The Stellar Black Hole

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Abstract

A black hole model is proposed in which a neutron star is surrounded by a neutral gas of electrons and positrons. The gas is in a completely degenerate quantum state and does not radiate. The pressure and density in the gas are found to be much less than those in the neutron star. The radius of the black hole is far greater than the Schwarzschild radius.

1. Introduction

In the following model, a neutron star forms the core of a stellar black hole. The collapse of a massive star creates the neutron star. Electron-positron pairs, which are produced during the collapse, cannot escape the intense gravitational field of the neutron star. They settle into a degenerate quantum state surrounding the core.

The Fermi energy for a completely degenerate ($T = 0$) gas of $N/2$ electrons is given by [1, 2]

$$\epsilon_F = \Big(\frac{3\pi^2}{2}\Big)^{2/3} \frac{\hbar^2}{2m} n^{2/3} \tag{1}$$

where n is the total number density of leptons. If the gas is non-relativistic, the Fermi level must be less than .5 MeV. Formula (1) then limits the number density to about 3×10^{30} cm^{-3}, with the corresponding mass density $\rho = mn = 3 \times 10^3$ g cm^{-3}. Therefore, the density of the lepton gas will be much less than that of the core ($\rho_c = 5 \times 10^{14}$ g cm^{-3}). Clearly, the gas envelope will have little direct impact on the core.

2. The black hole model

Under the assumption of spherical symmetry, the gravitational field equation and the equation of hydrostatic equilibrium are given by

$$\frac{1}{r^2}\frac{d}{dr}\Big(r^2\frac{d\psi}{dr}\Big) = 4\pi G\rho \tag{2}$$

and

$$\frac{dP}{dr} = -\rho\,\frac{d\psi}{dr} \tag{3}$$

If the lepton gas is assumed to be incompressible, then the density is constant and equation (2) gives

$$\frac{d\psi}{dr} = \frac{4\pi G}{3}\rho r + \frac{K}{r^2} \tag{4}$$

At the surface of the neutron star, $d\psi/dr = GM_c/R_c^2$, where M_c and R_c are the mass and radius of the core. Substitute this into (4) to find

$$K = GM_c\Big(1 - \frac{\rho}{\rho_c}\Big) \tag{5}$$

Since $\rho << \rho_c$, the constant of integration $K = GM_c$. Equations (3) and (4) then combine to give

$$P = \frac{GM_c}{R_c}\rho\,\Big(\frac{R_c}{r} - \frac{\rho}{2\rho_c}\frac{r^2}{R_c^2}\Big) + P' \tag{6}$$

At the outer surface of the gas, $r = R$ and $P = 0$, so that the integration constant is

$$P' = -\frac{GM_c}{R_c}\rho\,\Big(\frac{R_c}{R} - \frac{\rho}{2\rho_c}\frac{R^2}{R_c^2}\Big) \tag{7}$$

This yields the pressure formula for the gas

$$P = \frac{GM_c}{R_c}\rho\,\Big\{R_c\Big(\frac{1}{r} - \frac{1}{R}\Big) + \frac{\rho}{2\rho_c}\frac{1}{R_c^2}\Big(R^2 - r^2\Big)\Big\} \qquad (R_c < r < R) \tag{8}$$

The maximum pressure occurs at the surface of the core ($r = R_c$)

$$P_{\max} = \frac{GM_c}{R_c}\rho\,\Big\{\Big(1 - \frac{R_c}{R}\Big) + \frac{\rho}{2\rho_c}\Big(\frac{R^2}{R_c^2} - 1\Big)\Big\} \tag{9}$$

From the previous discussion regarding densities, it is clear that the radius of the gas envelope will be much greater than the core radius $R >> R_c$, so that (9) reduces to

$$P_{\max} = \frac{GM_c}{R_c}\rho\,\Big(1 + \frac{\rho}{2\rho_c}\frac{R^2}{R_c^2}\Big) \tag{10}$$

Since $\rho << \rho_c$, it follows that $\rho R^2 << 2\rho_c R_c^2$ for any reasonable black hole mass. This yields the formula

$$P_{\max} = \frac{GM_c}{R_c}\,\rho \tag{11}$$

which will be used to determine the physical conditions near the core.

The pressure in the electron-positron gas is given by

$$P_F = \Big(\frac{3\pi^2}{2}\Big)^{2/3}\,\frac{\hbar^2}{5m}\,n^{5/3} \tag{12}$$

Equate this to (11) and set $\rho = mn$ to obtain

$$\left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\hbar^2}{5m}\, n^{2/3} = \frac{GM_c m}{R_c} \tag{13}$$

or

$$n = 5^{1/2}\, \frac{10\, m^3}{3\pi^2\hbar^3} \left(\frac{GM_c}{R_c}\right)^{3/2} \tag{14}$$

A neutron star core, with $M_c = 1.5 M_\odot$ and $R_c = 1.1 \times 10^6$ cm, yields the number density for the lepton gas

$$n = 1.2 \times 10^{30}\ \mathrm{cm}^{-3} \tag{15}$$

and the mass density

$$\rho = mn = 1.1 \times 10^3\ \mathrm{g\ cm}^{-3} \tag{16}$$

Typical black hole dimensions are as follows: $M = 5M_\odot$, $R = 1.1 \times 10^{10}$ cm; $M = 10M_\odot$, $R = 1.5 \times 10^{10}$ cm; $M = 20M_\odot$, $R = 2.0 \times 10^{10}$ cm. They are nearly the size of a normal star.

3. Remarks

Formula (14) shows that the density of the gas is determined by the intense gravitational field of the neutron star. It is little affected by the mass of the lepton gas itself. This results in a nearly common gas density for all stellar black holes. According to (11), the gas pressure near the core is 2.0×10^{22} Pa. As expected, this is much less than the pressure within the neutron star (10^{34} Pa).

The very thick cloud of leptons is a conductor of electricity. It would shield the magnetic field of the neutron star. Moreover, the cloud would absorb much of the angular momentum of the parent star, thereby reducing the neutron star's rotation rate. In a more realistic treatment, the pressure and density would be related everywhere as in formula (12), so that they both decrease with elevation. The resulting decrease in Fermi energy enables the upper layers of gas to interact with radiation coming from the core. During a collision with another black hole or neutron star, the lepton cloud could absorb a great deal of the electromagnetic radiation. Gravitational radiation would pass through the cloud unhindered.

References

[1] L. Landau and E. Lifschitz, *Statistical Physics* Part 1
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[2] R. Pathria and P. Beale, *Statistical Mechanics* (Elsevier, 3rd ed., 2011)
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