standard part and "ε" its non-standard part, and (-0) = 0 − ε, where "0" is its standard part and "ε" its non-standard part. Then, we denote $]{-0}, 1{+}[$ to indicate a non-standard unit interval. Obviously, 0 and 1, and analogously non-standard numbers infinitely smaller but less than 0 or infinitely smaller but greater than 1, belong to the non-standard unit interval. It can be proven that S is a standard finite set if and only if every element of S is standard (See Reference [14]).

Definition 1. *In Reference [2], let T,I,F be real standard or non-standard subsets of the non-standard unit interval]−0,1+[, with*

$sup\ T = t_{sup}\ inf\ T = t_{inf},$
$sup\ I = i_{sup},\ inf\ I = i_{inf};$
$sup\ F = f_{sup}\ ,\ inf\ F = f_{inf}$ *and*
$n_{sup} = t_{sup} + i_{sup} + f_{sup}\ ,\ n_{inf} = t_{inf} + i_{inf} + f_{inf}.$

T, I, F are called neutrosophic components. Let U be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as x(T,I,F) and belongs to M in the following way: it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F. The set M is called a neutrosophic set (NS).

Definition 2. *In Reference [2], a neutrosophic set x (T,I,F) is called paraconsistent, if inf (T) + inf (I) + inf (F) > 1.*

Definition 3. *In Reference [11], the NSs* 0_N *and* 1_N *are defined as follows:*

0_N *may be defined as:*

(0_1) $0_N = x\ (0, 0, 1)$
(0_2) $0_N = x\ (0, 1, 1)$
(0_3) $0_N = x\ (0, 1, 0)$
(0_4) $0_N = x\ (0, 0, 0)$

1_N *may be defined as:*

(1_1) $1_N = x\ (1, 0, 0)$
(1_2) $1_N = x\ (1, 0, 1)$
(1_3) $1_N = x\ (1, 1, 0)$
(1_4) $1_N = x\ (1, 1, 1)$

Definition 4. *In Reference [11], let X be a non-empty set and* $A = x\ (T_A, I_A, F_A)$, $B = x\ (T_B, I_B, F_B)$ *be NSs. Then:*
$A \cap B$ *may be defined as:*

(I_1) $A \cap B = x\ (T_A \cdot T_B, I_A \cdot I_B, F_A \cdot F_B)$
(I_2) $A \cap B = x\ (T_A \wedge T_B, I_A \wedge I_B, F_A \vee F_B)$
(I_3) $A \cap B = x\ (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)$

$A \cup B$ *may be defined as:*

(U_1) $A \cup B = x\ (T_A \vee T_B, I_A \vee I_B, F_A \wedge F_B)$
(U_2) $A \cup B = x\ (T_A \vee T_B, I_A \wedge I_B, F_A \wedge F_B)$

Definition 5. *In Reference [11], let* $\{A_j | j \in J\}$ *be an arbitrary family of NSs in X, then:*

(1) $\cap A_j$ *may be defined as:*

(i) $\cap A_j = x\ (\wedge, \wedge, \vee)$
(ii) $\cap A_j = x\ (\wedge, \vee, \vee)$

(2) $\cup A_j$ *may be defined as:*

(i) $\cup A_j = x\ (\vee, \vee, \wedge)$
(ii) $\cup A_j = x\ (\vee, \wedge, \wedge)$

Definition 6. *In Reference [11], a neutrosophic topology on a non-empty set X is a family τ of NSs in X satisfying the following properties:*

(1) 0_N *and* $1_N \epsilon \tau_\tau$;
(2) $G_1 \cap G_2 \in \tau$, *for any* $G_1, G_2 \in \tau$;
(3) $\cup G_j \in \tau$ *or any subfamily* $\{G_j\}_{j \in J}$ *of* τ.

In this case, the pair **(X,τ)** is called a neutrosophic topological space.

3. Results

Proposition 1. *The set of paraconsistent NSs with the definitions above is not a bounded lattice.*

Proof.

(1) It is necessary to omit a definition of ∩, because we will need ∩ of paraconsistent NSs to be paraconsistent. Indeed, let **A** = x (1/2, 1/2, 1/2) and B = x (1/2, 1/3, 1/3) (both are paraconsistent NSs), but 1/4 + 1/6 + 1/6 is not > 1. Then, the case with product ((I_1), in Definition 4) must be deleted for paraconsistent NSs.
(2) The definitions of 0_N and 1_N also have problems for paraconsistent NSs:

    (a) Only (0_2) and (1_2), (1_3), (1_4) are paraconsistent;
    (b) If we want all NSs: $0_N \cup 0_N$, $0_N \cup 1_N$, $1_N \cup 1_N$, $0_N \cap 0_N$, and $0_N \cap 1_N$ to be paraconsistent NSs, it is necessary to delete 1_2 in Definition 3, because with this definition,

$0_N \cap 1_N$ is equal either to x (0, 0, 1) which is not paraconsistent, or to x (0,1,1) = 0_N.

The other cases have no problems: $0_N \cup 0_N$ = x (0, 1, 1) = 0_N,

$0_N \cup 1_N$ is equal either to x (1,0,1), or to x (1,1,0), or x (1,1,1), i.e equal to 1_N,
$1_N \cup 1_N$ is equal either to x (1,0,1), or to x (1,1,0), or x (1,1,1), i.e equal to 1_N,
$0_N \cap 0_N$ = x (0,1,1) = 0_N,
$1_N \cap 1_N$ is equal either to x (1,0,1), or to x (1,1,0), or x (1,1,1), i.e equal to 1_N.

Then, after these changes in Definitions 3 and 4, Definition 6 is suitable for Smarandache's paraconsistent NSs, and one can work on paraconsistent neutrosophic topological spaces. □

Definition 7. *Let X be a non-empty set. A family τ of neutrosophic paraconsistent sets in X will be called a paraconsistent neutrosophic topology if:*

(1) 0_N *= x (0,1,1), and* 1_N *= x (1,1,0) or x (1,1,1), are in* τ;
(2) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$ *(where ∩ is defined by (*I_2*) or (*I_3*));*
(3) $\cup G_j \in \tau$ *for any subfamily* $\{G_j\}_{j \in J}$ *of* τ *(where ∪ is defined by Definition 5).*

In this case, the pair (X,τ) is called a paraconsistent neutrosophic topological space.

Remark. *The above notion of paraconsistent neutrosophic topology generalizes Çoker's intuitionistic fuzzy topology when all sets are paraconsistent.*

4. Discussion

Definition 7 is suitable for the work on paraconsistent neutrosophic topological spaces. In fact:

Proposition 2. *The set of paraconsistent NSs with the following definitions,*

(a) 0_N *= x (0,1,1), and* 1_N *= x (1,1,0) or x (1,1,1)*
(b) *∩ defined by (*I_2*) or (*I_3*)*
(c) *∪ defined by Definition 5 is a bounded lattice.*

Proof. Obvious from proof of Proposition 1. □

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