Limitation on Effective Degree of Quantum Parallelism

Masataka Ohta
Email: mohta@necom830.hpcl.titech.ac.jp
Department of Computer Science, School of Information Science and Engineering
Tokyo Institute of Technology
2-12-1-W8-54, O-okayama, Meguro, Tokyo 1528552, JAPAN

Received on Nov. 1, 2017

PACS indexing codes: 03.67.Lx, 03.67.Pp

Abstract: Unlimited quantum parallelism is the key to make, in theory, quantum computers more powerful than classical ones. However, in practice, noisy quantum devices have limited quantum parallelism, which is directly derived from limited channel capacity of noisy quantum channels. As a result, in practice, quantum computers are only as powerful as classical ones.

I. INTRODUCION

In theory, quantum computers are believed to be more powerful than classical ones. For example, factorization of an integer is solved in polynomial (w.r.t. the number of digits of the integer) time by quantum computers [1], though, by classical computers, the best known algorism requires exponential time. It is essential that [1] use quantum parallelism. That is, a single computation step of a quantum device has quantum parallelism that exponentially many entangled terms are computed in parallel.

In practice, however, quantum devices suffer from noise. Just as noise of a classical channel limits capacity of the channel [2], noise of a quantum device may destroy subtle coherence and limit effective degree of quantum parallelism, precise definition of which on the simplest quantum device is given in the next section. If so, for medium sized or larger problems, there is no room for exponential parallelism in polynomial duration and, in practice, quantum computers are only as powerful as classical ones.

II. EFFECTIVE DEGREE OF QUANTUM PARALLELISM OF NOISY QUATUM DEVICES

Just as classical channel capacity depends on not only the number of bits sent in a unit time but also error probability of the bits, degree of quantum parallelism should depend on not only the number of entangled terms computed in parallel but also error probability of the computation results of the terms. In this paper, such degree is called EDQP (Effective Degree of Quantum Parallelism).

First, it is shown that EDQP of the simplest noisy quantum device is limited. The simplest noisy quantum device does nothing and input quantum state is output as is, except that some noise is added. That is, the simplest noisy quantum device is a noisy quantum channel. EDQP of a noisy quantum channel is the number of bits correctly carried over the channel during a quantum computation step, which is capacity of the channel multiplied by duration of the quantum computation step. According to [3], a quantum version of [2], channel capacity of a noisy quantum channel is limited. Thus, EDQP of a noisy quantum channel is limited. As is stated in [3] that "no coherent quantum information survives the transmission through the channel" for transmission beyond channel capacity, coherence beyond channel capacity is destroyed that no further quantum operations can restore lost information to improve quantum parallelism.

As for EDQP of noisy quantum devices in general, for the purpose of this paper, it is enough to point out that the noisy quantum devices are connected through noisy quantum channels. EDQP of a noisy quantum devices is limited by EDQP of its noisy output channel or channels and is limited. If noise of some channel has correlation to noise or signal of other channels, it may be possible to cancel noise through some operation to extract more parallelism. But, for the purpose of this paper, it is enough to consider noise component without such correlation only.

III. WHAT'S WRONG WITH QUANTUM THRESHOLD THEOREM?

Nothing. However, its applicability to quantum parallelism was not properly recognized. Just as information can be sent over a classical channel with exponentially small error probability if noise level of the channel is below certain threshold [2], quantum threshold theorem [4] assures that error probability of quantum calculation can be exponentially small if noise level of quantum devices is below certain threshold. The problem is that both threshold values depend on degree of parallelism. Just as the threshold of [2] must be lowered as the number of bits in a symbol increases, the threshold of [4] must be lowered as the number of entangled terms computed in parallel increases. Quantum threshold theorem works only if coherence of almost all the entangled terms is not destroyed by noise over almost all the elementally physical quantum devices, which, in practice, requires unlimited EDQP for all the elementally physical quantum devices, which is practically impossible.

IV. CONCLUSIONS

As noise of practical quantum devices limits degree of quantum parallelism of the device and destroy coherence for further parallelism, it is practically impossible to make quantum computers more powerful than classical ones. Computer scientists, including the Author, must concentrate on making classical computers faster, less power consuming with wider von Neumann bottlenecks.

REFERENCES

[1] P. W. Shor, in Proceedings of the 35th Annual Symposium on Foundations of Computer Science, 1994, p. 124.

[2] C. E. Shannon, in Proceedings of IRE, 37 (1), 1949, p. 10–21.

[3] S. Lloyd, Phys. Rev. A55 (1997) 1613, arXiv:quant-ph/9604015.

[4] D. Aharonov, M. Ben-Or, arXiv:quant-ph/9906129.