Journal of Experimental & Theoretical Artificial Intelligence

ISSN: 0952-813X (Print) 1362-3079 (Online) Journal homepage: http://www.tandfonline.com/loi/teta20

Projection and bidirectional projection measures of single-valued neutrosophic sets and their decision-making method for mechanical design schemes

Jun Ye

To cite this article: Jun Ye (2017) Projection and bidirectional projection measures of single-valued neutrosophic sets and their decision-making method for mechanical design schemes, Journal of Experimental & Theoretical Artificial Intelligence, 29:4, 731-740, DOI: 10.1080/0952813X.2016.1259263

To link to this article: http://dx.doi.org/10.1080/0952813X.2016.1259263

Published online: 23 Nov 2016.

JOURNAL OF EXPERIMENTAL & THEORETICAL ARTIFICIAL INTELLIGENCE, 2017
VOL. 29, NO. 4, 731–740
https://doi.org/10.1080/0952813X.2016.1259263

Taylor & Francis
Taylor & Francis Group

Projection and bidirectional projection measures of single-valued neutrosophic sets and their decision-making method for mechanical design schemes

Jun Ye

Department of Electrical and Information Engineering, Shaoxing University, Shaoxing, P.R. China

ABSTRACT

Projection measure is one of important tools for handling decision-making problems. First, the paper proposes projection and bidirectional projection measures between single-valued neutrosophic sets, and then the comparison of numerical examples shows that the bidirectional projection measure is superior to the general projection measure in measuring closeness degree between two vectors. Next, we develop their decision-making method for selecting mechanical design schemes under a single-valued neutrosophic environment. Through the projection measure or bidirectional projection measure between each alternative and the ideal alternative with single-valued neutrosophic information, all the alternatives can be ranked and the best one can be selected as well. Finally, the proposed decision-making method is applied to the selection of design schemes of punching machine and its effectiveness and advantages are demonstrated by comparison with relative methods.

ARTICLE HISTORY

Received 3 November 2015
Accepted 5 September 2016

KEYWORDS

Projection measure; bidirectional projection measure; decision-making; mechanical design scheme; single-valued neutrosophic set

1. Introduction

Projection measure is a suitable tool for dealing with decision-making problems because it can consider not only the distance but also the included angle between objects evaluated (Xu, 2005; Xu & Da, 2004; Yue, 2012). Therefore, some researchers have successfully applied projection measures to decision-making. For example, Xu and Hu (2010) presented the projection model-based approaches for multiple attribute decision-making problems with intuitionistic and interval-valued intuitionistic fuzzy information. Xu and Cai (2012) proposed projection model-based approaches for intuitionistic fuzzy multiple attribute decision-making problems. Yue (2013) and Zeng, Balezentis, Chen, and Luo (2013) developed projection methods for multiple attribute group decision-making problems with intuitionistic and interval-valued intuitionistic fuzzy information. Yue and Jia (2015) put forward a projection measure for handling a group decision-making problem with hybrid intuitionistic fuzzy information.

As the generalisation of an intuitionistic fuzzy set (IFS) (Atanassov, 1986) and an interval-valued intuitionistic fuzzy (IVIFS) (Atanassov & Gargov, 1989), single-valued neutrosophic sets (SVNSs) (Wang, Smarandache, Zhang, & Sunderraman, 2010) and interval neutrosophic sets (INSs) (Wang, Smarandache, Zhang, & Sunderraman, 2005) are the subclasses of the neutrosophic sets introduced by Smarandache (1998) and are very suitable for describing and handling indeterminate and inconsistent information, which IFSs and IVIFSs cannot describe and deal with, in science and engineering areas. Recently, many

CONTACT Jun Ye yehjun@aliyun.com

researchers have applied SVNSs and INSs to decision-making problems. Some methods have been developed to solve the multiple attribute decision-making problems with SVNS and INS information. For example, the correlation coefficients of SVNSs were used for multiple attribute decision-making (Ye, 2013). A TOPSIS method was extended to interval neutrosophic multiple attribute decision-making problems to rank alternatives (Chi & Liu, 2013). Various similarity measures of SVNSs and INSs were presented and applied to multicriteria (group) decision-making (Ye, 2014a, Ye, 2014b, Ye, 2014c). Single-valued and interval neutrosophic cross-entropy measures were developed for multiple attribute decision-making problems (Tian, Zhang, Wang, Wang, & Chen, 2016; Ye, 2014d). Some neutrosophic number aggregation operators were proposed and applied to multiple attribute decision-making problems (Liu, Chu, Li, & Chen, 2014; Liu & Wang, 2014). Outranking approaches were applied to multicriteria decision-making problems with simplified neutrosophic sets (including SVNSs and INSs) (Peng, Wang, Zhang, & Chen, 2014; Zhang, Wang, & Chen, 2016). Then, a multicriteria group decision-making method was introduced under a simplified neutrosophic environment (Peng, Wang, Wang, Zhang, & Chen, 2016). A multiple attribute decision-making method was proposed based on the possibility degree ranking method and ordered weighted aggregation operators of interval neutrosophic numbers (Ye, 2015). Zhang, Ji, Wang, and Chen (2015) proposed an improved weighted correlation coefficient based on integrated weight for INSs and applied it to multicriteria decision-making problems with interval neutrosophic information. Zavadskas, Baušys, and Lazauskas (2015) introduced the sustainable assessment of alternative sites for the construction of a waste incineration plant by the weighted aggregated sum product assessment method with SVNSs. Bausys, Zavadskas, and Kaklauskas (2015) presented a multicriteria decision-making method with SVNSs based on the complex proportional assessment method. A ranking method of single-valued neutrosophic numbers was applied to multiple attribute decision-making problems (Deli & Şubaş, in press). Furthermore, neutrosophic soft sets (Deli, in press), neutrosophic soft multi-sets (Deli, Broumi, & Ali, 2014), and power aggregation operators of multi-valued neutrosophic sets (Peng, Wang, Wu, Wang, & Chen, 2015) were applied to decision-making problems.

However, the existing projection methods cannot deal with decision-making problems with interval neutrosophic information and single-valued neutrosophic information. Furthermore, SVNSs and INSs are scarcely applied in mechanical engineering fields (Ye, in press). Therefore, it is essential to do the research on neutrosophic projection measures and their decision-making method of the mechanical design schemes. Therefore, this paper firstly presents a general projection between SVNSs as a generalisation of the projection measure of IFSs, and then proposes a bidirectional projection measure as an improvement of the general projection measure of SVNSs to overcome the drawback of the general projection in some case. Furthermore, their decision-making method is developed for selecting problems of mechanical design schemes (alternatives) under a single-valued neutrosophic environment.

The rest of the paper is organised as follows. Section 2 briefly describes some basic concepts of neutrosophic sets and SVNSs. Section 3 proposes general projection and bidirectional projection measures between SVNSs and gives their comparison of numerical examples. In Section 4, we develop the general projection and bidirectional projection measures-based decision-making method for selecting mechanical design schemes under a single-valued neutrosophic environment. In Section 5, the proposed decision-making method is applied to the selection of the design schemes of punching machine and its effectiveness and advantages are demonstrated by comparison with relative methods. Finally, Section 6 contains conclusions and future work.

2. Basic concepts of neutrosophic sets and SVNSs

The neutrosophic set proposed by Smarandache (1998) is a part of neutrosophy and extends the concept of fuzzy sets, interval valued fuzzy set, IFS, and IVIFS from a philosophical point of view. Smarandache (1998) originally gave the definition of a neutrosophic set.

Definition 1. (Smarandache, 1998). Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set N in X is characterised by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. The functions $T_N(x)$, $I_N(x)$ and $F_N(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_N(x)$: $X \to\ ]^-0, 1^+[$, $I_N(x)$: $X \to\ ]^-0, 1^+[$, and $F_N(x)$: $X \to\ ]^-0, 1^+[$. Hence, the sum of $T_N(x)$, $I_N(x)$ and $F_N(x)$ is no restriction and $^-0 \le \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \le 3^+$.

However, it is difficult to directly apply the neutrosophic set in real science and engineering fields (Wang et al., 2010) due to the nonstandard interval $]^-0, 1^+[$. Hence, Wang et al. (2010) introduced a SVNS in the real standard interval [0, 1] as a subclass of a neutrosophic set to suit its engineering applications under an indeterminate and inconsistent environment and gave the definition of a SVNS.

Definition 2. (Wang et al., 2010). Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS N in X is characterised by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then, a SVNS N can be expressed as $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle | x \in X\}$, where the sum of $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$ is $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$ for each point x in X.

For convenience, a basic element $\langle x, T_N(x), I_N(x), F_N(x)\rangle$ in $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle | x \in X\}$ is denoted by $e = (T, I, F)$ for short, which is called a single-valued neutrosophic value (SVNV).

Assume that $e_1 = (T_1, I_1, F_1)$ and $e_2 = (T_2, I_2, F_2)$ are two SVNVs. Then, the inclusion, equality, complement, union and intersection for SVNVs e_1 and e_2 are defined, respectively, as follows (Wang et al., 2010):

(1) Inclusion: $e_1 \subseteq e_2$ if and only if $T_1 \le T_2$, $I_1 \ge I_2$, $F_1 \ge F_2$;
(2) Equality: $e_1 = e_2$ if and only if $e_1 \subseteq e_2$ and $e_2 \subseteq e_1$;
(3) Complement: $e_1^c = (F_1, 1 - I_1, T_1)$;
(4) Union: $e_1 \cup e_2 = (T_1 \vee T_2, I_1 \wedge I_2, F_1 \wedge F_2)$;
(5) Intersection: $e_1 \cap e_2 = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2)$.

3. Projection and bidirectional projection measures of SVNSs

This section proposes a general projection measure and a bidirectional projection measure for SVNSs.

Based on the projection measure of IFSs (Xu & Hu, 2010) and the cosine measure of SVNSs (Ye, 2014c), we firstly give the definitions of a cosine measure and a general projection measure between SVNSs.

Definition 3. Let $N_1 = \{e_{11}, e_{12}, \dots, e_{1n}\}$ and $N_2 = \{e_{21}, e_{22}, \dots, e_{2n}\}$ be two SVNSs, where $e_{1j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{2j} = (T_{2j}, I_{2j}, F_{2j})$ ($j = 1, 2, \dots, n$) are the j-th SVNVs of N_1 and N_2 respectively. Then

$$N_1 \cdot N_2 = \sum_{j=1}^{n} \left(T_{1j}T_{2j} + I_{1j}I_{2j} + F_{1j}F_{2j}\right) \tag{1}$$

is called the inner product between SVNSs N_1 and N_2,

$$|N_1| = \sqrt{\sum_{j=1}^{n} \left(T_{1j}^2 + I_{1j}^2 + F_{1j}^2\right)}, \tag{2}$$

$$|N_2| = \sqrt{\sum_{j=1}^{n} \left(T_{2j}^2 + I_{2j}^2 + F_{2j}^2\right)} \tag{3}$$

are called the modules of N_1 and N_2, respectively, and then

$$\mathrm{Cos}(N_1, N_2) = \frac{N_1 \cdot N_2}{|N_1||N_2|} \tag{4}$$

is called the cosine of the included angle between N_1 and N_2.

Definition 4. Let $N_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$ and $N_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$ be two SVNSs, where $e_{1j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{2j} = (T_{2j}, I_{2j}, F_{2j})$ ($j = 1, 2, \ldots, n$) are the j-th SVNVs of N_1 and N_2 respectively. Then

$$\mathrm{Proj}_{N_2}(N_1) = |N_1|\mathrm{Cos}(N_1, N_2) = \frac{N_1 \cdot N_2}{|N_2|} = \frac{\sum_{j=1}^{n}\left(T_{1j}T_{2j} + I_{1j}I_{2j} + F_{1j}F_{2j}\right)}{\sqrt{\sum_{j=1}^{n}\left(T_{2j}^2 + I_{2j}^2 + F_{2j}^2\right)}} \tag{5}$$

is called the projection of N_1 on N_2.

The projection measure $\mathrm{Proj}_{N_2}(N_1)$ can include both the distance and the included angle between N_1 and N_2. In general, the larger the value of $\mathrm{Proj}_{N_2}(N_1)$ is, the closer N_1 is to N_2.

Based on the extension of the above projection measure of SVNSs, we further propose a bidirectional projection measure between SVNSs below.

Definition 5. Let $N_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$ and $N_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$ be two SVNSs, where $e_{1j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{2j} = (T_{2j}, I_{2j}, F_{2j})$ ($j = 1, 2, \ldots, n$) are the j-th SVNVs of N_1 and N_2 respectively. Then

$$\mathrm{BProj}(N_1, N_2) = \frac{1}{1 + \left|\frac{N_1 \cdot N_2}{|N_1|} - \frac{N_1 \cdot N_2}{|N_2|}\right|} = \frac{|N_1||N_2|}{|N_1||N_2| + \big||N_1| - |N_2|\big|N_1 \cdot N_2} \tag{6}$$

is called the bidirectional projection between N_1 and N_2, where $|N_1| = \sqrt{\sum_{j=1}^{n}(T_{1j}^2 + I_{1j}^2 + F_{1j}^2)}$ and $|N_2| = \sqrt{\sum_{j=1}^{n}(T_{2j}^2 + I_{2j}^2 + F_{2j}^2)}$ are the modules of N_1 and N_2, respectively, and $N_1 \cdot N_2 = \sum_{j=1}^{n}\left(T_{1j}T_{2j} + I_{1j}I_{2j} + F_{1j}F_{2j}\right)$ is the inner product between N_1 and N_2.

The bidirectional projection measure can include not only both the distance and the included angle between N_1 and N_2 but also the bidirectional projection magnitudes between N_1 and N_2. Obviously, the closer the value of BProj(N_1, N_2) is to 1, the closer the SVNS N_1 is to N_2. The bidirectional projection measure is a normalised measure, i.e., $0 \le \mathrm{BProj}(N_1, N_2) \le 1$.

For the comparison between the general projection measure and the bidirectional projection measure, we consider an example below to show their measuring performance.

Example 1. Let us consider the following two cases:

Case 1: Let $N_1 = \{(0, .5, .5), (.2, 0, .8)\}$ and $N_2 = N_3 = \{(.2, .4, .4), (.3, .3, .4)\}$ be three SVNSs.

According to Equation (5), since $N_1 \cdot N_2 = .78$ and $|N_2| = \sqrt{.7}$, we have that $\mathrm{Proj}_{N_2}(N_1) = .78/\sqrt{.7} = .9323$ and $\mathrm{Proj}_{N_2}(N_3) = .7/\sqrt{.7} = .8367$. In this case, since $\mathrm{Proj}_{N_2}(N_1)$ is larger than $\mathrm{Proj}_{N_2}(N_3)$, N_1 is much closer to N_2 than N_3. In fact, since $N_3 = N_2$, N_3 should be much closer to N_2 than N_1, and then $\mathrm{Proj}_{N_2}(N_3)$ should be equal to 1. Obviously, the closeness degree between two vectors indicated by the projection measure is not reasonable in this case.

According to Equation (6), since $N_1 \cdot N_2 = .78$, $|N_1| = \sqrt{1.18}$ and $|N_2| = \sqrt{.7}$, we have that BProj(N_1, N_2) = $1/(1+|.78/\sqrt{1.18} - .78/\sqrt{.7}|) = .8236$ and BProj(N_3, N_2)$=1/(1 + .7/\sqrt{.7} - .7/\sqrt{.7}) = 1$. Since BProj($N_3$, N_2) > BProj(N_1, N_2), N_3 is much closer to N_2 than N_1 and BProj(N_3, N_2) = 1 if and only if $N_2 = N_3$. So, the bidirectional projection measure is reasonable and effective.

Case 2: Let $N_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$, $N_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$, and $N_3 = \{e_{31}, e_{32}, \ldots, e_{3n}\}$ be three SVNSs. If $e_{1j} = e_{2j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{3j} = (2T_{1j}, 2I_{1j}, 2F_{1j})$ ($j = 1, 2, \ldots, n$) are the j-th SVNVs in N_1, N_2 and N_3 respectively, then their measures are as follows:

According to Equation (5), there are $P_{N_2}(N_1) = |N_1| \le P_{N_2}(N_3) = 2|N_1|$. In this case, N_3 is much closer to N_2 than N_1. In fact, since $N_1 = N_2$, N_1 should be much closer to N_2 than N_3, and then $P_{N_2}(N_1)$ should be equal to 1. Therefore, the results are not reasonable in this case.

According to Equation (6), we have that BProj(N_1, N_2) = $1/(1+|N_1| - |N_1|) = 1$ and BProj(N_3, N_2) = $1/(1 + 2|N_1| - |N_1|) = 1/(1 + |N_1|)$. In this case, N_1 is much closer to N_2 than N_3. Since $N_1 = N_2$, BProj(N_1, N_2) is equal to 1. Therefore, the results are reasonable and effective in this case.

From the example, we can see that the general projection measure is not always reasonable in some cases, while the bidirectional projection measure is reasonable and effective. Therefore, the proposed bidirectional projection measure is superior to the general projection measure and more suitable for pattern recognition, fault diagnosis, and decision-making.

If we consider the importance of each element in SVNSs, the weight of each element w_j (j = 1, 2, …, n) can be introduced with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Thus, we introduce the following definitions:

Definition 6. Let $N_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$ and $N_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$ be two SVNSs, where $e_{1j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{2j} = (T_{2j}, I_{2j}, F_{2j})$ (j = 1, 2, … , n) are the j-th SVNVs of N_1 and N_2 respectively. Then

$$(N_1 \cdot N_2)_w = \sum_{j=1}^{n} w_j^2 \left(T_{1j}T_{2j} + I_{1j}I_{2j} + F_{1j}F_{2j}\right) \tag{7}$$

is called the weighted inner product between SVNSs N_1 and N_2,

$$|N_1|_w = \sqrt{\sum_{j=1}^{n} w_j^2 \left(T_{1j}^2 + I_{1j}^2 + F_{1j}^2\right)}, \tag{8}$$

$$|N_2|_w = \sqrt{\sum_{j=1}^{n} w_j^2 \left(T_{2j}^2 + I_{2j}^2 + F_{2j}^2\right)} \tag{9}$$

are called the weighted modules of N_1 and N_2 respectively, and then

$$\mathrm{Cos}_w(N_1, N_2) = \frac{(N_1 \cdot N_2)_w}{|N_1|_w |N_2|_w} \tag{10}$$

is called the weighted cosine measure between N_1 and N_2.

Definition 7. Let $N_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$ and $N_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$ be two SVNSs, where $e_{1j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{2j} = (T_{2j}, I_{2j}, F_{2j})$ (j = 1, 2, … , n) are the j-th SVNVs of N_1 and N_2 respectively. Then

$$\mathrm{Proj}\, w_{N_2}(N_1) = |N_1|_w \mathrm{Cos}_w(N_1, N_2) = \frac{(N_1 \cdot N_2)_w}{|N_2|_w} = \frac{\sum_{j=1}^{n} w_j^2 \left(T_{1j}T_{2j} + I_{1j}I_{2j} + F_{1j}F_{2j}\right)}{\sqrt{\sum_{j=1}^{n} w_j^2 \left(T_{2j}^2 + I_{2j}^2 + F_{2j}^2\right)}} \tag{11}$$

is called the weighted projection of N_1 on N_2.

Definition 8. Let $N_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$ and $N_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$ be two SVNSs, where $e_{1j} = (T_{1j}, I_{1j}, F_{1j})$ and $e_{2j} = (T_{2j}, I_{2j}, F_{2j})$ (j = 1, 2, … , n) are the j-th SVNVs of N_1 and N_2 respectively. Then

$$\mathrm{BProj}_w(N_1, N_2) = \frac{1}{1 + \left| \frac{(N_1 \cdot N_2)_w}{|N_1|_w} - \frac{(N_1 \cdot N_2)_w}{|N_2|_w} \right|} = \frac{|N_1|_w |N_2|_w}{|N_1|_w |N_2|_w + \left| |N_1|_w - |N_2|_w \right| (N_1 \cdot N_2)_w} \tag{12}$$

is called the weighted bidirectional projection between N_1 and N_2, where $|N_1|_w = \sqrt{\sum_{j=1}^{n} w_j^2 (T_{1j}^2 + I_{1j}^2 + F_{1j}^2)}$ and $|N_2|_w = \sqrt{\sum_{j=1}^{n} w_j^2 (T_{2j}^2 + I_{2j}^2 + F_{2j}^2)}$ are the weighted modules of N_1 and N_2 respectively, and $(N_1 \cdot N_2)_w = \sum_{j=1}^{n} w_j^2 (T_{1j}T_{2j} + I_{1j}I_{2j} + F_{1j}F_{2j})$ is the weighted inner product between N_1 and N_2.

4. Decision-making method of mechanical design schemes

In this section, the projection measure and the bidirectional projection measure are used for the multiple attribute decision-making problems of mechanical design schemes with single-valued neutrosophic information.

In the conceptual design stage, designers usually propose various mechanical design schemes (alternatives) according to the function requirements of users and designers. The primal mechanical design schemes can be structured as a set of m alternatives $S = \{S_1, S_2, \dots, S_m\}$, which must satisfy the requirements of a set of attributes (criteria) $R = \{R_1, R_2, \dots, R_n\}$ by their suitability assessments for fuzzy concept "excellence". The weight w_j of the attribute R_j ($j = 1, 2, \dots, n$) is entered by the decision-maker with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. In this case, the characteristic of the alternative S_i ($i = 1, 2, \dots, m$) with respect to each attribute R_j ($j = 1, 2, \dots, n$) is expressed by a SVNS form:

$$S_i = \{\langle R_j, T_{S_i}(R_j), I_{S_i}(R_j), F_{S_i}(R_j)\rangle | R_j \in R\},$$

where $0 \le T_{S_i}(R_j) + I_{S_i}(R_j) + F_{S_i}(R_j) \le 3, T_{S_i}(R_j), I_{S_i}(R_j), F_{S_i}(R_j) \ge 0$ for $j = 1, 2, \dots, n$ and $i = 1, 2, \dots, m$. For convenience, a basic element in a SVNS S_i is denoted by a SVNV $e_{ij} = (T_{ij}, I_{ij}, F_{ij})$ for short. Here, the SVNV is usually obtained from the suitability evaluation to which an alternative S_i satisfies or does not satisfy an attribute R_j by means of a score law or appropriate membership functions in practical applications. Therefore, we can establish an single-valued neutrosophic decision matrix $D = (e_{ij})_{m\times n}$.

In multiple attribute decision-making environments, the concept of an ideal alternative has been used to help identify the best alternative in the decision set (Ye, 2014c). Hence, we define the ideal alternative (ideal solution) denoted by the following SVNS:

$$S^* = \{\langle R_j, e_j^*\rangle | R_j \in R\},$$

where an ideal SVNV is determined by $e_j^* = (T_j^*, I_j^*, F_j^*) = (\max_i(T_{ij}), \min_i(I_{ij}), \min_i(F_{ij}))$ for $j = 1, 2, \dots, n$.

Then, by applying Equation (11) or Equation (12) the weighted projection measure or weighted bidirectional projection measure between an alternative S_i and the ideal alternative S^* is given by

$$\mathrm{Proj}w_{S^*}(S_i) = \frac{(S_i \cdot S^*)_w}{|S^*|_w}, \tag{13}$$

$$\text{or } \mathrm{BProj}_w(S_i, S^*) = \frac{1}{1 + \left|\frac{(S_i\cdot S^*)_w}{|S_i|_w} - \frac{(S_i\cdot S^*)_w}{|S^*|_w}\right|} = \frac{|S_i|_w|S^*|_w}{|S_i|_w|S^*|_w + \left||S_i|_w - |S^*|_w\right|(S_i \cdot S^*)_w}, \tag{14}$$

where $|S_i|_w = \sqrt{\sum_{j=1}^{n} w_j^2(T_{ij}^2 + I_{ij}^2 + F_{ij}^2)}$ and $|S^*|_w = \sqrt{\sum_{j=1}^{n} w_j^2[(T_j^*)^2 + (I_j^*)^2 + (F_j^*)^2]}$, and $(S_i \cdot S^*)_w = \sum_{j=1}^{n} w_j^2(T_{ij}T_j^* + I_{ij}I_j^* + F_{ij}F_j^*)$.

The projection measure or the bidirectional projection measure provides the global evaluation for each alternative regarding all attributes. The bigger the measure value of $\mathrm{Proj}w_{S^*}(S_i)$ or $\mathrm{Bproj}_w(S_i, S^*)$ ($i = 1, 2, \dots, m$), the better the alternative S_i. According to the measure values between the ideal alternative and alternatives, all alternatives can be ranked and the best alternative can be easily selected as well.

5. Decision-making example of the design schemes of punching machine

This section provides a decision-making example about the selection of the design schemes (alternatives) of punching machine to demonstrate the application and effectiveness of the proposed decision-making method.

In the conceptual design stage, the designers usually need to give a group of primal design schemes to select a better one corresponding to some suitability evaluation for all the primal design schemes. The punching machine generally consists of the reducing mechanism, punching mechanism and feed intermittent mechanism to structure its motion scheme. Therefore, according to its motion scheme, designers propose a set of four potential design schemes (alternatives) $S = \{S_1, S_2, S_3, S_4\}$ by their knowledge and experiences, which are shown in Table 1. The chief designer (decision-maker) must take a decision according to the five attributes (criteria): (1) R_1 is the manufacturing cost; (2) R_2 is the structure complexity; (3) R_3 is the transmission effectiveness; (4) R_4 is the reliability; (5) R_5 is the maintainability. The weight vector of the five attributes is $\boldsymbol{w} = (.25, .2, .25, .15, .15)^T$. The four possible alternatives of S_i ($i = 1, 2, 3, 4$) are to be evaluated by the chief designer under the five attributes according to fuzzy concept "excellence" (suitability evaluation), and then the evaluation values are represented by the form of SVNVs.

To indicate the evaluation of an alternative S_i ($i = 1, 2, 3, 4$) with respect to an attribute R_j ($j = 1, 2, 3, 4, 5$), it can be obtained from the questionnaire or score law of a domain expert. For example, when we ask the opinion of the chief designer about an alternative S_1 with respect to an attribute R_1, he/she may say that the possibility in which the statement is suitable is .75 and the statement is unsuitable is .4 and the degree in which he/she is not sure is .1. By the neutrosophic notation, it can be expressed as $e_{11} = (.75, .1, .4)$. Thus, when the four possible alternatives with respect to the above five attributes are evaluated by the chief designer, Thus, the single-valued neutrosophic decision matrix $D = (e_{ij})_{4\times5}$ can be obtained as follows:

$$D = \begin{bmatrix} (.75,.1,.4) & (.8,.1,.3) & (.85,.1,.2) & (.85,.1,.3) & (.9,.1,.2) \\ (.7,.1,.5) & (.75,.1,.1) & (.75,.2,.1) & (.8,.1,.1) & (.8,.2,.3) \\ (.8,.2,.3) & (.78,.1,.2) & (.8,.1,.2) & (.8,.2,.2) & (.75,.1,.3) \\ (.9,.1,.2) & (.85,.1,.1) & (.9,.1,.2) & (.85,.1,.3) & (.85,.2,.3) \end{bmatrix}.$$

Then, we utilise the developed approach to obtain the most desirable alternative(s).

Firstly, according to $e_j^* = (T_j^*, I_j^*, F_j^*) = (\max_i(T_{ij}), \min_i(I_{ij}), \min_i(F_{ij}))$ for $j = 1, 2, 3, 4, 5$ in the decision matrix $D = (e_{ij})_{4\times5}$, we determine the ideal alternative (ideal solution) as follows:

$$S^* = \{(0.9,\ 0.1,\ 0.2),\ (0.85,\ 0.1,\ 0.1), (0.9,\ 0.1,\ 0.1),\ (0.85,\ 0.1,\ 0.1),\ (0.9,\ 0.1,\ 0.2)\}.$$

Secondly, according to Equation (13) or Equation (14), the measure values between an alternative S_i ($i = 1, 2, 3, 4$) and the ideal alternative S^* are shown in Table 2.

For convenient comparison of the example, we introduce the vector similarity measures of SVNSs in (Ye, 2014c) for the decision-making example to show the effectiveness of the proposed projection measures.

Firstly, the proposed projection measures for the decision-making problem are replaced by the cosine measure of Equation (10):

$$\begin{aligned} \mathrm{Cos}_w(S_i, S^*) &= \frac{(S_i \cdot S^*)_w}{|S_i|_w |S^*|_w} \\ &= \frac{\sum_{j=1}^n w_j^2 \left(T_{ij}T_j^* + I_{ij}I_j^* + F_{ij}F_j^*\right)}{\sqrt{\sum_{j=1}^n w_j^2 \left((T_{ij})^2 + (I_{ij})^2 + (F_{ij})^2\right)} \sqrt{\sum_{j=1}^n w_j^2 \left((T_j^*)^2 + (I_j^*)^2 + (F_j^*)^2\right)}} \end{aligned}. \tag{15}$$

Table 1. Four alternatives of punching machine.

Alternative	S_1	S_2	S_3	S_4
Reducing mechanism	Gear reducer	Gear head motor	Gear reducer	Gear head motor
Punching mechanism	Crank-slider mechanism	Six bar punching mechanism	Six bar punching mechanism	Crank-slider mechanism
Dial feed intermittent mechanism	Sheave mechanism		Ratchet feed mechanism	

Table 2. Various measure values and ranking orders.

	S_1	S_2	S_3	S_4	Ranking order
$\mathrm{Cos}_w(S_i, S^*)$	.9785	.9685	.9870	.9942	$S_4 > S_3 > S_1 > S_2$
$C_w(S_i, S^*)$	.9798	.9750	.9875	.9929	$S_4 > S_3 > S_1 > S_2$
$D_w(S_i, S^*)$	.9787	.9696	.9845	.9927	$S_4 > S_3 > S_1 > S_2$
$J_w(S_i, S^*)$	.9586	.9427	.9694	.9857	$S_4 > S_3 > S_1 > S_2$
$\mathrm{Proj}w_{S^*}(S_i)$	.3933	.3632	.3806	.4158	$S_4 > S_1 > S_3 > S_2$
$\mathrm{BProj}_w(S_i, S^*)$	.9883	.9636	.9728	.9958	$S_4 > S_1 > S_3 > S_2$

Then, the proposed projection measures for the decision-making problem are replaced by another cosine measure, the Dice and Jaccard measures introduced by Ye (2014c):

$$C_w(S_i, S^*) = \sum_{j=1}^{n} w_j \frac{T_{ij}T_j^* + I_{ij}I_j^* + F_{ij}F_j^*}{\sqrt{(T_{ij})^2 + (I_{ij})^2 + (F_{ij})^2}\sqrt{(T_j^*)^2 + (I_j^*)^2 + (F_j^*)^2}}, \tag{16}$$

$$D_w(S_i, S^*) = \sum_{j=1}^{n} w_j \frac{2\left(T_{ij}T_j^* + I_{ij}I_j^* + F_{ij}F_j^*\right)}{(T_{ij})^2 + (I_{ij})^2 + (F_{ij})^2 + (T_j^*)^2 + (I_j^*)^2 + (F_j^*)^2}, \tag{17}$$

$$J_w(S_i, S^*) = \sum_{j=1}^{n} w_j \frac{T_{ij}T_j^* + I_{ij}I_j^* + F_{ij}F_j^*}{\left((T_{ij})^2 + (I_{ij})^2 + (F_{ij})^2 + (T_j^*)^2 + (I_j^*)^2 + (F_j^*)^2\right) - \left(T_{ij}T_j^* + I_{ij}I_j^* + F_{ij}F_j^*\right)}. \tag{18}$$

Using Equations (15)–(18), we calculate the vector measures between an alternative S_i (i = 1, 2, 3, 4) and the ideal alternative S^*, and then all results are also shown in Table 2 for comparative convenience.

In Table 2, obviously the ranking orders based on the projection and bidirectional projection measures are identical, the ranking orders based on the cosine, Dice, and Jaccard measures are identical, and then the ranking orders between the projection and bidirectional projection measures and the cosine, Dice, and Jaccard measures only indicate the difference between S_1 and S_3. However, for all these measures, S_4 is their optimal choice among all alternatives (mechanical design schemes). In fact, from intuitional viewpoint, the alternative S_4 should also satisfy practical requirements from the designers' experience. Therefore, the proposed projection methods are effective.

Generally, the cosine measures defined in vector space are also not always reasonable in some cases. For example, when $e_1 = (T_1, I_1, F_1)$ and $e_2 = (2T_1, 2I_1, 2F_1)$ ($e_1 \neq e_2$), the values of the cosine measures between e_1 and e_2 are equal to 1. Then, if $e_1 = e_2 = (T_1, I_1, F_1)$, the cosine measure values of e_1 and e_2 are also equal to 1. Clearly, the cosine measures of Equations (15) and (16) are not reasonable with respect to the decision-making or pattern recognition problems in this case.

However, the bidirectional projection method is superior to the general projection method and the cosine, Dice, Jaccard measures (Ye, 2014c). The reason is the bidirectional projection method for decision-making reveals the following main advantages:

(1) The bidirectional projection method is more reasonable than the general projection method because the former can overcome the shortcoming of the latter, then the bidirectional projection measure value is bounded within [0, 1], which is a normalised measure.
(2) The bidirectional projection method is more comprehensive than the general projection, cosine, Dice, Jaccard measures because the bidirectional projection can consider not only the distance and the included angle between objects evaluated but also the bidirectional projection magnitudes.
(3) The bidirectional projection-based decision-making method provides an effective way for the decision-making of mechanical design schemes under a single-valued neutrosophic environment.

6. Conclusion

This paper proposed projection and bidirectional projection measures between two SVNSs. Then, the analysis of a numerical example demonstrated that the bidirectional projection measure is superior to the general projection measure in measuring closeness degree between two vectors. Further, a decision-making method based on the weighted projection or bidirectional project measure was developed and applied to the decision-making problem of mechanical design schemes (alternatives) under a single-valued neutrosophic environment. Through the projection measure or bidirectional projection measure between the ideal alternative and each alternative, we can determine the ranking order of all alternatives and the best one. Finally, a decision-making example on choosing mechanical design schemes of punching machine was provided to demonstrate the applications and effectiveness of the developed approach. Similarly, these proposed projection measures can be also extended to INSs. In the future, we shall further apply the projection and bidirectional projection measures of SVNSs and INSs to group decision-making, medical diagnosis and fault diagnosis problems.

Disclosure statement

No potential conflict of interest was reported by the author.

References

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*, 87–96.

Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 31*, 343–349.

Bausys, R., Zavadskas, E. K., & Kaklauskas, A. (2015). Application of neutrosophic set to multicriteria decision making by COPRAS. *Journal of Economic Computation and Economic Cybernetics Studies and Research, 49*, 91–106.

Chi, P. P., & Liu, P. D. (2013). An extended TOPSIS method for multiple attribute decision making problems based on interval neutrosophic set. *Neutrosophic Sets and Systems, 1*, 63–70.

Deli, I. (in press). Interval-valued neutrosophic soft sets and its decision making. *International Journal of Machine Learning and Cybernetics*. doi:10.1007/s13042-015-0461-3

Deli, I., & Şubaş, Y. (in press). A ranking method of single valued neutrosophic numbers and its applications to multiattribute decision making problems. *International Journal of Machine Learning and Cybernetics*. doi:10.1007/s13042-016-0505-3

Deli, İ., Broumi, S., & Ali, M. (2014). Neutrosophic soft multi-set theory and its decision making. *Neutrosophic Sets and Systems, 5*, 65–76.

Liu, P. D., & Wang, Y. M. (2014). Multiple attribute decision making method based on single valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications, 25*, 2001–2010. doi:10.1007/s00521-014-1688-8

Liu, P. D., Chu, Y. C., Li, Y. W., & Chen, Y. B. (2014). Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making. *International Journal of Fuzzy Systems, 16*, 242–255.

Peng, J. J., Wang, J. Q., Zhang, H. Y., & Chen, X. H. (2014). An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. *Applied Soft Computing, 25*, 336–346.

Peng, J. J., Wang, J. Q., Wu, X. H., Wang, J., & Chen, X. H. (2015). Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. *International Journal of Computational Intelligence Systems, 8*, 345–363.

Peng, J. J., Wang, J. Q., Wang, J., Zhang, H. Y., & Chen, X. H. (2016). Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International Journal of Systems Science, 47*, 2342–2358.

Smarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic*. Rehoboth, MA: American Research Press.

Tian, Z. P., Zhang, H. Y., Wang, J., Wang, J. Q., & Chen, X. H. (2016). Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets. *International Journal of Systems Science, 47*, 3598–3608.

Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2005). *Interval neutrosophic sets and logic: Theory and applications in computing*. Phoenix, AZ: Hexis.

Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure, 4*, 410–413.

Xu, Z. (2005). On method for uncertain multiple attribute decision making problems with uncertain multiplicative preference information on alternatives. *Fuzzy Optimization and Decision Making, 4*, 131–139.

Xu, Z. S., & Cai, X. Q. (2012). Projection model-based approaches to intuitionistic fuzzy multiattribute decision making. In *Intuitionistic Fuzzy Information Aggregation* (pp. 249–258). Springer.

Xu, Z. S., & Da, Q. L. (2004). Projection method for uncertain multi-attribute decision making with preference information on alternatives. *International Journal of Information Technology & Decision Making, 3*, 429–434.

Xu, Z., & Hu, H. (2010). Projection models for intuitionistic fuzzy multiple attribute decision making. *International Journal of Information Technology & Decision Making, 9*, 267–280.

Ye, J. (2013). Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *International Journal of General Systems, 42*, 386–394.

Ye, J. (2014a). Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *Journal of Intelligent and Fuzzy Systems, 26*, 165–172.

Ye, J. (2014b). Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. *Journal of Intelligent and Fuzzy Systems, 27*, 2927–2935.
Ye, J. (2014c). Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. *International Journal of Fuzzy Systems, 16*, 204–211.
Ye, J. (2014d). Single valued neutrosophic cross-entropy for multicriteria decision making problems. *Applied Mathematical Modelling, 38*, 1170–1175.
Ye, J. (2015). Multiple attribute decision-making method based on the possibility degree ranking method and ordered weighted aggregation operators of interval neutrosophic numbers. *Journal of Intelligent & Fuzzy Systems, 28*, 1307–1317.
Ye, J. (in press). Single valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Computing*. doi:10.1007/s00500-015-1818-y
Yue, Z. L. (2012). Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling, 36*, 2900–2910.
Yue, Z. L. (2013). An intuitionistic fuzzy projection-based approach for partner selection. *Applied Mathematical Modelling, 37*, 9538–9551.
Yue, Z. L., & Jia, Y. Y. (2015). A group decision making model with hybrid intuitionistic fuzzy information. *Computers & Industrial Engineering, 87*, 202–212.
Zavadskas, E. K., Baušys, R., & Lazauskas, M. (2015). Sustainable assessment of alternative sites for the construction of a waste incineration plant by applying WASPAS method with single-valued neutrosophic set. *Sustainability, 7*, 15923–15936.
Zeng, S. Z., Balezentis, T., Chen, J., & Luo, G. F. (2013). A projection method for multiple attribute group decision making with intuitionistic fuzzy information. *Informatica, 24*, 485–503.
Zhang, H. Y., Ji, P., Wang, J. Q., & Chen, X. H. (2015). An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision-making problems. *International Journal of Computational Intelligence Systems, 8*, 1027–1043.
Zhang, H. Y., Wang, J. Q., & Chen, X. H. (2016). An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. *Neural Computing and Applications, 27*, 615–627.