Will we ever have a purely mathematical proof of the existence of the universe?

Suppose for a moment that physicists discovered the formulation of the ultimate theory of everything (ToE) which explains all physical phenomenas. This is a remarkable achievement no doubt. But, what is to stop someone from asking; “Why this theory and not another?” In other words, can it prove its own uniqueness? Surely this question will be asked. Some, like myself, would consider this question to be one of the most philosophically important questions of the theory. So, what requirements must an eventual ToE have to be able to provide an answer to this very question?
In the early 2000s, I developed the intuition that the ultimate theory of everything in physics should be able to answer this question, and to be able to do so it should be formulated independently of any specific assumptions (or axioms). This, I believed, would be the only way to explain why specifically it exist and not something else.

In other words, there should be at least one thing which exists without assumptions and it should be the universe.

For many years, none of this was formal. But in 2014, I had an idea regarding time and axioms. This was the first discovery I made that allowed me to formalize part of my intuition. Informally speaking, I had the idea that the number of axioms in the universe should grow with time, and that the familiar laws of physics should be interpreted as the “glue” between these axioms. In this interpretation, the general laws of physics would be the same for all possible universes, but the instantaneous state of the universe would be encoded by a long list of axioms. Unknown to me at the time, this idea would provide me with an entirely new interpretation of quantum mechanics and time. It would also connect the notions of axioms to that of entropy and eventually to the familiar laws of physics.

Picture this. Say we formulate the ToE with a long list of axioms, quite possibly infinitely long. This would be somewhat equivalent to encoding the state of the universe with a series of yes or no questions. “Is there milk in the fridge?-yes.” “Did the chicken cross the road?-no.” etc. The specific questions are not important but the fact that we can encode the universe as such, is. The questions and their answers can be abstract. For example, we can label the questions as Question1, Question2, Question3, etc without ever defining them explicitly. And the answers as Answer1, Answer2, Answer3, etc. The question paired with its verifiable answer would be an axiom of the universe. If any question is without answer, then it is simply left out of the set of axioms. After-all, not all questions need to have an answer.

Notation alert: I will use the shorter notation QA to refer to a Question and Answer pair. For example, Q1A1 refers to Question1 paired with Answer1.

I noticed that this idea could be connected to the arrow of time when I realized that any group of n axioms contained the answers to any
questions of its subgroups but cannot by itself determine additional answers found in its supergroups. For example, the group of Q1, Q2, Q3 and Q4 and their answers contains at least the information that is contained in the subgroup of Q1, Q2 and Q3. But, as the QA pairs are logically independent axioms, the smaller groups cannot determine the answer to Q5. All that was left to do was to define each instant of time as a specific group of QA pairs of a certain size. As time moves forward, the size of the group would increase to accommodate more information. The result is that the information of the past would be encoded in the group associated with the present, but the future would be out of reach until it occurs—yielding the arrow of time. At that point I was hooked to the idea.

In the second half of 2017, I was done with most of my work. By then, I was able to show that encoding the universe as a series of such QA pairs is enough to exactly recover the familiar laws of physics. Here, I present what I believe to be the most fundamental equation governing the whole of the universe:

\[ Z(x, t) = \sum_{p}^{\infty} 2^{-\beta D|p| - \beta Pt} \]

Physicists will no doubt recognize it as a Gibbs ensemble of statistical physics. It connects an infinite set of recursively enumerable axioms to a physical entropy.

In a paper, I have shown explicitly that Space, Time, Inertial mass, Special relativity, General relativity, Dark energy, the Schrödinger equation, the Dirac equation and the holographic principle are solutions of this equation. The equations are derived from pure reason with no appeal to physical observations or experimentations. You can read the paper here for free but be warned that a formal training both in physics and in computer science is likely required to properly understand the paper. In here, my goal is to explain the main results using a minimal amount of technical jargon.
First, allow me to do a brief summary of the results.

An important result of the equation is indeed, as I originally intuited, that the familiar laws of physics are simply the glue that binds the axioms of the universe together. So what then are the axioms? Before we answer that, let us see what their properties are. The axioms are numerous, their quantity grows with time and, as they are logically independent from one another, they cannot be known until they occur. Furthermore, they encode the state of the universe independently from its laws.

The equation connects the result of a quantum measurement to an axiom defining the universe. The presence or the absence of an axiom in the group cannot logically be determined because all axioms are logically independent. To find out if an axiom will be part of a future group, we must wait until time runs its course and see if the axiom has entered the larger group or not yet. Likewise, the value of the quantum measurement is random for the same reason—it connects to such an axiom. To exactly define the universe, one must catalogue all values of all quantum measurements that have occurred since its inception. This is the state of the universe. As the quantum measurements are random, this information cannot be compressed.

Indeed, in this scenario, the formulation of the laws of physics are independent of the axioms themselves but the state of the universe at an instant in time isn't. This dynamic is fully recovered in the fundamental equation I have derived.

The conclusion is that the universe exist in precisely the way it does because, if there are to be any verifiable answers to any questions, then the glue between them must in fact be the familiar laws of physics. For completeness of the thought, I have also shown that the reverse cannot
occur. Indeed, if the universe was composed of different laws it would contain gaps in the type of QA that it could verify. Meaning the “glue” would not stick to all questions or all answers and the universe would be incomplete.

The derivation of the laws of physics from this equation is certainly of some significance and warrant its own discussion. This is done in more detail in my paper. However, it is not the point I want to focus on in this article. What I believe to be the most important result is that the derivation of the equation contains a purely mathematical proof of the existence of the universe. I will explain what the proof is and why this equation governs the universe and not another.

**I think, therefore I am—an irrefutable statement?**

To truly understand the proof, we have to recall the philosophy of René Descartes (1596–1650), the famous 17th century French philosopher most directly responsible for the mind-body dualism ever so present in western culture. As I will show, the fundamental equation is naturally obtained when I modernize his main result into a formal logic system. But first, let us recall the derivation of *the cogito*.

![Image of René Descartes](image.png)

His main idea was to come up with a test that every statement must pass before it will be accepted as true and use this test to identify that which is irrefutable. But this will not be just any test mind you, it will be the strictest test imaginable. Any reason to doubt a statement will be a sufficient reason to reject it.
Using this test, and for a few years, Descartes rejected every statement he considered. The laws and customs of society, as they have no logical justifications, are obviously the first to be rejected as necessarily true statements. Then, he rejects any information that he collects with his senses; vision, taste, hearing, etc, on the grounds that a “demon” (think LSD) could trick his senses without him knowing. He also rejects the theorems of mathematics on the grounds that axioms are required to derive them, and such axioms could be wrong. For a while, his efforts were fruitless and he doubted if he would ever find an irrefutable statement.

But, eureka! He finally found one which he published in 1641. He doubts of things! The logic goes that if he doubts of everything, then it must be true that he doubts. Furthermore, to doubt he must think and to think, he must exist (at least as a thinking being). Hence, cogito ergo sum, or I think, therefore I am.

This quite magnificent argument is, almost by itself, responsible for the mind-body dualism of western culture.

**Why this equation, and not something else?**

The derivation of the cogito is not without a few technical subtleties. Nothing too major that would make it incorrect of course, but nonetheless are noteworthy. For example, Descartes must assume that his capacity to correctly reason is not impaired. Meaning, he must be of sound mind. Rejecting this assumption, he cannot derive anything with certainty including the cogito. This is better explained by the Church-Turing thesis in that Descartes must be able to correctly follow the steps of an argument in his head and not make mistakes in the application of the logic. This subtle point will prove to be quite important in deriving the fundamental equation as it significantly increases the number of statements that he cannot reject by his methodology. One might forgive Descartes for not seeing it as the idea of a Turing machine was first published much later than his time—in 1936.

I have found that modernizing the derivation of the cogito into a formal language such as first order logic is sufficient to derive the fundamental equation.
\[ Z(x, t) = \sum_{p}^{\infty} 2^{-\beta|p| - \beta Pt} \]

The fundamental equation repeated here

This equation acts as the irrefutable statement which replaces the informal cogito in the formalized derivation. The equation explains the physical process by which \textit{I think} implies \textit{therefore I am} by acting as the fundamental equation describing the physics of a universe permitting Descartes to think.

René Descartes initially derived \textit{the cogito} by showing that a contradiction occurs when it is assumed to be false. Indeed, one cannot both \textit{think} and \textit{not exist} at the same time. Hence, to think one must exist. This is a non-constructive proof.

It is quite common in mathematics to obtain a non-constructive proof first, then later a constructive proof is found. The constructive proof always contains more insight than the non-constructive one. This is because it produces an object having the required properties and such object can be studied in more detail. The drawback of the constructive proof is that it is often significantly harder to obtain.

I believe the equation I have obtained is best understood when interpreted as a constructive proof of the cogito.

Simply put, the cogito is the non-constructive proof of one’s own existence \textit{-I exist because I think-}, and the fundamental equation is the constructive proof \textit{-I exists because I think, and for me to think, then my existence must be bounded by this equation which I can derive purely from the application of my mind-}.

**Modernizing the derivation of the cogito**

For Descartes to derive the cogito and be confident in his derivation he must assume that he cannot doubt of his own ability to think clearly. Meaning, he must be of sound mind. Otherwise, everything he deduces is in doubt even the cogito. Formally and in a more modern language, we would say that Descartes is able to think with the accuracy of a computer algorithm, of if you will, that Descartes, on a good day, is at
least as good as a Turing machine. As a result, Descartes can verify a logical argument and such verification will be correct.

We are now ready to repeat the derivation of the cogito in formal logic. To do so, we will be taking the constructivist project of mathematics to the extreme. In my paper, I have given it the name of minimalist logic if only to illustrate the purpose. The method removes from first order logic any formal axioms it has. Then it further removes any rules of inference with the exception of the rule by deduction. The result is a system of logic which, essentially, does not lie to its user. Since, as a precaution, we have taken every tool out, we are assured that any remaining theorems must be entirely irrefutable. This method parallels Descartes’ universal doubt method but within a first order logic.

In minimalist logic, no theorem stands on its own. Any theorem must include, within its description, the list of assumptions that are required to prove it. The user of minimalist logic is always reminded that the theorems that he proves are of the form “Assume A, then A proves C”. Hence, by the rule of deduction, A → C is a theorem, but C by itself never is. In real life, no theorem can be proven without first posing an assumption, a fact that minimalist logic never forgets. As the ancient Greeks might have said:

To get to something one must first start with something, a first principle.

This is what I mean when I say minimalist logic does not lie to its user. All other logic systems allows theorems to be proven without first forcing a listing of the assumptions required by the proof. Therefore, the user of a non-minimalist logic system might be fooled into thinking he proven something which is actually true.

The theorems of minimalist logic form the group of all statements that cannot be rejected by Descartes universal doubt method. There are infinitely many such statements. Indeed, to reject such a statement, Descartes would have to admit that his reason is flawed, hence the derivation of the cogito, which requires him to have a sound mind, would no longer hold. This is a unique property of the minimalist logic system. If we had used any other system (e.i. a system that has axioms), Descartes could have rejected all the theorems on the ground the axioms could be doubted.
We apply the same reasoning for all possible statement similar to A and C and we group them. The group of such statements can be represented as the following logical formula:

\[ \forall k \forall t [ k \vdash t \implies T \vdash (k \vdash t)] \]

This grouping, represented by T, acts as a micro-state of the Gibbs ensemble.

Formally, this is what Descartes means when he says he can think. This logical formula means that if assuming k allows Descartes to prove t, then it must be the case that the theory T which explains Descartes’ existence must be able to prove that k proves t. This is because Descartes proved it in his head, and since Descartes’ head is in the universe, then whatever he proved in it must also be provable in the universe itself.

In other words, the logical formula can be understood as the group of all statements whose intellectual provability is isomorphic (or identical) to their physical provability. As we derive the fundamental equation, we will see why this group, its properties and the “glue” that ties its elements together are and must be identical in every aspect to the laws that govern the universe. Failure to have that, Descartes could not think hence he could not exist.

**Deriving the fundamental equation**

First, we consider that each statement of the form \( k \vdash t \) is a sentence of a language. For simplicity we can pose the language to be the decimal numbers. In this case, we can associate each statement \( k \vdash t \) to a natural. There are infinitely many \( k \vdash t \) statements and infinitely many naturals. Both sets have the same cardinality hence they can be placed in a one to one correspondence as shown in this table:
We can represent this association as a real number by constructing it as the following sum:

$$\Omega = \sum_{n=1}^{\infty} 2^{-E(n)-n} \quad \text{where, } E(n) = \begin{cases} 0 & [k \vdash t]_n \\ \infty & \text{otherwise} \end{cases}$$

Here, if \( k \) does prove \( t \), then \( E(n)=0 \), otherwise it is infinite. This sum is the definition of the halting probability of a universal Turing machine (UTM) with a unary code. The definition of \( \Omega \) for a general prefix-free code was first given by Gregory Chaitin in 1975.

The group of all statements that Descartes’ universal doubt method cannot refute is now encoded as an infinitely long and complex real number whose value is the sum. As an example, let us unpack the sum for some example values of \( E(n) \).

$$= 2^{-\infty} 2^{-1} + 2^{-0} 2^{-2} + 2^{-0} 2^{-3} + 2^{-0} 2^{-4} + 2^{-\infty} 2^{-5} + ... = 0.01110..._b$$

Here the subscript \( _b \) denotes a binary number.

In this example, the sum is the real number 0.01110...\( _b \). Each bit of this number tells us whether a sentence is an axiom or not. For example, if its second bit is a 1, then the sentence \( k \vdash t \) associated with the number 2 is an axiom. Otherwise, it is not.

This construction, called \( \Omega \), is known to be non-computable, algorithmically random and is a normal number. Each digit of \( \Omega \) can be interpreted as a logically independent axiom.
The next step is to connect this number to a physical entropy. This can be done by injecting a function $F$ and a factor $\beta$, which can be interpreted as a decompression term, applicable to the exponential term $n$ of the sum. This converts the sum to a Gibbs ensemble of statistical physics. We obtain this equation:

$$Z_\Omega = \sum_{n=1}^{\infty} 2^{-\beta[E(n)+F\cdot n]}$$

When $\beta F$ is equal to 1, we recover the definition of $\Omega$.

The entropy of the axiomatic system, as it is described by a Gibbs ensemble, is now “physically-compatible” with the universe.

Finally, we want both time and space to emerge from this entropy. How will we do that? Well, physics already provide us with a framework that allows various properties, called thermodynamic observables, to emerge from information. “Coincidentally”, this framework happens to be the Gibbs ensemble of statistical physics—and that’s precisely what we have. We define space and time each as an observable that emerges from the entropy of the system. This can be done by introducing the conjugate pairs $Pt$ and $Fx$ into the sum. We will respectively call them, entropic time and entropic space. Here $P$ is a Power in Watt, $t$ is a time in seconds, $F$ is a Force in Newton and $x$ is a distance in meters. For technical reasons, $E(n)$ can be removed from the sum as it is recovered by $Pt$ when $t\to\infty$. We also rename $n$ to $x$.

Within a first order approximation, we obtain the final form of the fundamental equation, which is:

$$Z_\Omega = \sum_{x=1}^{\infty} 2^{-\beta[F\cdot x - P\cdot t]}$$

The Gibbs ensemble of space and time. In the end, we obtain something quite simple!

This construction connects a set of recursively enumerable axioms to a physical entropy. It connects the notion of “thinking” in the sense of a universal Turing machine to the notion of “existing” in the sense of being bound to a set of physical laws which limits the performance and scope of the “thinking”.
This equation can also be interpreted as a prefix-free universal Turing machine which maximizes the entropy during a convergent calculation of its halting probability, $\Omega$.

It can also be interpreted as the Gibbs ensemble of space and time. Its non-computable micro-states are directly responsible for converting classical special relativity to the Dirac equation.

Adding a time conjugate to a Gibbs ensemble such as the time $t$ adds a whole new dynamic to a thermodynamic system. The system now becomes aware of future, past and present entropy and can translate from time to space and from space to time for an entropic cost and provided that various limits are respected. By studying thermodynamic cycles involving space and time, I was able to look at what happens to the entropy when a system is translated forward or backward in time and draw conclusions in regards to the arrow of time. In the model presented, space serves as an entropy sink that encourages a forward arrow of time, the future is non-computable and the past is singular. The number of axioms, represented in the equation by the entropy, grows with time. Space encodes the questions and the quantum measurements are the answers.

**An example—special relativity**

The equation along with specific examples and solutions is explained in more detail in my paper. Here, as the simplest case, I will show how special relativity comes out of the equation.

Using the rules of statistical physics, we can extract the following state equation from the Gibbs ensemble.

$$T \, dS = F \, dx - P \, dt$$

By posing $dS=0$, we obtain the fundamental relation of special relativity $dx=P/F \, dt$ relating space to time. The relation is enough to recover special relativity. Here, $P/F$ is a constant having the units of meters per second and is therefore a speed. When the Power is the Planck power, and the Force $F$ is the Planck force, then the ratio $P/F$ is the speed of light, and $dx=cdt$. 
Therefore, the equation, obtained purely from the application of the mind, predicts that the universe admits a maximal speed.

General relativity, dark energy and the holographic principles can be recovered in similar ways. The equations are converted to their quantum mechanical counterparts by taking into account the non-computable effects of the micro-states of the ensemble. Those effects are enough to exactly recover the Schrödinger equation as a special case and the Dirac equation in the general case. The constant switching across the available micro-states, a characteristic feature of any systems of statistical physics, produces a random walk effect on the dx and dt variable. It is a well known result of the literature (now know for more than 60 years) that such random walk effects on the coordinates produces the Schrödinger equation when such effect occurs on dx, and the Dirac equation when it occurs on both dx and dt.

**Could we have missed anything?**

The fundamental equation connects to a physical entropy any set of recursively enumerable axioms of sufficient complexity to embed a universal Turing machine. This makes sense as the universe must at least be as complex as its most complex element—arguably the human mind. Therefore, the axioms of the universe must contain the whole of that which can be verified by *it*, namely by *reason*. As we took all of reason to build this equation, there is nothing left for us to use that could increase its scope.