ON THE VARIATION OF VACUUM PERMITTIVITY IN GENERAL RELATIVITY

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Abstract. Vacuum permittivity is the scalar in Maxwell's equations that determines the speed of light and the strength of electrical fields. In 1907 Albert Einstein found that vacuum permittivity changes with gravity. Møller, Landau & Lifshitz, and Sumner found that it changes with spacetime curvature.

The wavelengths of both photons and atomic emissions decrease with vacuum permittivity but the photons emitted from atoms decrease more than photons themselves do. This changes the interpretation of gravitational redshift to one where blueshifted photons are compared to greater blueshifted atomic emissions.

For Schwarzschild geometry, redshifts can be calculated most accurately using the metric. The equation derived using time dilation and the one derived here are identical. Their weak field approximation is the same as the redshift derivation using special relativity with Doppler shift.

For Friedmann geometry, redshifts calculated by comparing blueshifted atomic emissions to those of photons flips the meaning of Hubble redshift. The universe is accelerating in collapse, a result confirmed by supernova redshift observations. The equations and logic used for Friedmann and Schwarzschild redshifts are identical. Only their vacuum permittivities are different.

1. Introduction

In 1907 Einstein [1] [2, p 252] summarized the status of special relativity and its implications in the two years since its publication. At the end of this survey he concluded with a speculative section on "The Principle of Relativity and Gravitation." He considered a uniformly accelerated coordinate system and assumed that locally it is equivalent to a gravitational field. Einstein concluded that Maxwell's equations in the accelerated coordinate system (and hence in a gravitational field) are exactly the same as they are in the inertial coordinate systems of special relativity, except that "The principle of the constancy of the velocity of light does not hold ... the velocity of light in the gravitational field is a function of place ..." [3] [4, p 385].

In Maxwell's equations, vacuum permittivity, ε, is the scalar that determines the speed of light and the strength of electrical fields. Einstein's discovery means that both the wavelengths of photons and the wavelengths of photons emitted by atoms change with gravity in special relativity and with spacetime curvature in general relativity.

Two exact solutions to general relativity are examined, Schwarzschild's solution [5] for a spherical mass in an otherwise empty universe and Friedmann's closed solution [6] for a matter-filled, homogeneous universe. The Friedmann solution rapidly expands from a singularity, then slows until it reaches a maximum size before accelerating back to a singularity. Both are solutions to Einstein's [7] theory of general relativity without a cosmological constant.

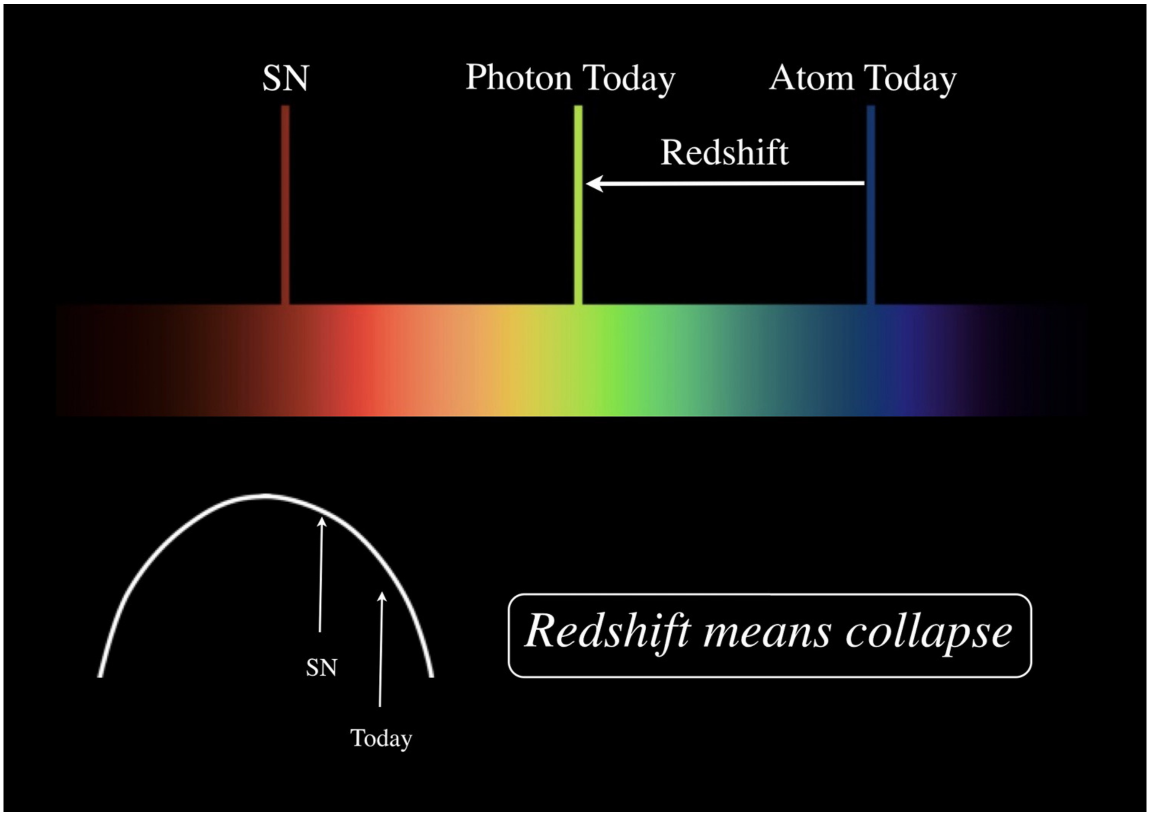

FIGURE 1. Photons blueshift in a collapsing universe. Atomic spectra blueshift about twice as much. Hubble redshift is observed.

In a Friedmann universe, vacuum permittivity is directly proportional to the Friedmann radius [8] and is therefore a function of time. As the size of the universe evolves, the changing strength of the electrical force between charges shifts atomic energy levels, changing the wavelengths of emitted light. This shift in photon emission due to the evolution of electrical attraction in the atom is in the same direction but about twice as large as the evolutionary shift in photon wavelength. The evolution of atoms and photons considered together reverses the interpretation of Hubble redshift to imply that the Friedmann universe is now collapsing. This mathematical result is confirmed by modern supernova redshift observations.

In Schwarzschild spacetime, ε is a function of the distance r from a central mass. At long distances, $\varepsilon(r)$ is close to its flat space value and increases as r gets smaller. Atomic sizes and the wavelengths of their emitted photons are functions of and change with $\varepsilon(r)$. Wavelengths of photons also change with $\varepsilon(r)$. The redshift equation obtained using time dilation and the one derived here are mathematically identical despite making different assumptions. Their weak field approximation is equivalent to one derived using special relativity commonly used for optical clock experiments.

2. Mathematical Models

2.1. **Einstein's Solution.** In his study of Maxwell's equations in an uniformly accelerated coordinate system, Einstein [1] [2, p 252] concluded that the velocity of light in special relativity, c, is reduced to c^*, the local coordinate velocity of light in the accelerated system. In an accelerated system corresponding locally to the gravitational field of a point mass, Einstein [2, p 310] found

$$c^* = c\left(1+\frac{\Phi}{c^2}\right), \tag{1}$$

where Φ is the Newtonian gravitational potential,

$$\Phi = -\frac{k\,m}{r}. \tag{2}$$

m is the mass of the object creating the gravitational field at a distance r. k is the gravitational constant.

The connection between Einstein's result, equation (1), and the strength of the electrical field follows from the relationship of relative vacuum permittivity ε to c^* and c,

$$\varepsilon = \frac{c}{c^*}. \tag{3}$$

Combining equations (1), (2), and (3) gives Einstein's value for ε,

$$\varepsilon\,(r) = \frac{1}{\left(1 - \dfrac{km}{rc^2}\right)}. \tag{4}$$

Einstein's reasoning about gravity evolved, leading to his theory of general relativity without a cosmological constant [7].

2.2. **Schwarzschild Solution.** The Schwarzschild solution to general relativity may be written [9, p 301]

$$ds^2 = \left(1-\frac{2km}{rc^2}\right)c^2dt^2 - \frac{dr^2}{\left(1-\dfrac{2km}{rc^2}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2). \tag{5}$$

Møller [10, p 308] and Landau & Lifshitz [9, p 258] studied the effects of curved spacetime on Maxwell's equations. Both proved that in a static gravitational field the electromagnetic field equations take the form of Maxwell's phenomenological equations in a medium at rest with

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}}. \tag{6}$$

$g_{\mu\nu}$ is the metric tensor. [1]

Einstein's pre-general relativity result, equation (4), is the first approximation to the exact equations (5) and (6),

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}} = \frac{1}{\sqrt{\left(1 - \frac{2km}{rc^2}\right)}} \approx \frac{1}{\left(1 - \frac{km}{rc^2}\right)}. \tag{7}$$

2.3. **Friedmann Solution.** Friedmann [6] found a closed universe solution to Einstein's theory of general relativity without a cosmological constant. The Friedmann universe rapidly expands from a singularity, slowing until it reaches a maximum size before accelerating back to a singularity.

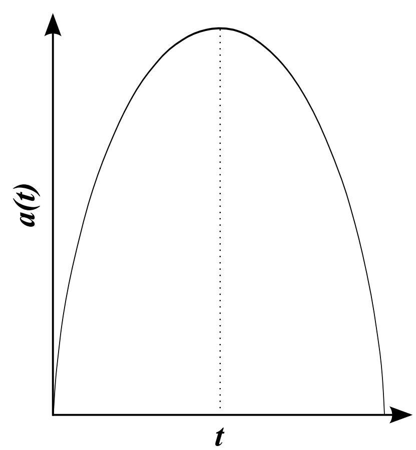

FIGURE 2. Friedmann's solution for a closed universe.

Friedmann assumed the metric,

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{(1-r^2)} + r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right)\right], \tag{8}$$

and assumed homogeneous, incoherent matter, conserved in amount and exerting negligible pressure. His solution is the cycloid shown in Figure 2,

$$a = \frac{\alpha}{2}(1 - \cos\psi) \qquad ct = \frac{\alpha}{2}(\psi - \sin\psi), \tag{9}$$

where α is a constant and $0 \le \psi \le 2\pi$ [11].

[1] This result is valid for every static metric, not just the Schwarzschild metric.

Sumner [8] studied Maxwell's equations in this closed Friedmann universe without a cosmological constant and showed that ε changes with spacetime curvature,

$$\varepsilon\,(t) \;=\; a(t). \tag{10}$$

$a(t)$ is radius of the Friedmann universe defined above.

2.4. **Changes in Atoms and Photons.** Photon wavelengths, atomic sizes, and the wavelengths of photons they emit change with ε. In the following equations κ (the Greek letter kappa) is to be replaced by either the radial coordinate r for Schwarzschild geometry or the time coordinate t for Friedmann geometry. The logic and the math are exactly the same for each geometry.

The Bohr radius a_o of a hydrogen atom in its ground state at κ is [2]

$$a_o(\kappa) \;=\; \frac{4\pi\varepsilon_o\,\varepsilon\,(\kappa)\,\hbar^2}{me^2}. \tag{11}$$

ε_o has the defined value $\varepsilon_o \;=\; 8.854187817\ldots x10^{-12}$ F/m (farads per meter). m is the mass of the electron, e is the charge of the electron, and $\hbar$ is Planck's constant h divided by 2π. These are assumed to remain constant as spacetime curvature changes.

The change in Bohr radius a_o as κ changes is

$$\frac{a_o(\kappa_1)}{a_o(\kappa_2)} = \frac{\varepsilon\,(\kappa_1)}{\varepsilon\,(\kappa_2)}. \tag{12}$$

The characteristic wavelength λ_e emitted by a hydrogen atom during the transition between the principle quantum numbers n_2 and n_1 is

$$\lambda_e(\kappa) \;=\; \frac{8\,\varepsilon_o^2\,\varepsilon^2\,(\kappa)\,h^3\,c}{me^4}\left(\frac{n_1^2\,n_2^2}{n_2^2 - n_1^2}\right). \tag{13}$$

c in equation (13) comes from the defining relationship between λ and ν, $\lambda\,\nu \;=\; c$.

The change in $\lambda_e(\kappa)$ as κ changes is

$$\frac{\lambda_e(\kappa_1)}{\lambda_e(\kappa_2)} = \frac{\varepsilon^2\,(\kappa_1)}{\varepsilon^2\,(\kappa_2)}. \tag{14}$$

Consider the Compton wavelength, λ_c, of a particle with mass m_p,

$$\lambda_c(\kappa) = \frac{h}{m_p\,c^*(\kappa)} = \frac{h\,\varepsilon\,(\kappa)}{m_p\,c}. \tag{15}$$

The change in $\lambda_c(\kappa)$ as κ changes is

$$\frac{\lambda_c(\kappa_1)}{\lambda_c(\kappa_2)} = \frac{\varepsilon\,(\kappa_1)}{\varepsilon\,(\kappa_2)}. \tag{16}$$

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[2] See standard texts, e.g. Leighton [12].

The Compton wavelength of a particle is equivalent to the wavelength of a photon of the same energy as the particle. Compton and photon wavelengths have the same $\varepsilon(\kappa)$ dependency that the Bohr radius has. The wavelength change for a photon is

$$\frac{\lambda(\kappa_1)}{\lambda(\kappa_2)} = \frac{\varepsilon\,(\kappa_1)}{\varepsilon\,(\kappa_2)}. \tag{17}$$

2.5. **Gravitational Redshift.** The following notation is used. The wavelength of a photon λ emitted at κ_1 and examined at the time of emission at κ_1 will be written $\lambda(\kappa_1, \kappa_1)$. The wavelength of a photon λ emitted at κ_1 and examined at κ_2 will be written $\lambda(\kappa_1, \kappa_2)$.

The traditional redshift z formula assumes that photons evolve, equation (17), but that atomic emissions do not evolve,

$$z = \frac{\lambda(\kappa_1, \kappa_2) - \lambda(\kappa_1, \kappa_1)}{\lambda(\kappa_1, \kappa_1)} = \frac{\varepsilon(\kappa_2)}{\varepsilon(\kappa_1)} - 1. \tag{18}$$

κ_2 is the observer's location and κ_1 is the location at the time of emission.

Since atomic emissions do evolve with spacetime geometry, equation (14), a new redshift variable ζ (the Greek letter zeta) is defined[3]

$$\zeta = \frac{\lambda(\kappa_1, \kappa_2) - \lambda(\kappa_2, \kappa_2)}{\lambda(\kappa_2, \kappa_2)}. \tag{19}$$

$\lambda(\kappa_1, \kappa_2)$ is the wavelength observed today from a distant source and $\lambda(\kappa_2, \kappa_2)$ is the wavelength emitted today by a local reference atom.

$$\zeta = \frac{\varepsilon(\kappa_1)}{\varepsilon(\kappa_2)} - 1. \tag{20}$$

2.6. **Schwarzschild Redshift.** The Schwarzschild metric describes the spacetime geometry around spherical masses.

Substituting $\varepsilon(r)$ from equation (7) in equation (20) gives[4]

$$\zeta = \frac{\sqrt{\left(1 - \dfrac{2km}{r_2 c^2}\right)}}{\sqrt{\left(1 - \dfrac{2km}{r_1 c^2}\right)}} - 1. \tag{21}$$

For sources and receivers separated by small distances, a weak field approximation to the exact equation (21) gives

$$\frac{\Delta\nu}{\nu} \approx \frac{km}{r^2 c^2} h, \tag{22}$$

[3]This equation also describes blueshifts and frequency shifts as well as redshifts.

[4]This agrees with traditional derivations using time dilation with the Schwarzschild metric. See standard texts, e.g. Weinberg [13, p 80] or Wikipedia [14].

where k is the gravitational constant, m is the mass of the earth, r is the distance from the center of the earth to the source, c the speed of light, and h is the vertical separation between source and receiver.[5]

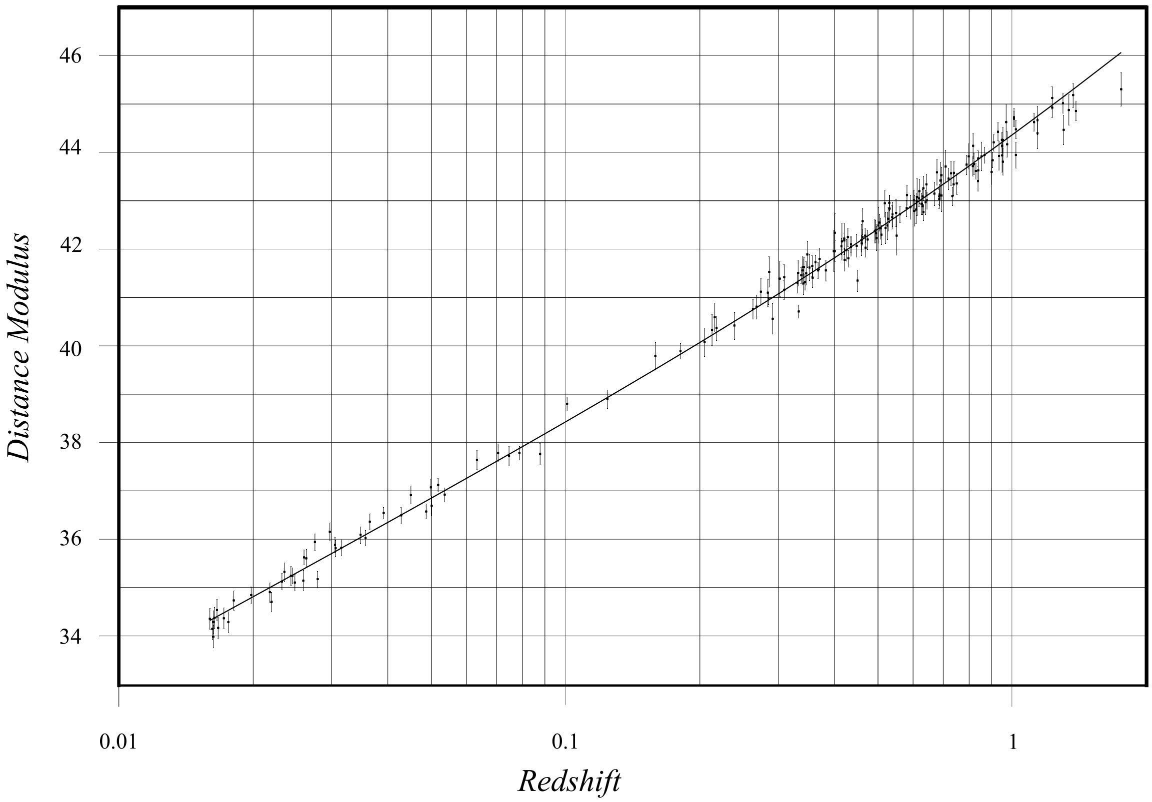

FIGURE 3. Supernovae redshift data is from Davis et al. [19] which combines data from Wood-Vasey et al. [20] and Riess et al. [21]. The solid line fit uses the Friedman solution, with only two free parameters, $H_o = -66.5\,km\,s^{-1}Mpc^{-1}$ and $q_o = \frac{1}{2} + (0.001)$. The Hubble constant H_o is negative because the universe is collapsing. The deceleration parameter q_o is close to the value for a precisely flat universe, $q_o = \frac{1}{2}$. The average data error is 0.231 and for these fit parameters the standard deviation is 0.234.

[5]Will [15, p 15] has a good explanation how this approximate equation is derived using special relativity and Doppler shifts. Two experiments using this equation are those by Pound and Rebka [16, p 439] [17], who measured the redshift of gamma rays, and Chou et al. [18], who compared the frequencies of two optical clocks at different elevations.

2.7. Friedmann Redshift. The redshift mathematics are identical for Schwarzschild and Friedmann geometries, only their vacuum permittivities $\varepsilon(\kappa)$ are different. The result that observed Hubble redshift implies that the Friedmann Universe is now collapsing comes directly from equation (20). The Hubble redshift observed means $\zeta > 1$. This implies that $\varepsilon(\kappa_1) > \varepsilon(\kappa_2)$ or $a(t_1) > a(t_2)$ from equation (10). The universe was larger in our past, t_1, putting us somewhere on the collapsing half of the curves in Figures 1 and 2.

A fit to modern data is shown in Figure 3. See Sumner [22] for details.

3. Physical measurements

The mathematics of general relativity isn't a physical theory until mathematical concepts such as $g_{\mu\nu}$ and x^μ are linked by axioms to specific laboratory methods that measure things like distances and times. Albert Einstein took this step, just as he did for special relativity, by asserting that measurements made with rigid meter sticks and balance clocks are equivalent to the mathematical distances and times of general relativity. Assuming a rigid meter stick is equivalent to assuming that atoms never change. Even as he did this Einstein had qualms about this choice.

In his 1921 Nobel Lecture Einstein said:

> ... it would be logically more correct to begin with the whole of the laws and ... to put the unambiguous relation to the world of experience last instead of already fulfilling it in an imperfect form for an artificially isolated part, namely the space-time metric. We are not, however, sufficiently advanced in our knowledge of Nature's elementary laws to adopt this more perfect method without going out of our depth. [23, p 483]

It is intriguing that it was Einstein who discovered vacuum permittivity depends on gravity. In 1907, there was no general relativity, no Bohr atom, and no real understanding of photons. When these theories were later in place, the connection provided by vacuum permittivity between spacetime curvature and atomic structure was overlooked. Einstein knew that the "tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations." [24, p 685] What he did not know was that the solution was already in his 1907 paper and that there was no need of "going out of our depth" to create the more complete general relativity he wanted, where the "tools for measurement" depend on spacetime exactly as "other objects implicated by the field-equations."

Feynman [25, p 55] was correct when he noted that "Physics is not mathematics, and mathematics is not physics ... mathematicians prepare abstract reasoning that's ready to be used if you will only have a set of axioms about the real world ..."

Assuming meter sticks are rigid and atoms never change should not be one of those axioms about the real world.

4. Conclusions

Einstein assumed that gravitational fields are equivalent to uniformly accelerating coordinate systems. Using Maxwell's equations, Einstein showed that the coordinate speed of light and vacuum permittivity change with the strength of the gravitational field. They change with spacetime curvature in general relativity.

The strength of electrical fields changes with location, changing atoms and photons. The wavelengths of both photons and atomic emissions change with vacuum permittivity but the photons emitted from atoms change about twice as much as photons do.

For Schwarzschild geometry, redshifts are calculated more accurately using general relativity. Redshift derived here is the same as the traditional result using time dilation and the metric. A weak field approximation to these exact results agrees with the one derived using special relativity and Doppler shift.

For Friedmann geometry, a comparison of yesterday's photons emitted by yesterday's atoms against today's photons emitted by today's atoms shows that Hubble redshift means the universe is collapsing. This is confirmed by supernova Hubble redshift observations.

Acknowledgments

The author thanks Dawn Sumner and Tom Hornbein for their continuing encouragement, surgical insights, and helpful suggestions.

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